AN ORDERING POLICY UNDER TWO-LEVEL TRADE CREDIT POLICY WITH ALLOWABLE SHORTAGE AND PERMISSIBLE DELAY IN PAYMENTS

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Abstract

In this paper, a model is developed to determine retailer's inventory policy under two-level trade credit system. It is assumed that the retailer has a powerful position so that he can obtain the full trade credit offered by supplier whereas customer can obtain just the partial trade credit from the retailer. The proposed model also allows fully backlogged shortages. This robust Mathematical model considers price dependent demand and is developed for various cases in which the length of the cycle is divided into two parts, i.e., length of the period with positive stocks and length of the period with negative stocks. The ultimate aim of the study is to minimize the total cost of the retailer by finding the optimum order quantity. Numerical examples and Sensitivity analysis are presented to prove the validity of the model.

Keywords: Inventory, Price dependent demand, Shortages, Two-level trade credit policy

1. Introduction

Inventory theory deals with the management of stock levels of goods with the aim of ensuring that demand for these goods is met. Most models are designed to address two fundamental decision issues: when a replenishment order should be placed, and what the order quantity should be. Their complexity depends on the assumptions made about demand, the cost structure and physical characteristics of the system. The costs involved include ordering/production costs, setup cost, costs for holding the product in stock and penalty costs for not being able to satisfy demand when it occurs.[1]

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Trade credit is an essential tool for financing growth. In most business transactions, the supplier would allow a specified credit period to the retailer for payment without penalty to stimulate the demand of his/her products. The numberof days for which a credit is given is determined by the company allowing the credit, and is agreed upon by both the company allowing the credit and the company receiving it.Before end of the trade credit period, retailer can sell goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by end of the trade credit period.

Fisman [2] indicates that trade credit i.e. permissible delay in payment is an important form of financing for businesses in a broad range of industries. Goyal [3] presented a single-item inventory model under trade credit. Chung [4] established an alternative approach to determine the economic order quantity under condition of trade credit. Jamal et al.[5] considered an ordering policy for deteriorating items with allowable shortage and permissible delay in payment. Table 1 in Jamal et al. [5] reveal that they are first to incorporate both concepts of allowable shortage and permissible delay in payment to establish a new inventory model. Furthermore, Sarker et al. [6] address a model to determine an optimal ordering policy for deteriorating items under inflation, permissible delay of payment and allowable shortage.Optimal payment time Jamal et al. [7] was addressed under permissible delay in payment with commodity deterioration.Teng [8] assumed that the selling price is not equal to the purchasing price to modify the Goyal's model [3].

Chung et al. [9] discussed that the selling price is not equal to the purchasing price and different payment rules are allowed. Shinn & Hwang [10] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. Chung & Huang [11] extended this type of problem-solving within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy. Huang & Chung [12] extended Goyal's model [3] to allow for cash discount for early payment. Salameh et al. [13] extended this issue to inventory decision under continuous review. Huang [14] assumed that the retailer will adopt a similar trade credit policy to stimulate demand from customer to develop the retailer's replenishment model. Huang et al. [15] determined ordering decision-making under two-level trade credit policy for a retailer with a powerful position. Recently, Chung & Huang [16] discussed ordering policy with allowable shortage and permissible delay in payments.

Present study extends Huang's model [15] and Chung & Huang's model [16]to investigate that the retailer can obtain the full trade credit offered by the suppliers and the retailer just offers the partial trade credit to customers with shortage and price-dependent demand. This paper presents a model on retailer's inventory decision-making as a cost minimization problem to determine the retailer's optimal ordering policies.

The remainder of this paper is organized as follows. In Section 2, notations and assumptions of the mathematical model is described. Section 3 gives the formulation of the mathematical model.An optimal solution of the model is determined in Section 4.In Section 5, a numerical example is given to illustrate the model and sensitivity analysis has been made for different changes in the parameter values.

