

A decorative wooden sign featuring a carved figure of a person in traditional attire on the left and a dark rectangular plaque on the right with the word "WELCOME" in raised, golden letters. The sign is set against a dark background.

WELCOME

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**COMPACT SUBSETS OF THE METRIC SPACE ARE  
CLOSED**

# PROOF

Let  $x$  be a metric space.

Let  $k$  be a compact subset of the metric space  $x$ .

To prove  $k$  is closed.

(i. e) It is enough to prove  $k^c$  is open.



Let  $p \in X$  and  $p \in K$  and  $q \in K \dots \dots (1)$

Let  $V_p$  and  $W_q$  are the neighbourhoods of  $p$  and  $q$  whose radius are less than

$$\frac{1}{2} d(p, q).$$

Since  $K$  is compact, there are finitely many points  $q_1, q_2, \dots, q_n \in K$  such that  $K \subset W_{q_1} \cup W_{q_2} \cup \dots \cup W_{q_n}$ .

$$\text{let } W = W_{q_1} \cup W_{q_2} \cup \dots \cup W_{q_n}$$

Then  $K \subset W$ .

Let  $V = V_{q1} \cap V_{q2} \cap \dots \cap V_{qn}$

Where  $v$  is the neighbourhood of  $p$  and  
 $v$  does not intersect  $k$ .

$$(i. e) V \cap K = \Phi$$

$$(i. e) V \cap k^c$$

From equation (1),  
 $p \in k^c$ , and the neighbourhood of  $p$ ,  $V \subset k^c$ .  
Therefore  $p$  is an interior point of  $k^c$ .  
Since  $p$  is arbitrary,  $k^c$  is open.

By the theorem

“ $E$  is open iff  $E^c$  is closed”,

Therefore  $K$  is closed.

Therefore Compact subset of the metric space are closed.

