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A Study on Membrane Structure of Contextual Hexagonal Arrays

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Abstract:
A method of generating hexagonal arrays based on parallel contextual hexagonal array grammars is developed in [6].In the area of membrane computing, a computability model known as P systems was introduced by Gh. Paun [2].In this paper, a new method of P system called Z-direction Parallel Contextual Array Psystem is introduced with hexagonal array objects and parallel array contextual rules based on choice mappings and this new class of hexagonal array languages is compared with the existing families of hexagonal array languages.

Keywords: P system, Hexagonal Array, Contextual array operation, Contextual array P system.

1. Introduction

Hexagonal patterns are known to occur in the literature on picture processing and scene analysis. Motivated by the fact that hexagonal arrays on triangular grids can be thought of as two dimensional representations of three dimensional blocks or parallelepipeds, in the late seventies, Siromoney et al. [5] introduced formal models of generation of hexagonal arrays, extending their earlier studies of picture languages of rectangular arrays. A novel feature of the study is the notion of "arrow head catenation" [4, 5, 6]. On the other hand, P systems introduced by Paun [3] are a class of distributed parallel computing devices of biochemical inspiration. These systems are based on a structure of finitely many cell membranes which are hierarchically arranged. For formal definition and generation about P system we refer to [1, 2]. A new method of applying the parallel contextual operation on image of hexagonal arrays that involve Z-directions is introduced in [7] in this paper, we introduced a new P system called Z-direction Parallel Contextual Hexagonal Array P system with hexagonal array objects and parallel array contextual rules based on choice mappings.

2. Preliminaries

In this section we recall some basic definitions to be used throughout the rest of this paper.

2.1. Definition

Let V be a finite alphabet of symbols. A hexagonal picture p over V is a hexagonal array of symbols of V.

2.2. Example

A hexagonal picture over the alphabet $V = \{a, b, c, d\}$ is shown in Figure 1.

aab
abdc
bccac
abda
aad

Figure 1: A hexagonal picture

The set of all hexagonal arrays over the alphabet V is denoted by V^{**H}. A hexagonal picture language L over V is a subset of V^{**H}. With respect to a triad of triangular axes x, y, z the coordinates of each element of the pictures in Figure 1 are shown in Figure 2.

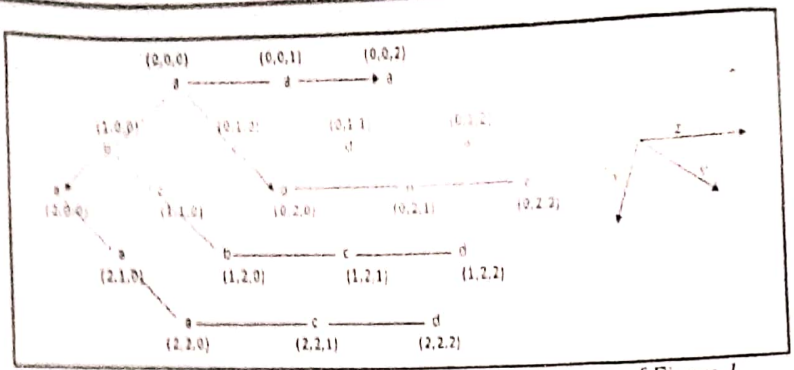

Figure 2: Coordinates of elements of hexagonal picture of Figure 1

Given a picture $p \in V^{**H}$, let $l_1(p)$ denote the number of elements in the border of p from upper left vertex to left most vertex in the direction ↙ called x- direction. $l_2(p)$ denote the number of elements in the border of p from upper left vertex to right most vertex in the direction ↘ called y- direction and $l_3(p)$ denote the number of elements in the border of p from upper left vertex to upper right vertex in the direction → called z- direction. The triple $(l_1(p), l_2(p), l_3(p))$ is called the size of p.

2.3. Definition

If $x \in V^{**H}$, then $\hat{x}$ is the hexagonal array obtained by surrounding x with a special boundary symbol $\# \notin V$.