2. Notations and Assumptions

2. 1. Notations

$D(p)$	Price dependent demand per year. $D(p)=a-bp$ where a and b are positive constants
s	Shortage cost per item per year
h	Unit stock-holding cost per year excluding interest charges
A	Ordering cost per year
c	Unit purchase cost
p	Unit selling price
α	Customer's fraction of the total amount payable at atime of placing an order within the delay period to retailer, $0 \le \alpha \le 1$
μ	Mark-up rate, $\mu > 1$
I_d	Rate of interest earned per year
I_k	Rate of interest payable per year to the supplier
M	Retailer's trade credit period as measured by years offered by supplier

N	Customer's trade credit period as measured by years offered by retailer
T_1	Length of the period with positive stock of item
T_2	Length of the period with negative stock of item
T	Length ofthe inventory cycle. $T = T_1 + T_2$.
$TRC(T_1, T_2)$	Annual total relevant cost, function of T_1, T_2
T_{ij}^*	Optimal cycle time of $TRC(T_1, T_2)$, $i = 1, 2, 3;\ j = 1, 2$
Q^*	Optimal order quantity, $Q^* = DT^*$

2. 2. Assumptions

- Demand rate is assumed to be a function of selling price i.e. $D(p) = a - bp$ which is a function of selling price p, where a, b are positive constants and $0 < b < a / p$.

- Unsatisfied demand / Shortages are allowed and completely backlogged.

- Time horizon of the inventory systemis infinite.

- $I_k \geq I_d$, $M \geq N$.

- Replenishments are instantaneous and lead-time is negligible.

- Supplier offers the full trade credit to retailer whereas retailer offers the partial trade credit to customers.

- Up to the time theaccount is not settled, generated sales revenue is deposited to retailer's interest bearing account. At the end of this period, the account is settled and we start paying for the interest charges on the items in stock

3. Model Formulation

Based on the above assumptions, the considered inventory system goes like this:

The total annual relevant cost consists of the following elements:

a) Cost of placing orders $= \dfrac{A}{T} = \dfrac{A}{T_1 + T_2}$

b) Stock-holding cost $= \dfrac{DhT_1^{\,2}}{2T} = \dfrac{DhT_1^{\,2}}{2(T_1+T_2)}$ (Since, the average stock $= \dfrac{DT_1}{2}$)

c) Shortage cost $= \dfrac{DsT_2^{\,2}}{2T} = \dfrac{DsT_2^{\,2}}{2(T_1+T_2)}$ (Since, the average shortage $= \dfrac{DT_2}{2}$)

d) Cost of interest charges for the items kept in stock: As the items are sold and before the replenishment account is settled, the sales revenue is used to earn interest. When the replenishment account is settled, the situation is reversed and effectively the items in the stock have to be charged at interest rate I_k.

There are two cases to be considered in annual interest payable. When $T_1 \geq M$, the account is settled at M and retailer starts paying for the interest charges on the items in stock with rate I_k. When $T_1 \leq M$, all items are sold at T and retailer can pay the amount of purchasing cost to supplier at M. Hence, retailer does not need to pay any interest charge when $T_1 \leq M$.

Case 1 ($T_1 \geq M$):

Annual interest paid $= \dfrac{cI_k D(T_1 - M)^2}{2T}$

$$= \dfrac{cI_k D(T_1 - M)^2}{2(T_1 + T_2)}$$

Case 2 ($T_1 \leq M$):

Annual interest paid $= 0$

e) Cost of interest earned during the permissible settlement period: There are three cases to be considered in annual interest earned. At the beginning of the time interval, the backordered quantity DT_2 should be replenished first. The maximum accumulated amount earning interest at the beginning of the time interval equals DpT_2. The maximum amount earning interest within the settlement period equals DpM if $T_1 \geq M$ and DpT_1 if $T_1 \leq M$

.Retailer can accumulate interest from customer's partial payment during $(0, N]$ and from the total amount of payment on $[N, M]$ with interest I_d.

