Solving Intuitionistic Fuzzy Transportation Problem with Ranking Method using Matlab Code

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Abstract **- Transportation Problem has various applications in the real world because of their wide, applicability in Production Industry, Commerce, Management Science, etc., In the real world, in many case the decision maker has no crisp information about the co-efficient belonging to the Transportation Problem due to change of Economic and Environment conditions. Hence the use of Intuitionistic Fuzzy Transportation Problem is more appropriate way to model real world Transportation Problem. In this paper, the Intuitionistic Fuzzy Transportation Problem with Intuitionistic triangular fuzzy numbers is formulated to crisp Transportation Problem using Ranking Technique and MATLAB coding has been applied to find an optimal solution. Numerical example is provided to illustrate the method.**

Keywords: **Intuitionistic triangular fuzzy numbers, Ranking technique, Intuitionistic fuzzy transportation problem, MATLAB**

I INTRODUCTION

The theory of fuzzy set introduced by L.A.Zadeh[5]in 1965 as an extension of representing or vagueness in day to day life. The concept of fuzzy (IFS) proposed by Attanssov[4] in 1986 as generalization of fuzzy sets. The major advantage of IFS over fuzzy set is that IFS separate the degree of membership(belongingness) and the degree of non membership (non-belongingness) of an element in the set. The frame work of Decision making in fuzzy environment proposed by Bellman and Zadeh [6]. In [1],Nagoor gani et al., presented a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. S.Ismail mohideen and P.Senthil kumar[8] investigated a comparative study on transportation problem in fuzzy environment.K.Pramila and G.Uthra[3] proposed an algorithm to find solution of an intuitionistic fuzzy transportation problem. B.Srinivas and G.Ganesan[2] presented a method for solving intuitionistic fuzzy assignment problem using branch and bounded method. R.Jahirhussain and P.Senthil kumar[7] presented an algorithm for solving Intuitionistic Fuzzy Transportation Problem. In this paper, the Intuitionistic Fuzzy Transportation Problem with Intuitionistic triangular fuzzy numbers is formulated to crisp Transportation Problem using Ranking Technique and MATLAB code has been applied to find an optimal solution. Numerical example is provided to illustrate the method.

II PRELIMINARIES

A. *Fuzzy Set*

A fuzzy set is built from a reference set called universe of discourse. The reference set is never fuzzy. Assume that $U=\{x_1, x_2, ..., x_n\}$ is the universe of discourse, then a fuzzy set A in U $(A \subset U)$ is defined as a set of ordered pairs $\{(x_i, \mu_A(x_i))\}$, where $x_i \in U$, $\mu_A : U \to [0,1]$ is the membership function of A and $\mu_A(x) \in [0,1]$ is the degree of membership of x in A.

B. Intuitionistic Fuzzy Set

An intuitionistic fuzzy set A on X, a universe of discourse is defined by $\tilde{A}^I = \left\{ \left(x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x) \right) / x \in X \right\}$, where $\mu_{\tilde{A}^I} : X \to [0,1]$ and $\nu_{\tilde{A}^I} : X \to [0,1]$ such that $0 \le \mu_{\tilde{A}^i}(x) + \vartheta_{\tilde{A}^i}(x) \le 1,$ for all $x \in X$.

The numbers $\mu_{\tilde{A}^I}(x)$ and $\nu_{\tilde{A}^I}(x)$ denote the degree of membership and the degree of non-membership of x to A, respectively. For an IFS A in X, the intuitionistic index of an element $x \in X$ in A is defined as follows: $\prod_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x)$, $\prod_{\tilde{A}^I}(x)$ is called as a hesitancy degree of x in A. It is evident that $0 \le \prod_{\tilde{A}^I}(x) \le 1, \forall x \in X.$

C. Intuitionistic Fuzzy Number (IFN)

An Intuitionistic fuzzy number $\tilde{A}$ is

1. An Intuitionistic fuzzy subset of the real line,
2. Normal, that is, there is some $x_o \in$ R such that $\mu_{\tilde{A}}(x_o) = 1, \nu_{\tilde{A}}(x_o) = 0,$
3. Convex for the membership function $\mu_{\tilde{A}}(x),$ that is $\mu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda)x_2\right) \ge \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right)$ for every $x_1, x_2 \in R, \lambda \in [0,1],$
4. Concave for the non-membership function $\nu_{\tilde{A}}(x)$,that is,

$\nu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda)x_2\right) \ge \max\left(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)\right)$ for every $x_1, x_2 \in R, \lambda \in [0,1].$

