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AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH TIME DEPENDENT DEMAND IN BOTH FUZZY AND CRISP SENSE

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Abstract. An inventory model is developed in which the items are considered as deteriorating items and independent of time with time dependent demand. This research work considers the demand rate of an item as exponential distribution. We develop the model by subdividing the time period (o,T) in to (o,t_1) and (t_1 ,T). In the former time period the constant production rate is considered. After that the production stops. The aim of this investigation is to determine the optimum total cost so as to yield profit for manufacturers. Here the model is explained both in crisp and fuzzy sense. To develop the model in fuzzy sense , we fuzzify some of the parameters using different fuzzy numbers . Defuzzification is done using graded mean method and signed distance method for fuzzy numbers. Numerical example is given , to compare the total cost between crisp and various fuzzy numbers. Finally sensitivity analysis has been worked out.

Keywords : Fuzzy Production, Time Dependent Demand, Deterioration, Defuzzifiction.

Introduction: In real life situation uncertainty arises in decision making problem due to either lack of knowledge or inherent vagueness. An inventory consists of raw materials, work in progress or finished goods. Effective inventory control is essential for manufacturing organisations for many reasons. The objective of many inventory problems is to deal with minimisation of carrying cost . Thus it is essential to determine a suitable inventory model to meet the future demand. The most widely used inventory model is the Economic Order Quantity(EOQ) model, in which the successive operations are classified as supply and demand. The first quantitative treatment of inventory was the simple EOQ model. This model was developed by Harris et al.[1]. Wilson [2] showed interest in developing EOQ model in academics and industries. Hadely et al. [3] analysed many inventory models.

Uncertainties and imprecision is inherent in real inventory problems. This can be approached by probabilistic methods. But there are uncertainties that cannot be appropriately treated by usual probabilistic models. To define inventory optimization tasks in such environment and to interpret optimal solution, fuzzy set theory is considered as more convenient than probability theory.

In certain situation uncertainties are due to fuzziness, introduced by Zadeh [4] is applied. He proposed some strategies for decision making in fuzzy environment. Jain [5] worked on decision making in the presence of fuzzy variables. Kacpryzk et al. [6] discussed some long term inventory policy making through fuzzy decision making models. The application of fuzzy set theory can be found in Zimmerman [7] Dutta and Kumar [8] developed fuzzy inventory model without shortages using fuzzy trapezoidal number and used Signed distance method for defuzzification. Jaggi et al. [9] developed fuzzy inventory model with deterioration where demand was taken as time-varying. Kumar and Rajput [10] developed a fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. Saha and chakra borty [11] developed inventory model with time dependent demand and deterioration with shortages. Trailokyanath Singh and HadibandhuPattanayak [12] developed an EOQ model for a deteriorating item with time dedpendentexponentially declining demand under permissible delay in payments.NanbenduSen, BimanKantiNath and SumitSaha [13] developed fuzzy inventory model for deteriorating items based on different defuzzification techniques. In this model he considered demand as constant demand.

In present paper, a production inventory model for deteriorating item with time dependent demand is discussed. More over the holding cost, deterioration cost, purchase cost, and demand are taken as triangular fuzzy number and trapezoidal number. For defuzzification many methods are available. In this investigation graded mean method and signed distance methods are used. The aim of this paper is to optimize the proposed production inventory model.

Preliminaries

2.1 Definition: Let X be a nonempty set. Then a fuzzy set A in X (ie., a fuzzy subset A of X) is characterized by a function of the form $\mu_A : X \to [0,1]$. Such a function μ_A is called the membership function and for each $x \in X$, $\mu_{\tilde{A}}(x)$ is the degree of membership of x (membership grade of x) in the fuzzy set A.

In other words, A fuzzy set $\tilde{A} = \{(x, \mu_A(x))/ x \in X\}$ where $\mu_A: X \to [0,1]$. F(X) denotes the collection of all fuzzy sets in X, called the fuzzy power set of X.

2.2 Definition : A fuzzy set is a fuzzy number if it satisfies the following four conditions

(i) It is a convex set

(ii) It is normalised

(iii) It is defined on the real number R

(iv) It is piecewise continuous

2.3 Definition (Trapezoidal fuzzy number)

A trapezoidal fuzzy number $\tilde{A}$ =(a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a}, & when\, a \le x \le b \\ 1 & when\ b \le x \le c \\ R(x) = \dfrac{d-x}{d-c} & when\, c \le x \le d \\ 0 & otherwise \end{cases}$$

2.4 Definition (Triangular fuzzy number): A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ with $a_1 < a_2 < a_3$ is triangular if its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & when\, a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & when\, a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

2.5 Definition : Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers then the arithmetical operations are defined as

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

$$\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$$

$$\widetilde{A} \Theta \widetilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

$$\widetilde{A} \oslash \widetilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1} \right)$$

$$\alpha \otimes \widetilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \alpha \geq 0 \\ (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1) \alpha < 0 \end{cases}$$

similarly we can define arithmetic operations for Triangular Fuzzy number.