2.4. Definition

A Z-direction parallel internal contextual hexagonal array grammar with choice is an ordered system

$G_{ZI} = (V, B, X, Y, Z, T, \varphi_{xy}, \varphi_{yx}, \varphi_{xx}, \varphi_{yy})$

where

(i) V is a finite alphabet,
(ii) B is a finite subset of V^{**H} called the base of G_{ZI},
(iii) X is a finite subset of $PA_x \$_{xy} PA_y$ called set of xy array contexts,
(iv) Y is a finite subset of $PA_y \$_{yx} PA_x$ called set of yx array contexts,
(v) Z is a finite subset of $PA_x \$_{xx} PA_x$ called set of xx array contexts,
(vi) T is a finite subset of $PA_y \$_{yy} PA_y$ called set of yy array contexts,
(vii) $\varphi_{xy}: xyTR \to 2^X$, $\varphi_{yx}: yxTR \to 2^Y$, $\varphi_{xx}: PA_x \to 2^Z$, $\varphi_{yy}: PA_y \to 2^T$ are choice mappings which perform the parallel internal xy, yx, yy, xx contextual operations respectively. When $\varphi_{xy}, \varphi_{yx}, \varphi_{xx}, \varphi_{yy}$ are omitted we call G_{ZI} as a Z-direction parallel internal contextual hexagonal array grammar without choice.

The direct derivation with respect to G_{ZI} is a binary relation $\Rightarrow_{z-in}$ on V^{**H}. It is defined as $H \Rightarrow_{z-in} H'$ where $H, H' \in V^{**H}$ with $H = X_1 \circledS X_2 \circledS X_3$,
$H' = X_1 \circledS L \circledS X_2 \circledS R \circledS X_3$ for some $X_2 \in V^{**H}$, X_1 is xy arrow head or xy arrow, X_3 is a yx arrow head or yx arrow and L, R are contexts which are respectively xy arrow head or xy arrow and yx arrow head or yx arrow, performed by the parallel internal xy, yx, yy, xx contextual operations according to the choice mappings.

$\Rightarrow^{*}_{z-in}$ is a reflexive and transitive closure of the relation $\Rightarrow_{z-in}$.

Let $G_{ZI} = (V, B, X, Y, Z, T, \varphi_{xy}, \varphi_{yx}, \varphi_{xx}, \varphi_{yy})$ be a Z-direction parallel internal contextual hexagonal array grammar. The language generated by G_{ZI}, denoted by $L(G_{ZI})$ is defined as

$L(G_{ZI}) = \{H' \in V^{**H} \mid \text{there exists } H \in B \text{ such that } H \Rightarrow^{*}_{z-in} H'\}$.

The family of all Z-direction parallel internal contextual hexagonal array languages is denoted by $ZPAIHAL$.

2.5. Definition

A Z- direction parallel external contextual hexagonal array grammar with choice is an ordered system $G_{ZE} = (V, B, X, Y, Z, T, \varphi_{xy}, \varphi_{yx}, \varphi_{xx}, \varphi_{yy})$ where the components are same as in definition 3. The direct derivation with respect to G_{ZE} is a binary relation $\Rightarrow_{z-ex}$ on V^{**H}. It is defined as $H \Rightarrow_{z-ex} H'$ where $H, H' \in V^{**H}$ if and only if $H' = L \circledS H \circledS R$ and L, R are contexts which are respectively xy arrow head or xy arrow and yx arrow head or yx arrow, performed by the parallel external xy, yx, yy, xx contextual operations according to the choice mappings. $\Rightarrow^{*}_{z-ex}$ is a reflexive and transitive closure of the relation $\Rightarrow_{z-ex}$. The language generated by G_{ZE}, denoted by $L(G_{ZE})$ is defined as

$$L(G_{ZE}) = \{H' \in V^{**H} \mid \text{there exists } H \in B \text{ such that } H \Rightarrow^{*}_{z-ex} H'\}.$$

The family of all Z-direction parallel external contextual hexagonal array languages is denoted by $ZPAEHAL$.

3. Z-Direction Parallel Contextual Hexagonal Array P System

In this section, we introduce a new P system called Z-direction Parallel Contextual Hexagonal Array P system where the objects are hexagonal arrays and the evolution rules are contexts array choice mappings.

3.1. Definition

A $ZPIHA$ P system of degree $m, (m \geq 1)$ is a construct

$$\Pi = \{V \cup \{\$_{xy}, \$_{yx}, \$_{xx}, \$_{yy}\}, \mu, H_1, H_2, \ldots\ldots\ldots H_m, A_1, A_2, \ldots\ldots\ldots A_m, (R_1, \rho_1), (R_2, \rho_2), \ldots\ldots\ldots (R_m, \rho_m), i_o\}$$