D. Triangular Intuitionistic Fuzzy Numbers [TIFN]

A triangular intuitionistic fuzzy number $\tilde{A}^I$ is denoted by $\tilde{A}^I = ((a_1, a_2, a_3); (a_1^{'}, a_2, a_3^{'}))$, where $a_1^{'} \le a_1 \le a_2 \le a_3 \le a_3^{'}$ with the following membership function $\mu_{\tilde{A}^I(x)}$ and the non membership function $\nu_{\tilde{A}^I(x)}$:

$$\mu_{\tilde{A}^1(X)} = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & for\ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & for\ a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{A}^I(X)} = \begin{cases} \frac{a_2 - x}{a_2 - a_1'}, & for\ a_1' \le x \le a_2 \\ \frac{x - a_2}{a_3' - a_2}, & for\ a_2 \le x \le a_3' \\ 0, & otherwise \end{cases}$$

E. Operations on Triangular Fuzzy Number

Let $\tilde{a}^I = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\tilde{b}^I = (b_1, b_2, b_3; b_1', b_2, b_3')$ be two triangular intuitionistic fuzzy numbers, the arithmetic operations of $\tilde{a}^I$ and $\tilde{b}^I$ is given below:

ADDITION:

$$(a_1, a_2, a_3; a_1', a_2, a_3') + (b_1, b_2, b_3; b_1', b_2, b_3') = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, a_3' + b_3')$$

SUBTRACTION:

$$(a_1, a_2, a_3; a_1', a_2, a_3') - (b_1, b_2, b_3; b_1', b_2, b_3') = (a_1 - b_3, a_2 - b_2, a_3 - b_1; a_1' - b_3', a_2 - b_2, a_3' - b_1')$$

MULTIPLICATION:

$$(a_1, a_2, a_3; a_1', a_2, a_3') \times (b_1, b_2, b_3; b_1', b_2, b_3') = (a_1 b_1, a_2 b_2, a_3 b_3; a_1' b_1', a_2 b_2, a_3' b_3')$$

SCALAR MULTIPLICATION:

$$k(a_1, a_2, a_3; a_1', a_2, a_3') = (ka_1, ka_2, ka_3; ka_1', ka_2, ka_3') \text{ if } k > 0$$

$$= (ka_3, ka_2, ka_1; ka_3', ka_2, ka_1') \text{ if } k < 0$$

F. Ranking of Triangular Intuitionistic Fuzzy

$$R(\tilde{A}) = \frac{1}{3}\left[\frac{(a_3' - a_1')(a_2 - 2a_3' - a_1') + (a_3 - a_1)(a_1 + a_2 + a_3) + 3(a_3'^2 - a_1'^2)}{a_3' - a_1' + a_3 - a_1}\right].$$

III INTUITIONISTIC FUZZY TARNSPORTATION PROBLEM (IFTP)

Consider a transportation problem with m intuitionistic fuzzy (IF) origins and n intuitionistic fuzzy destination. Let $C_{ij}(i = 1, 2, ...m, j = 1, 2, ...n)$ be the cost of transporting one unit of the product from i^{th} origin and j^{th} destination. Let $\tilde{a}_i^I (i = 1, 2, ...m)$ be the quantity of commodity available at IF origin i. Let $\tilde{b}_j^I (j = 1, 2, ...n)$ be the quantity of commodity needed of IF destination j. Mathematical Model of intuitionistic Fuzzy Transportation Problem is

Minimize $\tilde{Z}^I = \sum_{i=1}^{m} \sum_{j=i}^{n} \tilde{x}_{ij}^I \tilde{c}_{ij}^I$

Subject to $\sum_{j=1}^{n} \tilde{x}_{ij}^{I} = \tilde{a}_{i}^{I} \qquad i = 1, 2, \ldots\ldots m$

$$\sum_{i=1}^{m} \tilde{x}_{ij}^{I} = \tilde{b}_{j}^{I} \qquad j = 1, 2, \ldots\ldots\ldots n,$$

$$\overline{x}_{ij}^{I} \geq 0 \forall i, j.$$

ALGORITHM:

Step 1: Defuzzify the quantities of the problem into crisp by using ranking technique.

Step2: Test whether the given IFTP is balanced or not. If not change this unbalanced into balanced one by adding the number of dummy row(s)/column(s) and the values for the entries are zero.

Step3: Convert the transportation problem into the linear programming problem and then solve the problem using MATLAB code.

IV NUMERICAL EXAMPLE

Consider an intuitionistic fuzzy transportation problem whose quantities are triangular intuitionistic fuzzy number.