2.6 Definition (Signed distance method):

Let $\widetilde{A}$ be a fuzzy set defined on R. then the signed distance of $\widetilde{A}$ is defined as

$d_F(\widetilde{A}) = \dfrac{A_1 + 2A_2 + A_3}{4}$ for defuzzifying the triangular fuzzy number.

$d_F(\widetilde{A}) = \dfrac{A_1 + A_2 + A_3 + A_4}{4}$ for defuzzifying the trapezoidal fuzzy number.

2.7 Definition (graded mean method):

Graded mean integration representation for defuzzifying the triangular fuzzy number is def as $d_F(\widetilde{A}) = \dfrac{a_1 + 4a_2 + a_3}{6}$.

Graded mean integration representation for defuzzifying the trapezoidal fuzzy numbe defined as $d_F(\widetilde{A}) = \dfrac{a_1 + 2a_2 + 2a_3 + a_4}{6}$.

Assumptions and Notations:

3.1 Assumptions:

1. The inventory system involves production of single item.
2. Lead time is zero and shortages are not allowed.
3. Demand is time dependent.
4. Replenishment is instantaneous.

3.2 Notations

A - set up cost per cycle
$\widetilde{A}$ - fuzzy set up cost
θ - deterioration rate independent of time
$\widetilde{\theta}$ - fuzzy deterioration
T - cycle length
P - production rate
$\widetilde{P}$ - fuzzy production rate
h - holding cost per unit per unit time
$\widetilde{h}$ - fuzzy holding cost per unit per unit time
d - deterioration cost per unit per unit time
$\widetilde{d}$ - fuzzy deterioration cost per unit per unit time
D - demand rate which depends exponentialy over time
$\widetilde{D}$ - fuzzy demand rate
t_1 - duration of production
$I_1(t)$ - inventory level at time t, $0 \leq t \leq t_1$
$I_2(t)$ - inventory level at time t, $t_1 \leq t \leq T$

C - total cost for the period [o,T]

$\tilde{C}$ - fuzzy total cost for the period [o,T]

$d_F\tilde{C}$ - defuzzified value of $\tilde{C}$

Description of Crisp Model: At t = o, the inventory level is zero and it increases in [o,t_1] due to the production at
the constant rate P.
At t = T again it reaches the inventory level zero. This is due to demand and deterioration of the item. This can be represented by the following figure 3

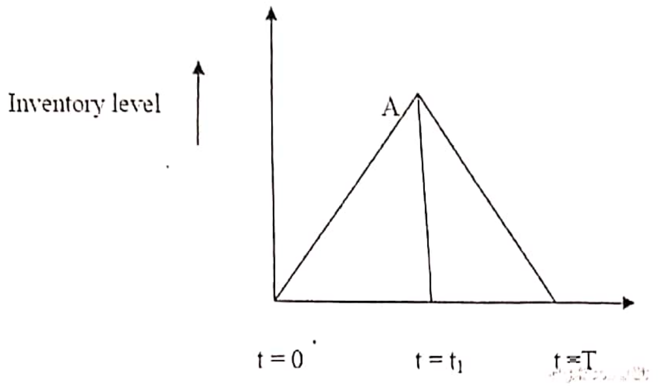

Fig 1

For $o \le t \le t_1$
The differential equation governing the situation is

$$\frac{d}{dt}I_1(t) = P - D - \theta I_1(t)$$

$$\frac{d}{dt}I_1(t) + \theta I_1(t) = P - Ke^{-\lambda t}$$, where K is the initial demand and λ is the decreasing rate of demand. K > o and $o < \lambda < \theta$

$$\frac{d}{dt}I_1(t) + \theta I_1(t) = P - Ke^{-\lambda t}$$

The solution of the above linear equation is

$$I_1(t) = \frac{P}{\theta} - K\frac{e^{(\theta-\lambda)t}e^{-\theta t}}{\theta-\lambda} + ce^{-\theta t}$$