where

- i. V is a finite alphabet; its elements are called objects, $\$_{xy}, \$_{yx}, \$_{xx}, \$_{yy}$ are called special symbols not in V.
- ii. μ is a membrane structure consisting of m membranes (labeled with 1,2,……,m).
- iii. $H_i\ (1 \leq i \leq m) \subseteq V^{**H}$ are hexagonal arrays called axiom arrays over V associated with regions 1,2,…… m of μ.
- iv. $A_i\ (1 \leq i \leq m) \subseteq V^{**H}$ are finite set of array contexts associated with the regions 1,2,……m of μ given in the following form $\{X_i, Y_i, T_i, Z_i\}$ where $X_i\ (1 \leq i \leq m)$ is a finite subset of $PA_x\$_{xy}PA_y$ called set of xy array contexts, $Y_i\ (1 \leq i \leq m)$ is a finite subset of $PA_y\$_{yx}PA_x$ called set of yx array contexts,$T_i\ (1 \leq i \leq m)$ is a finite subset of $PA_y\$_{yy}PA_y$ called set of yy array contexts, $Z_i\ (1 \leq i \leq m)$ is a finite subset of $PA_x\$_{xx}PA_x$ called set of xx array contexts.
- v. $R_i (1 \leq i \leq m)$ are finite sets of evolution rules associated with the regions 1,2,……m of μ given in the following forms: $\{\varphi_{xy}, \varphi_{yx}, \varphi_{xx}, \varphi_{yy}, tar\}$ where $\varphi_{xy}: xyTR \rightarrow 2^X$, $\varphi_{yx}: yxTR \rightarrow 2^Y$, $\varphi_{xx}: PA_x \rightarrow 2^Z$, $\varphi_{yy}: PA_y \rightarrow 2^T$ are choice mappings which perform the parallel internal xy, yx, yy, xx contextual operations respectively, and $tar \in \{here, out\} \cup \{in_j | 1 \leq j \leq m\}$, ρ_i is a partial relation over $R_i, (1 \leq i \leq m)$ specifying a priority relation among rules of ρ_i.
- vi. i_o is the output membrane.

When an object is present in a region of our system, it is assumed to appear in arbitrarily many copies. Any m – tuple $(H_1, H_2, \ldots\ldots\ldots H_m)$ of Hexagonal array language over V is called configuration of Π.For any two configurations $(H_1, H_2, \ldots\ldots\ldots H_m), (H_1', H_2', \ldots\ldots H_m')$ we write $(H_1, H_2, \ldots\ldots\ldots H_m) \Rightarrow_{z-in} (H_1', H_2', \ldots\ldots H_m')$ if we can pass from $(H_1, H_2, \ldots\ldots\ldots H_m)$ to $(H_1', H_2', \ldots\ldots H_m')$ by applying the parallel internal contextual operations from each region of μ, in parallel to all possible arrays of the corresponding regions, and following target indicates associated with the rules.

More precisely, if $X_i, Y_i, T_i, Z_i \in A_i$ and $\{\varphi_{xy}, \varphi_{yx}, \varphi_{xx}, \varphi_{yy}, tar\} \subseteq R_i$ such that we can have $H_i \Rightarrow_{z-in} H_i'$ where $H_i, H_i' \in V^{**H}$ with $H_i = X_1 \circledS X_2 \circledS X_3$, $H_i' = X_1 \circledS L \circledS X_2 \circledS R \circledS X_3$ for some $X_2 \in V^{**H}$, X_1 is xy arrow head or xy arrow, X_3 is a yx arrow head or yx arrow, performed by the xy arrow head or xy arrow and yx arrow head or yx arrow and L, R are contexts which are respectively xy arrow head or xy arrow and yx arrow head or yx arrow. By applying these rules, we can generate the parallel internal xy, yx, yy, xx contextual operations according to the choice mappings. By applying these rules, we can generate the language of hexagonal arrays p of sizes $\left(l_1(p), l_2(p), l_3(p)\right)$ where $l_1(p)$ and $l_2(p)$ are fixed and $l_3(p)$ varies.Then H_i' is sent to the region indicated by tar. If $tar = here$, then the generated array remains in the same membrane where it is generated. If $tar = out$, then the generated array is moved to the region immediately outside the membrane. If $tar = in$,then the generated array is sent the region immediately inside the membrane. If not, the rule cannot be applied. The result of a computation consists of all hexagonal arrays over V which are sent to the output membrane i_o at any time during the computation.The set of all hexagonal arrays computed by $ZPIHAP$ system Π is denoted by $ZPIHAPL(\Pi)$. The family of all hexagonal array languages $ZPIHAPL\ (\Pi)$ generated by such systems Π, with atmost m membranes, is denoted by $ZPIHAP_m$.