Case 1 ($T_1 \geq M$):

Annual interest earned

$$= \frac{PI_d[\frac{\alpha DN^2}{2} + \frac{(DN + DM)(M - N)}{2}] + DT_2 pMI_d}{T}$$

$$= \frac{pI_d D[M^2 - (1 - \alpha)N^2 + 2T_2 M]}{2T}$$

Case 2 ($N \leq T_1 \leq M$):

Annual interest earned

$$= \frac{PI_d[\frac{\alpha DN^2}{2} + \frac{(DN + DT_1)(T_1 - N)}{2} + DT_1(M - T_1)] + DT_2 pMI_d}{T}$$

$$= \frac{pDI_d[2MT - T_1^2 - (1 - \alpha)N^2]}{2T}$$

Case 3 ($T_1 \leq N$):

Annual interest earned

$$= \frac{PI_d[\frac{\alpha DT_1^2}{2} + \alpha DT_1(N - T_1) + DT_1(M - N)] + DT_2 pMI_d}{T}$$

$$= \frac{pDI_d[2MT - 2(1 - \alpha)T_1 N - \alpha T_1^2]}{2T}$$

Note that the interest earned should be subtracted from other variable costs in order to get the total variable cost per year. From the above arguments, annual total relevant cost for retailer can be expressed as,

$TRC(T_1,T_2)$ = Ordering cost+ Stock-holding cost+ Shortage cost+ Interest payable

- Interest earned.

Therefore,

$$TRC(T_1,T_2) = \begin{cases} TRC_1(T_1,T_2) & \text{if } T_1 \geq M \\ TRC_2(T_1,T_2) & \text{if } N \leq T_1 \leq M \\ TRC_3(T_1,T_2) & \text{if } 0 \leq T_1 \leq N \end{cases}$$

Where

$$TRC_1(T_1,T_2) = \frac{A}{T_1+T_2} + \frac{DhT_1^2}{2(T_1+T_2)} + \frac{DsT_2^2}{2(T_1+T_2)} + \frac{cI_kD(T_1-M)^2}{2(T_1+T_2)}$$
$$- \frac{pDI_d[M^2-(1-\alpha)N^2+2T_2M]}{2(T_1+T_2)} \tag{1}$$

$$TRC_2(T_1,T_2) = \frac{A}{T_1+T_2} + \frac{DhT_1^2}{2(T_1+T_2)} + \frac{DsT_2^2}{2(T_1+T_2)}$$
$$- \frac{pDI_d[2MT-(1-\alpha)N^2-T_1^2]}{2(T_1+T_2)} \tag{2}$$

$$TRC_3(T_1,T_2) = \frac{A}{T_1+T_2} + \frac{DhT_1^2}{2(T_1+T_2)} + \frac{DsT_2^2}{2(T_1+T_2)}$$
$$- \frac{pDI_d[2MT-(1-\alpha)2T_1N-\alpha T_1^2]}{2(T_1+T_2)} \tag{3}$$

From now on, for convenience, we treat $TRC_1(T_1,T_2)$, $TRC_2(T_1,T_2)$ and $TRC_3(T_1,T_2)$ all are defined on $T_1 \geq 0$, $T_2 \geq 0$ and $T_1+T_2 > 0$. The following Theorem has been proposed as Theorem 1 of Chung and Huang [16].

Theorem 1

i) $TRC_1(T_1,T_2)$ is convex on $T_1 \geq 0$, $T_2 \geq 0$, $T_1+T_2 > 0$

ii) $TRC_2(T_1,T_2)$ is convex on $T_1 \geq 0$, $T_2 \geq 0$, $T_1+T_2 > 0$

iii) $TRC_3(T_1,T_2)$ is convex on $T_1 \geq 0$, $T_2 \geq 0$, $T_1+T_2 > 0$

4. Determination of optimal solutions of $TRC(T_1, T_2)$

Simplifying equations (1), (2) and (3), we get

$$TRC_i(T_1, T_2) = \frac{Z_i(T_1, T_2)}{2(T_1 + T_2)}, i = 1, 2, 3$$

Where

$$Z_1(T_1, T_2) = 2A + DhT_1^2 + DsT_2^2 + CI_k D(T_1 - M)^2 - pDI_d(M^2 - (1-\alpha)N^2 + 2T_2 M)$$

$$Z_2(T_1, T_2) = 2A + DhT_1^2 + DsT_2^2 - pDI_d(2MT_1 + 2MT_1 - (1-\alpha)N^2 - T_1^2)$$

$$Z_3(T_1, T_2) = 2A + DhT_1^2 + DsT_2^2 - pDI_d(2MT_1 + 2MT_1 - 2(1-\alpha)T_1 N - \alpha T_1^2)$$