Now apply the initial condition $I_1(t)$ =o when t = o we get

$$I_1(t) = \frac{K}{\theta-\lambda}\left(e^{-\theta t} - e^{-\lambda t}\right) + \frac{P}{\theta}(1 - e^{-\theta t})$$

For $t_1 \le t \le T$
The differential equation governing the above condition is

$$\frac{d}{dt}I_2(t) + \theta I_2(t) = - Ke^{-\lambda t}$$, where K is the initial demand and λ is the decreasing rate of demand. K > o and $o < \lambda < \theta$

$$\frac{d}{dt}I_2(t) + \theta I_2(t) = -Ke^{-\lambda t}$$

The solution of the linear equation after applying the condition I_2 (t)= o at t = T is

$$I_2(t)=\frac{K}{\theta-\lambda}\left(e^{\theta T-\lambda T-\theta t}-e^{-\lambda t}\right)$$

Since at $t = t_1$, $I_2(t_1)=I_1(t_1)$ implies

$$\frac{K}{\theta-\lambda}e^{\theta T-\lambda T-\theta t_1}-\frac{K}{\theta-\lambda}e^{-\lambda t_1}=\frac{K}{\theta-\lambda}e^{-\theta t_1}-\frac{K}{\theta-\lambda}e^{-\lambda t_1}+\frac{P}{\theta}-\frac{P}{\theta}e^{-\theta t_1}$$

After simplification we get

$$t_1=\frac{1}{\theta}\log\left\{\frac{K\theta}{P(\theta-\lambda)}e^{T(\theta-\lambda)}-\frac{K\theta}{P(\theta-\lambda)}+1\right\}$$

Holding cost can be calculated by using the formula $H.C=h\left[\int_0^{t_1}I_1(t)dt+\int_0^{t_2}I_2(t)dt\right]$

We get

$$h\left[\frac{K}{\theta-\lambda}\left(\frac{1}{\theta}-\frac{1}{\lambda}\right)\left(1-e^{-\lambda T}\right)+\frac{P}{\theta}t_1\right]$$

The deteriorating cost can be found out by the formula

$$DC=d\left[\int_0^{t_1}\theta I_1(t)dt+\int_{t_1}^{T}\theta I_2(t)dt\right]=d\left[\frac{K\theta}{\theta-\lambda}\left(\frac{1}{\theta}-\frac{1}{\lambda}\right)\left(1-e^{-\lambda T}\right)+Pt_1\right]$$

Total cost can be defined as

$$C=\frac{1}{T}(A+HC+DC)$$

Using the formula for e^x, $\log(1+x)$ and neglecting higher powers of λt, $(\theta-\lambda)$, $T(\theta-\lambda)$ we

$$\frac{A}{T}+\left(\frac{h+d\theta}{2}\right)KT\left[1-\frac{K}{P}\right]$$

$$Total\ cost=C=\frac{1}{T}\left(A+\frac{h+d\theta}{2}KT^2-\frac{1}{2}(h+d\theta)\frac{K^2T^2}{P}\right)$$

The optimum value of T can be found out by differentiating with respect to T

$$\frac{\partial C}{\partial T}=\frac{-A}{T^2}+\frac{(h+d\theta)K}{2}\left(1-\frac{K}{P}\right)\quad and\quad \frac{\partial^2 C}{\partial T^2}=\frac{2A}{T^3}>0$$

Now equate first derivative to zero to find optimum value T^*

$$\frac{-A}{T^2}+\frac{(h+d\theta)K}{2}\left(1-\frac{K}{P}\right)=0$$

$$T^*=\sqrt{\frac{A}{\frac{(h+d\theta)K}{2}\left(1-\frac{K}{P}\right)}}$$

Fuzzy Model: It is not always possible to define certain parameters with certainty for which we fuz some parameters A , h, d , θ , P , D , K.