3.2. Example

Consider the $ZPIHAP_2$

$$\Pi = \left\{V \cup \{\$_{xy}, \$_{yx}, \$_{xx}, \$_{yy}\}, \mu, H_1, H_2, A_1, A_2, (R_1, \rho_1), (R_2, \rho_2), i_o\right\}$$

where $V = \{a\}$, $\mu = [_2[_1]_1]_2$, $H_1 = \left\{\begin{array}{ccccccccc} & a & & a & & a & & a & \\ a & & a & & a & & a & & a \\ & a & & a & & a & & a & a \\ & & a & & a & & a & & a \end{array}\right\}$, $H_2 = \phi$,

$$A_1 = \left\{X_1 = \begin{bmatrix} & a \\ a & \end{bmatrix} \$_{xy} \begin{bmatrix} a & \\ & a \end{bmatrix},\ Y_1 = \begin{bmatrix} a & \\ & a \end{bmatrix} \$_{yx} \begin{bmatrix} & a \\ a & \end{bmatrix},\ Z_1 = \phi,\quad T_1 = \begin{bmatrix} a & \\ & a \end{bmatrix} \$_{yy} \begin{bmatrix} a & \\ & a \end{bmatrix}\right\}$$

$$R_1 = \left\{ \varphi_{xy}\left(\left[\begin{array}{ccccc} & a & & a & \\ a & & a & & a \end{array}\right]\right) = \left[\begin{array}{cc} & a \\ a & \end{array}\right] \$_{xy} \left[\begin{array}{cc} a & \\ & a \end{array}\right], \varphi_{yx} = \left[\begin{array}{cc} a & \\ & a \end{array}\right] \$_{yx} \left[\begin{array}{cc} & a \\ a & \end{array}\right], \quad \varphi_{xx} = \phi, \right.$$

$$\left. \varphi_{yy}\left(\left[\begin{array}{cccc} a & a & a & \\ & a & a & a \end{array}\right]\right) = \left[\begin{array}{cc} a & \\ & a \end{array}\right] \$_{yy} \left[\begin{array}{cc} a & \\ & a \end{array}\right], (here, out) \right\},$$

$\rho_1 = \phi, (R_2, \rho_2) = \phi, i_o = 2.$

Initially in membrane 1, the axiom hexagonal array is given by H_1. A new hexagonal array H_1' will be obtained from the axiom hexagonal array H_1 by using the parallel internal contextual operations as given in rule R_1 as follows:

$$H_1 = \begin{array}{c} a\ a\ a\ a \\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a \\ a\ a\ a\ a \end{array} \Rightarrow_{z-in} H_1' = \begin{array}{c} a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a \end{array}$$

$H_1' = X_1 \circledS L \circledS X_2 \circledS R \circledS X_3$ where

$$X_1 = \begin{array}{cccc} a & & & \\ & a & & \\ & & a & \\ & & & a \end{array}, \quad L = \begin{array}{cccc} a & & & \\ & a & & \\ & & a & \\ & & & a \end{array}, \quad X_2 = \begin{array}{c} a\ a \\ a\ a\ a \\ a\ a\ a \\ a\ a \end{array},$$

$$R = \begin{array}{cccc} a & & & \\ & a & & \\ & & a, & \\ & a & & \end{array} \quad X_3 = \begin{array}{ccc} a & & \\ & a & \\ & & a. \\ a & & \end{array}$$

The generated hexagonal array will be sent to membrane 2 by using $tar = out$.It will remain in the same membrane if $tar = here$. By applying the rule R_1 iteratively and sending the generated arrays to the output membrane, we get a language of hexagonal arrays given by

$$L(\Pi) = \left\{ \begin{array}{c} a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a \end{array}, \begin{array}{c} a\ a\ a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a\ a\ a\ a \\ a\ a\ a\ a\ a\ a\ a\ a \end{array}, \ldots\ldots\ldots \right\}$$

Hence the system Π generates the language of hexagonal arrays of size $(2, 3, 2n), n \geq 2$.

3.3. Theorem

The family $ZPIHAP_2$ intersects with the family $ZPAIHAL$.

Proof: The language consisting of hexagonal arrays over $\{a\}$ of sizes $(2, 3, 2n), n \geq 2$ are generated by $ZPAIHAL$ which also can be generated by the $ZPIHAP_2$.

3.4. Definition

A $ZPEHAP$ system of degree $m, (m \geq 1)$ is a construct

$$\Pi = \{V \cup \{\$_{xy}, \$_{yx}, \$_{xx}, \$_{yy}\}, \mu, H_1, H_2, \ldots\ldots H_m, A_1, A_2, \ldots\ldots A_m, (R_1, \rho_1), (R_2, \rho_2), \ldots\ldots (R_m, \rho_m), i_o\}$$

where the components are same as in $ZPIHAP$ system.