The optimal solutions (T_{i1}^*, T_{i2}^*), $i = 1, 2, 3$ of $TRC_i(T_1, T_2)$ as determined by equations,

$$\frac{\partial TRC_i(T_1, T_2)}{\partial T_1} = 0 \text{ and } \frac{\partial TRC_i(T_1, T_2)}{\partial T_2} = 0, i = 1, 2, 3.$$

are

$$T_{12} = \frac{T_{11}(h + cI_K) + Mc(\mu I_d - I_K)}{s}$$

$$T_{11} = \frac{-Mc(\mu I_d - I_k)}{h + s + cI_k}$$

$$+ \left\{ \frac{M^2 c^2 (\mu I_d - I_k)^2}{(h + s + cI_k)^2} + \frac{\frac{2As}{D} + spI_d(1-\alpha)N^2 - M^2 c(\mu I_d - I_k)(s + c(\mu I_d - I_k))}{(h + cI_k)(h + s + cI_k)} \right\}^{1/2}$$

$$T_{22} = \frac{(h + pI_d)T_{21}}{s}$$

$$T_{21} = \left\{ \frac{\frac{2As}{D} + pI_d(1-\alpha)N^2 s}{(h + pI_d)(h + pI_d + s)} \right\}^{1/2}$$

$$T_{32} = \frac{T_{31}(h + pI_d\alpha) + pI_d(1-\alpha)N}{s}$$

$$T_{31} = \frac{-pI_dN(1-\alpha)}{h+s+pI_d\alpha} + \left\{ \frac{p^2I_d^2N^2(1-\alpha)^2}{(h+s+pI_d\alpha)^2} + \frac{\frac{2As}{D} - p^2I_d^2(1-\alpha)^2N^2}{(h+pI_d\alpha)(h+s+pI_d\alpha)} \right\}^{1/2}$$

Also, we have

$$\frac{d\overline{TRC}_i(T_1)}{dT_1} = \begin{cases} < 0 \ if \ T_1 < T_{i1} \\ = 0 \ if \ T_1 = T_{i1} \\ > 0 \ if \ T_1 > T_{i1} \end{cases}$$

$TRC_i(T_1)$ is decreasing on $(0, \ T_{i1}]$ and increasing on $[T_{i1}, \ \infty)$.

Let

$$\overline{TRC}(T_1) = \begin{cases} \overline{TRC}_1(T_1) & if & T_1 \geq M \\ \overline{TRC}_2(T_1) & if & N \leq T_1 \leq M \\ \overline{TRC}_3(T_1) & if & 0 < T_1 \leq N \end{cases}$$

Theorem 2

Let $\Delta_1 = \dfrac{DM^2(h+s+pI_d)(h+pI_d) - spDI_d(1-\alpha)N^2}{2s} - A$ and

$$\Delta_2 = \frac{\left(\begin{array}{l} D(h+cI_k)[N^2(h+s+cI_k) + 2MNc(\mu I_d - I_k)] \\ -spI_d(1-\alpha)N^2 + M^2c(\mu I_d - I_k)(s + c(\mu I_d - I_k)) \end{array} \right)}{2s} - A \ , \text{ then}$$

i) $T_{11} \geq M$ if and only if $\Delta_1 \leq 0, \Delta_2 \leq 0$.

ii) $N \leq T_{11} \leq M$ if and only if $\Delta_1 \geq 0, \Delta_2 \leq 0$.

iii) $T_{11} \leq N$ if and only if $\Delta_1 \geq 0, \Delta_2 \geq 0$.