We consider triangular fuzzy numbers of the above parameters as

$$\tilde{A}=(A_1,A_2,A_3)\ ,\ \tilde{h}=(h_1,h_2,h_3)\ ,\ \tilde{d}=(d_1,d_2,d_3)\ ,\ \tilde{\theta}=(\theta_1,\theta_2,\theta_3),$$

$$\tilde{P}=(P_1,P_2,P_3),\ \tilde{D}=(D_1,D_2,D_3),\ \tilde{K}=(K_1,K_2,K_3)$$

$$\tilde{C}=\frac{1}{T}\left[\tilde{A}+\frac{\tilde{h}+d\tilde{\theta}}{2}\tilde{K}T^2-\frac{1}{2}\left(\tilde{h}+\tilde{d}\tilde{\theta}\right)\frac{\tilde{K}^2T^2}{\tilde{P}}\right]=(C_1,C_2,C_3)$$

Where $C_i = \frac{1}{T}\left(A_i + \frac{h_i + d_i\theta_i}{2}K_i T^2 - \frac{1}{2}(h_i + d_i\theta_i)\frac{K_i^2 T^2}{P_{4-i}}\right), i = 1, 2, 3$

$\frac{d}{dt}(C_i) = \frac{-A_i}{T^2} + \frac{1}{2}(h_i + d_i\theta_i)K_i - \frac{1}{2}(h_i + d_i\theta_i)\frac{K_i^2}{P_{4-i}}$, i = 1,2,3 and $\frac{d^2}{dt^2}(C_i) = \frac{2A_i}{T^3}$, i = 1,2,3

To defuzzify the value of total cost, we use Graded mean integration method as explained below

$$d_F\tilde{C} = \frac{1}{6}(C_1 + 4C_2 + C_3) \text{.........}(3)$$

To find the optimum value differentiating (3) with respect to T

$$\frac{d}{dt}(d_F\tilde{C}) = \frac{1}{6}\left(\frac{dC_1}{dt} + 4\frac{dC_2}{dt} + \frac{dC_3}{dt}\right)$$

$$\frac{d^2}{dt}(d_F\tilde{C}) = \frac{1}{6}\left[\frac{d^2C_1}{dt^2} + 4\frac{d^2C_2}{dt^2} + \frac{d^2C_3}{dt^2}\right] = \frac{1}{6}\left[\frac{2A_1}{T^3} + \frac{4A_2}{T^3}2 + \frac{2A_3}{T^3}\right]$$

$= \frac{A_1 + 4A_2 + A_3}{3T^3}$ which is greater than zero .Therefore we get the minimum total cost.

Now let us find optimum solution of total cost by putting $\frac{d}{dt}(d_F\tilde{C})$=0

$$\frac{1}{2}(h_1 + d_1\theta_1)K_1 + \frac{4}{2}(h_2 + d_2\theta_2)K_2 + \frac{1}{2}(h_3 + d_3\theta_3)K_3 - \frac{1}{2}(h_1 + d_1\theta_1)\frac{K_1^2}{P_3} - \frac{4}{2}(h_2 + d_2\theta_2)\frac{K_2^2}{P_2} - \frac{1}{2}(h_3 + d_3\theta_3)\frac{K_3^2}{P_1} = \frac{A_1 + 4A_2 + A_3}{T^2}$$

$$T = \frac{\sqrt{A_1 + 4A_2 + A_3}}{\sqrt{\frac{(h_1 + d_1\theta_1)K_1}{2}\left(1 - \frac{K_1}{P_3}\right) + 2(h_2 + d_2\theta_2)K_2\left(1 - \frac{K_2}{P_2}\right) + \frac{(h_3 + d_3\theta_3)K_3}{2}\left(1 - \frac{K_3}{P_1}\right)}}$$

Signed distance method for defuzzifying triangular fuzzy number is

$$d_F\tilde{C} = \frac{1}{4}(C_1 + 2C_2 + C_3)$$

$$\frac{d}{dt}(d_F\tilde{C}) = \frac{1}{4}\left(\frac{dC_1}{dt} + 2\frac{dC_2}{dt} + \frac{dC_3}{dt}\right)$$

$\frac{d^2}{dt}(d_F\tilde{C}) = \frac{1}{4}\left[\frac{d^2C_1}{dt^2} + 2\frac{d^2C_2}{dt^2} + \frac{d^2C_3}{dt^2}\right] = \frac{1}{4}\left[\frac{2A_1}{T^3} + \frac{4A_2}{T^3} + \frac{2A_3}{T^3}\right] = \frac{A_1 + 2A_2 + A_3}{2T^3}$ which is greater than zero for all values . Therefore we get minimum total cost. Now let us find optimum solution of total cost by putting $\frac{d}{dt}(d_F\tilde{C})$=0

After simplification We get

$$T = \frac{\sqrt{A_1 + 2A_2 + A_3}}{\sqrt{\frac{(h_1 + d_1\theta_1)K_1}{2}\left(1 - \frac{K_1}{P_3}\right) + (h_2 + d_2\theta_2)K_2\left(1 - \frac{K_2}{P_2}\right) + \frac{(h_3 + d_3\theta_3)K_3}{2}\left(1 - \frac{K_3}{P_1}\right)}}$$