	D1	**D2**	**D3**	**D4**	**Supply**
O1	(14,16,18; 12,16,20)	(0,1,2; -1,1,3)	(7,8,9; 6,8,10)	(11,13,15; 10,13,16)	(2,4,6; 1,4,7)
O2	(8,11,14; 7,11,15)	(3,4,5; 2,4,6)	(5,7,9; 4,7,10)	(8,10,12; 6,10,14)	(5,6,7; 4,6,8)
O3	(6,8,10; 5,8,11)	(13,15,17; 12,15,18)	(7,9,11; 6,9,12)	(1,2,3; 0,2,4)	(7,8,9; 5,8,11)
O4	(5,6,7; 4,6,8)	(11,12,13; 10,12,14)	(3,5,7; 1,5,9)	(12,14,16; 11,14,17)	(8,10,12; 6,10,14)
Demand	3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

By defuzzifying the quantities we get

$$a_{11} = R(14,16,18;12,16,20)$$

$$= \frac{1}{3}\left[\frac{(20-12)(16-2(20)-2(12))+(18-14)(14+16+18)+3(20^2-12^2)}{20-12+18-14}\right.$$

$$= \frac{1}{3}\left(\frac{576}{12}\right) = 16$$

Similarly $a_{12}=1$, $a_{13}=8$, $a_{14}=3.09$, $a_{21}=11$, $a_{22}=4$, $a_{23}=7$, $a_{24}=10$, $a_{31}=9.6$, $a_{32}=15$, $a_{33}=9$, $a_{34}=2$, $a_{41}=6$, $a_{42}=11.11$, $a_{43}=5$, $a_{44}=14$.

Rank of all demand $d_1=4$, $d_2=5$, $d_3=12$, $d_4=7$.

Rank of all supply $s_1=4$, $s_2=6$, $s_3=8$, $s_4=10$.

	D1	**D2**	**D3**	**D4**	**Supply**
O1	16	1	8	3.09	4
O2	11	4	7	10	6
O3	9.6	15	9	2	8
O4	6	11.11	5	14	10
Demand	4	5	12	7	

EXISTING METHOD

Using the algorithm proposed by **K.Pramila et al., [3]**

	D1	**D2**	**D3**	**D4**	**Supply**
O1	4 16	1	8	3.09	4
O2	11	5 4	1 7	10	6
O3	9.6	15	8 9	2	8
O4	6	11.11	3 5	7 14	10
Demand	4	5	12	7	

The intuitionistic fuzzy optimum cost

$$= (4\times 16)+(5\times 4)+(1\times 7)+(3\times 5)+(9\times 8)+(7\times 14) = 276$$

PROPOSED METHOD

Step: 1 We convert the transportation into linear programming problem become *as*

$$
\begin{aligned}
Min\ z = {} & 16x_{11} + 1x_{12} + 8x_{13} + 3.09x_{14} + \\
& 11x_{21} + 4x_{22} + 7x_{23} + 10x_{24} + \\
& 9.6x_{31} + 15x_{32} + 9x_{33} + 2x_{34} + \\
& 6x_{41} + 11.11x_{42} + 5x_{43} + 14x_{44}
\end{aligned}
$$

Subject to

$$
\begin{aligned}
& x_{11} + x_{12} + x_{13} + x_{14} = 4 \\
& x_{21} + x_{22} + x_{23} + x_{24} = 6 \\
& x_{31} + x_{32} + x_{33} + x_{34} = 8 \\
& x_{41} + x_{42} + x_{43} + x_{44} = 10 \\
& x_{11} + x_{21} + x_{31} + x_{41} = 4 \\
& x_{12} + x_{22} + x_{32} + x_{42} = 5 \\
& x_{13} + x_{23} + x_{33} + x_{34} = 12 \\
& x_{14} + x_{24} + x_{34} + x_{44} = 7 \\
& x_{ij} \geq 0
\end{aligned}
$$

Then use the MATLAB CODE in optimization tool

SYNTAX

$$>> [x, fval, exitflag, output, lambda] = linprog(f, [\ ], [\ ], Aeq, beq, lb)$$

The minimum transportation cost $f = 119.6$

RESULT

	Existing Method **Pramila et al.,(2014)**	**Proposed Method**
Example	$f = 276$	$f = 119.6$

Method 2 produces better result than method 1.

VI CONCLUSION

In this paper, the transportation cost is taken as intuitionistic fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of intuitionistic triangular numbers has been transformed into crisp transportation problem using ranking technique and MATLAB coding has been applied to find an optimal solution. By comparing the result of the proposed method and existing method, the result shown that it is better to use the proposed method instead of existing method.

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