Proof:

i) $T_{11} \geq M => T_{11} \geq N$ (since $M \geq N$)

If $T_{11} \geq M$

$$\Leftrightarrow \frac{-Mc(\mu I_d - I_k)}{h+s+cI_k}$$

$$+\left\{ \frac{M^2 c^2(\mu I_d - I_k)^2}{(h+s+cI_k)^2} + \frac{\frac{2As}{D} + spI_d(1-\alpha)N^2 - M^2 c(\mu I_d - I_k)(s+c(\mu I_d - I_k))}{(h+cI_k)(h+s+cI_k)} \right\}^{1/2} \geq M$$

$$\Leftrightarrow \left\{ \frac{M^2 c^2(\mu I_d - I_k)^2}{(h+s+cI_k)^2} + \frac{\frac{2As}{D} + spI_d(1-\alpha)N^2 - M^2 c(\mu I_d - I_k)(s+c(\mu I_d - I_k))}{(h+cI_k)(h+s+cI_k)} \right\}$$

$$\geq \left(\frac{M(h+s+pI_d)}{h+s+cI_k} \right)^2$$

$$\Leftrightarrow \frac{2As}{D} \geq M^2(h+s+pI_d)(h+pI_d) - spI_d(1-\alpha)N^2$$

$$\Leftrightarrow 0 \geq \frac{DM^2(h+s+pI_d)(h+pI_d) - spDI_d(1-\alpha)N^2}{2s} - A$$

$$\Leftrightarrow 0 \geq \Delta_1$$

If $T_{11} \geq N$

$$\Leftrightarrow \frac{-Mc(\mu I_d - I_k)}{h+s+cI_k}$$

$$+\left\{ \frac{M^2 c^2(\mu I_d - I_k)^2}{(h+s+cI_k)^2} + \frac{\frac{2As}{D} + spI_d(1-\alpha)N^2 - M^2 c(\mu I_d - I_k)(s+c(\mu I_d - I_k))}{(h+cI_k)(h+s+cI_k)} \right\}^{1/2} \geq N$$

$$\Leftrightarrow \left\{ \frac{M^2 c^2(\mu I_d - I_k)^2}{(h+s+cI_k)^2} + \frac{\frac{2As}{D} + spI_d(1-\alpha)N^2 - M^2 c(\mu I_d - I_k)(s+c(\mu I_d - I_k))}{(h+cI_k)(h+s+cI_k)} \right\}$$

$$\geq \left(\frac{Mc(\mu I_d - I_k) + N(h+s+cI_k)}{h+s+cI_k} \right)^2$$

$$\Leftrightarrow \left(\frac{2As}{D} + spI_d(1-\alpha)N^2 - M^2c(\mu I_d - I_k)(s + c(\mu I_d - I_k)) \right)(h + s + cI_k)$$

$$+ M^2c^2(\mu I_d - I_k)^2(h + cI_k) \geq (h + cI_k)[N(h + s + cI_K) + Mc(\mu I_d - I_k)]^2$$

$$\Leftrightarrow \left(\frac{2As}{D} + spI_d(1-\alpha)N^2 - M^2c(\mu I_d - I_k)(s + c(\mu I_d - I_k)) \right)$$

$$\geq (h + cI_k)[N^2(h + s + cI_K) + 2MNc(\mu I_d - I_k)]$$

$$\Leftrightarrow 0 \geq \frac{\begin{array}{c} D(h + cI_k)[N^2(h + s + cI_k) + 2MNc(\mu I_d - I_k)] - spI_d(1-\alpha)N^2 \\ + M^2c(\mu I_d - I_k)(s + c(\mu I_d - I_k)) \end{array}}{2s} - A$$

$$\Leftrightarrow 0 \geq \Delta_2$$

Therefore, (i) is proved.

Similarly, (ii) and (iii) can be proved.

The following Theorem has been proposed as Theorem 2 of Chung and Huang [16].

Theorem 3

i) If, $\Delta_1 \leq 0, \Delta_2 \leq 0$ then $T_1 = T_{11}$. Hence $\left(T_1, \frac{(h + cI_k)T_1 + Mc(\mu I_d - I_k)}{s} \right)$ will be the

optimal solution.

ii) If $\Delta_1 \geq 0, \Delta_2 \leq 0$ then $T_1 = T_{21}$. Hence $\left(T_1, \frac{(h + pI_d)T_1}{s} \right)$ will be the optimal

solution.

iii) If $\Delta_1 \geq 0, \Delta_2 \geq 0$ then $T_1 = T_{31}$. Hence $\left(T_1, \frac{(h + pI_d\alpha)T_1 + pI_d(1-\alpha)N}{s} \right)$ will be

the optimal solution.