Here the choice mappings perform parallel external contextual operations, and we can have $H_i \Rightarrow_{z-ex} H_i'$ where $H_i, H_i' \in V^{**H}$ with $H_i' = L \circledS H_i \circledS R$ where L, R are xy arrow head or xy arrow and yx arrow head or yx arrow respectively.Then H_i' is sent to the region indicated by tar.The set of all hexagonal arrays computed by $ZPEHAP$ system Π is denoted by $ZPEHAPL(\Pi)$.

3.5. Example

Consider the $ZPEHAP$ system of degree 2

$$\Pi = \left\{ V \cup \{\$_{xy}, \$_{yx}, \$_{xx}, \$_{yy}\}, \mu, H_1, H_2, A_1, A_2, (R_1, \rho_1), (R_2, \rho_2), i_o \right\}$$

where $V = \{a\}, H_1 = \left\{ \begin{matrix} a\ a \\ a\ a\ a \\ \ \ a\ a\ a \\ a\ a \end{matrix} \right\}, H_2 = \phi,$

$$A_1 = \left\{ X_1 = \begin{bmatrix} & a \\ a & \end{bmatrix} \$_{xy} \begin{bmatrix} a & \\ & a \end{bmatrix}, Y_1 = \begin{bmatrix} a & \\ & a \end{bmatrix} \$_{yx} \begin{bmatrix} & a \\ a & \end{bmatrix}, \right.$$

$$\left. Z_1 = \phi, T_1 = \begin{bmatrix} a & \\ & a \end{bmatrix} \$_{yy} \begin{bmatrix} a & \\ & a \end{bmatrix} \right\},$$

$$R_1 = \left\{ \varphi_{xy} \left(\begin{bmatrix} a^n \\ a^{n+1} \end{bmatrix}_{n \geq 2} \right) = \begin{bmatrix} & a \\ a & \end{bmatrix} \$_{xy} \begin{bmatrix} a & \\ & a \end{bmatrix}, \right.$$

$$\varphi_{yx} \left(\begin{bmatrix} a^{n+1} & \\ & a^n \end{bmatrix}_{n \geq 2} \right) = \begin{bmatrix} a & \\ & a \end{bmatrix} \$_{yx} \begin{bmatrix} & a \\ a & \end{bmatrix},$$

$$\left. \varphi_{xx} = \phi,\ \varphi_{yy} \left(\begin{bmatrix} a^n & \\ & a^n \end{bmatrix}_{n \geq 2} \right) = \begin{bmatrix} a & \\ & a \end{bmatrix} \$_{yy} \begin{bmatrix} a & \\ & a \end{bmatrix}, (here, out) \right\},$$

where $\begin{bmatrix} a^n \\ a^{n+1} \end{bmatrix} = \begin{bmatrix} a & \dots & a \\ a\ a & \dots & a \end{bmatrix}, \begin{bmatrix} a^n \\ a^n \end{bmatrix} = \begin{bmatrix} a & \dots & a \\ a & \dots & a \end{bmatrix},$

$$\begin{bmatrix} a^n & \\ & a^n \end{bmatrix} = \begin{bmatrix} a & \dots & a & \\ & a & \dots & a \end{bmatrix}, \begin{bmatrix} a^{n+1} & \\ & a^n \end{bmatrix} = \begin{bmatrix} a & a & \dots & a \\ & a & \dots & a \end{bmatrix}$$

$\rho_1 = \phi, (R_2, \rho_2) = \phi, i_o = 2.$

Hence the language $L(\Pi)$ of hexagonal arrays of size $(2, 3, 2n), n \geq 2$.

3.6. Theorem

The family $ZPEHAP_2$ intersects with the family $ZPAEHAL$.

Proof: The language consisting of hexagonal arrays over $\{a\}$ of sizes $(2, 3, 2n), n \geq 2$ are generated by $ZPEIHAL$ which also can be generated by the $ZPEHAP_2$.

4. Conclusion

A new method of generating Hexagonal Arrays based on an extension of Parallel Contextual Array Grammars called Z-direction Parallel Contextual Array P system is presented in this paper. In the P system, we introduced hexagonal array objects and array contextual rules based on choice mappings which are finite. The choice mappings are performed into two ways namely Parallel internal contextual operations and Parallel external contextual operations. The new model is compared with other generative models. It is worth examining further properties such as closure under set as well as language operations of the system. Comparisons with other such generative models could also be done as future work.

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