Similarly we can use trapezoidal fuzzy number for fuzzifying the parameters

A, h, d, θ, P, D, K and defuzzification can be done by signed distance method. By pplying trapezoidal fuzzy number to the above said parameters and defuzzificztion using signed distance method we get the total cost as

$$\tilde{C}=\frac{1}{4}\left[\begin{array}{l}\frac{A_1+A_2+A_3+A_4}{T}+\frac{(h_1+d_1\theta_1)K_1}{2}\left(1-\frac{K_1}{P_4}\right)T+\frac{1}{2}(h_2+d_2\theta_2)K_2\left(1-\frac{K_2}{P_3}\right)T\\+\frac{(h_3+d_3\theta_3)K_3}{2}\left(1-\frac{K_3}{P_2}\right)T+\frac{(h_4+d_4\theta_4)K_4}{2}\left(1-\frac{K_4}{P_1}\right)T\end{array}\right]$$

follow the procedure as in triangular fuzzy number we get the optimum value of T as follows

And

$$T=\frac{\sqrt{A_1+A_2+A_3+A_4}}{\sqrt{\frac{(h_1+d_1\theta_1)K_1}{2}\left(1-\frac{K_1}{P_4}\right)+\frac{1}{2}(h_2+d_2\theta_2)K_2\left(1-\frac{K_2}{P_3}\right)+\frac{(h_3+d_3\theta_3)K_3}{2}\left(1-\frac{K_3}{P_2}\right)+\frac{(h_4+d_4\theta_4)K_4}{2}\left(1-\frac{K_4}{P_1}\right)}}$$

Now we use trapezoidal fuzzy number for calculating the optimum cost and defuzzification will be done by graded mean method.

$$T=\frac{\sqrt{A_1+2A_2+2A_3+A_4}}{\sqrt{\frac{(h_1+d_1\theta_1)K_1}{2}\left(1-\frac{K_1}{P_4}\right)+(h_2+d_2\theta_2)K_2\left(1-\frac{K_2}{P_3}\right)+\frac{(h_3+d_3\theta_3)K_3}{1}\left(1-\frac{K_3}{P_2}\right)+\frac{(h_4+d_4\theta_4)K_4}{2}\left(1-\frac{K_4}{P_1}\right)}}$$

Numerical example

Crisp Model: Suppose A = 54, h = 8, θ = 0.010, P = 550, d = 1.5, K = 500

By using the formula derived above we get $T^* = 0.54$

And total cost C = 198.37

Fuzzy sense

Triangular fuzzy number(Graded mean method):

$\tilde{A}=(30,34,38)$, $\tilde{h}=(4,6,8)$, $\tilde{\theta}=(0.002,0.006,0.010)$

$\tilde{P}=(500,550,600)$, $\tilde{K}=(450,500,550)$, $\tilde{d}=(1,1.3,1.6)$

Then T = 0.6086 and total cost = 111.7391

Triangular fuzzy number(signed distance method):

$\tilde{A}=(30,34,38)$, $\tilde{h}=(4,6,8)$, $\tilde{\theta}=(0.002,0.006,0.010)$

$\tilde{P}=(500,550,600)$, $\tilde{K}=(450,500,550)$, $\tilde{d}=(1,1.3,1.6)$ T= 0.6997 and total cost = 97.1784

Trapezoidal fuzzy number (Signed distance method):

$\tilde{A}=(30,34,38,42)$, $\tilde{h}=(4,6,8,10)$, $\tilde{\theta}=(0.002,0.006,0.010,0.014)$

$\tilde{P}=(500,550,600,650)$, $\tilde{K}=(400,450500,550)$, $\tilde{d}=(1,1.3,1.6,1.9)$

T = 0.5106 and total cost = 140.998

Trapezoidal fuzzy number (Graded mean method):

$\widetilde{A} = (30,34,38,42)$, $\widetilde{h} = (4,6,8,10)$, $\widetilde{\theta} = (0.002,0.006,0.010,0.014)$

$\widetilde{P} = (500,550,600,650)$, $\widetilde{K} = (400,450500,550)$, $\widetilde{d} = (1,1.3,1.6,1.9)$

T = 0.4488 and total cost = 160.4261

Sensitivity Analysis:

7.1 Sensitivity analysis have been done by considering one parameter as Triangular fuzzy Number and the remaining are considered as constants.