5. Sensitivity Analysis

To illustrate the results obtained in this paper, let us apply the proposed method to efficiently solve the numerical examples. For convenience, the values of the parameters are selected at random.

$$For\ A = 1000, c = 400, I_k = 0.13, I_d = 0.09, h = 50, s = 200, M = 1/4, a = 1000, b = 1, \alpha = 0.5, N = 0.15$$

Table 1

μ	p	D	Δ_1	Δ_2	Theorem	T_1	T_2	$T = T_1 + T_2$	$Q = DT$	$TRC(T_1, T_2)$
1	400	600	>0	<0	3-(ii)	0.1743	0.0750	0.2493	149.058	3404.7
1.1	440	560	>0	<0	3-(ii)	0.1760	0.0788	0.2548	142.688	3152.7
1.2	480	520	>0	<0	3-(ii)	0.1781	0.0830	0.2611	135.772	2931.1
1.3	520	480	>0	<0	3-(ii)	0.1808	0.0875	0.2683	128.784	2739.7
1.4	560	440	>0	<0	3-(ii)	0.1841	0.0925	0.2765	121.66	2578.6
1.5	600	400	>0	<0	3-(ii)	0.1883	0.0979	0.2892	114.48	2447.1
1.6	640	360	>0	<0	3-(ii)	0.1936	0.1042	0.2978	107.208	2343.9
1.7	680	320	>0	<0	3-(ii)	0.2003	0.1113	0.3116	99.712	2266.7
1.8	720	280	>0	<0	3-(ii)	0.2087	0.1198	0.3285	91.98	2211.8
1.9	760	240	>0	<0	3-(ii)	0.2198	0.1301	0.3499	83.976	2173.1
2	800	200	>0	<0	3-(ii)	0.2346	0.1431	0.3777	75.54	2141.9

$For\ A = 1000, I_k = 0.13, I_d = 0.09, h = 50, s = 200, M = 1/4, a = 1000, b = 1, \mu = 1.5.$

Table 2

α	N	c	Δ_1	Δ_2	Theorem	T_1	T_2	$T = T_1 + T_2$	$Q = DT$	$TRC(T_1, T_2)$
0.1	0.05	200	>0	<0	3-(ii)	0.1654	0.0637	0.2291	160.37	4216.5
		400	>0	<0	3-(ii)	0.1800	0.0936	0.2736	109.44	2100.4
		600	<0	<0	3-(i)	0.3063	0.1998	0.5061	50.61	1971
	0.15	200	>0	<0	3-(ii)	0.1787	0.0688	0.2475	173.25	4927
		400	>0	<0	3-(ii)	0.1963	0.1021	0.2984	119.36	2778.6
		600	<0	<0	3-(i)	03174	0.2069	0.5243	52.43	2112.5
	0.25	200	>0	>0	3-(iii)	0.1777	0.0772	0.2549	178.43	5542.9
		400	>0	>0	3-(iii)	0.2024	0.1168	0.3192	127.68	3681.2
		600	<0	<0	3-(i)	0.3384	0.2203	0.5587	55.87	2381.7
0.5	0.05	200	>0	<0	3-(ii)	0.1646	0.0634	0.228	159.6	4175.3
		400	>0	<0	3-(ii)	0.1790	0.0931	0.2721	108.84	2060.9
		600	<0	<0	3-(i)	0.3057	0.1994	0.5051	50.51	1963
	0.15	200	>0	<0	3-(ii)	0.1722	0.0663	0.2385	166.95	4578.7
		400	>0	<0	3-(ii)	0.1883	0.0979	0.2862	114.48	2447.1
		600	<0	<0	3-(i)	0.3119	0.2034	0.5153	51.53	2042.3
	0.25	200	>0	>0	3-(iii)	0.1706	0.0710	0.2416	169.12	4907.4
		400	>0	>0	3-(iii)	0.1886	0.1063	0.2949	117.96	2940.9
		600	<0	<0	3-(i)	0.3240	0.2111	0.5351	53.51	2196.6
0.9	0.05	200	>0	<0	3-(ii)	0.1639	0.0631	0.227	158.9	4133.9
		400	>0	<0	3-(ii)	0.1781	0.0926	0.2707	108.28	2021.1
		600	<0	<0	3-(i)	0.3051	0.1190	0.4241	42.41	1954.9
	0.15	200	>0	<0	3-(ii)	0.1654	0.0637	0.2291	160.37	4216.5
		400	>0	<0	3-(ii)	0.1800	0.0936	0.2736	109.44	2100.4
		600	<0	<0	3-(i)	0.3063	0.1998	0.5061	50.61	1971
	0.25	200	>0	>0	3-(iii)	0.1649	0.0646	0.2295	160.62	4281.6
		400	>0	>0	3-(iii)	0.1797	0.0953	0.275	110	2203.6
		600	<0	<0	3-(i)	0.3088	0.2014	0.5102	51.02	2002.8