$\widetilde{A}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(20,26,32)	0.5332	**97.7130**	0.6119	**84.9801**
(30,34,38)	0.6086	**111.7391**	0.6997	**97.1784**
(50,52,54)	0.7526	**138.1870**	0.8654	**120.1800**
(60,70,80)	0.8732	**160.3299**	1.004	**139.4374**

$\widetilde{d}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(1,1.6,2.2)	0.6086	**111.7389**	0.6998	**97.1639**
(1,1.3,1.6)	0.6086	**111.7391**	0.6997	**97.1784**
(1.3,1.7,2.1)	0.6085	**111.7507**	0.6997	**97.1794**
(2,2.5,3)	0.6083	**111.7778**	0.6997	**97.1880**

$\widetilde{\theta}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(0.002,0.006,0.010)	0.6086	**111.7391**	0.6997	**97.1784**
(0.003,0.006,0.009)	0.6085	**111.7492**	0.6996	**97.1960**
(0.005,0,010,0.015)	0.6083	**111.7817**	0.6995	**97.2106**
(0.008,0.015,0.022)	0.6081	**111.8276**	0.6993	**97.238**

$\tilde{h}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(2,7,12)	0.6976	97.4828	1.16	58.516
(4,6,8)	0.6086	111.7391	0.6997	97.1784
(3,9,15)	0.5957	114.1451	0.9069	74.9834
(10,14,18)	0.3900	174.3191	0.4396	154.7006

$\tilde{K}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(450,500,550)	0.6086	111.7391	0.6997	97.1784
(400,425,450)	0.3562	190.8992	0.3638	186.9053
(350,410,470)	0.3507	193.907	0.3630	187.3028
(200,230,260)	0.2931	232.015	0.2942	231.111

$\tilde{P}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(500,550,600)	0.6086	111.7391	0.6997	97.1784
(550,575,600)	0.4497	151.2109	0.4695	144.8291
(600,700,800)	0.2982	228.0372	0.3078	220.95416
(625,705,785)	0.2918	233.0312	0.2989	227.5021

7.2 Similarly sensitivity analysis have been done by considering one parameter as **Trapezoidal fuzzy number** and the remaining are considered as constants.

$\tilde{A}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(20,26,32,38)	0.4028	143.9868	0.4583	126.5497
(30,34,38,42)	0.4488	160.4261	0.5106	140.998
(50,52,54,56)	0.5446	194.4749	0.6196	162.5608
(60,70,80,90)	0.6478	231.5552	0.7371	203.5133

$\widetilde{d}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(1,1.3,1.6,1.9)	0.4488	160.4261	0.5106	140.998
(1,1.6,2.2,2.8)	0.4488	160.4358	0.5107	140.9842
(1.3,1.7,2.1,2.5)	0.4487	160.4495	0.5106	140.998
(2,2.5,3,3.5)	0.4486	160.4997	0.5105	141.0314

$\widetilde{K}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(400,460,520,580)	0.5497	130.9790	0.7188	100.1680
(400,450,500,550)	0.4488	160.4261	0.5106	140.998
(250,350,450,550)	0.3716	193.7487	0.4248	169.505
(300,330,360,390)	0.2859	251.8303	0.2912	247.2341

$\widetilde{h}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(2,7,12,17)	0.4601	156.4860	0.6387	112.7375
(4,6,8,12)	0.4488	160.4261	0.5106	140.998
(3,9,15,21)	0.4002	179.9076	0.5363	134.2517
(10,14,18,22)	0.2906	247.7857	0.3253	221.3460

$\widetilde{P}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(500.530,560,590)	0.5403	133.2611	0.6316	113.9971
(500,550,600,650)	0.4488	160.4261	0.5106	140.998
(520,560,600,640)	0.4162	172.9541	0.4556	158.0322
(550,575,600,625)	0.3816	188.9004	0.4028	178.743

$\tilde{\theta}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(0.002,0.006,0.010,0.014)	0.4488	160.4261	0.5106	140.998
(0.003,0.006,0.009,0.012)	0.4488	160.4342	0.5106	141.0166
(0.005,0,010,0.015,0.020)	0.4486	160.4908	0.5105	141.0481
(0.008,0.015,0.022,0.029)	0.4484	160.5657	0.5103	141.0967

Conclusion: From the above calculation we observe that the total cost is minimum with corresponding value of T when signed distance method of defuzzification for triangular fuzzy number is used. When we fuzzify using trapezoidal fuzzy numberalso,thesigned distance method gives optimum solution.

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