For A=1000, I_k=0.13, I_d=0.09, h=50, s=200, M=1/4, N=0.15, c=400, α=0.5, μ=1.5

Table 3

a	b	Δ_1	Δ_2	Theorem	T_1	T_2	$T = T_1 + T_2$	$Q = DT$	$TRC(T_1, T_2)$
1200	1.5	>0	<0	3-(ii)	0.2145	0.115	0.3295	98.85	2648.9
	1	>0	>0	3-(iii)	0.1606	0.0821	0.2427	145.62	1786.6
	0.5	>0	>0	3-(iii)	0.1278	0.0695	0.1973	177.57	346.2
1000	1.5	<0	<0	3-(i)	0.3630	0.1876	0.5506	55.06	2402.7
	1	>0	<0	3-(ii)	0.1883	0.0979	0.2862	114.48	2447.1
	0.5	>0	>0	3-(iii)	0.1474	0.0770	0.2244	157.08	1339.4
800	1.5	-	-	-	-	-	-	-	-
	1	<0	<0	3-(i)	0.2592	0.1347	0.3939	78.78	2688
	0.5	>0	<0	3-(ii)	0.1707	0.0888	0.2595	129.75	2143.5

From Table 1, we can observe the optimal cycle time with various parameters of μ. The following inference can be made based on Table 1. When the value of μ (i.e. Mark-up rate) is increasing, the optimal cycle time will increase. But, the demand per year will decrease when the retailer increases the μ value.

From Table 2, we can observe the optimal cycle time with various parameters of α, N, c. The following inferences can be made based on Table 2.

- When the value of α (i.e. Customer's fraction of the total amount payable at a time of placing an order within delay period to retailer) is increasing, the optimal cycle time will decrease. The Ordering quantity and the total relevant cost decreases.

- When the value of N (i.e. Customer's trade credit period as measured by years offered by retailer) is increasing, the optimal cycle time will increase. The Ordering quantity and the total relevant cost increases.

- When the value of c (i.e. Unit purchase cost) is increasing, the optimal cycle time will increase. The Ordering quantity and the total relevant cost decreases.

From Table 3, we can observe the optimal cycle time with various parameters of a, b. The following inferences can be made based on Table 3.

- When the value of a is increasing, the optimal cycle time will increase. The Ordering quantity decreases and the total relevant cost increases.

- When the value of b is increasing, the optimal cycle time will decrease. The Ordering quantity increases and the total relevant cost decreases.

Conclusion

This paper incorporated both Huang's model [15] and Chung's model [16] by considering the price dependent demand. We provide three theoremsto help the retailer in quickly determining the optimal cycle time.Theorem 1 proves the convexities of the functions $TRC_1 (T_1, T_2)$, $TRC_2 (T_1, T_2)$ and $TRC_3 (T_1, T_2)$ when $T_1 \geq 0$, $T_2 \geq 0$, $T_1 + T_2 > 0$. Theorem 3 helps retailer accurately and speedily determine the optimal ordering policy after computing for the numbers Δ_1 and Δ_2. Finally, numerical examples are given to illustrate the result developed in this paper. There are several managerial insights as follows:(1) When the Mark-up rate is increasing, the optimal cycle time will increase. But, the demand per year will decrease when the retailer increases the μ value. In the long run, the retailer has a disadvantage if the demand keeps on decreasing. The retailer must not only focus on decreasing the total relevant cost but should also increase the annual demand. So, the retailer should be very careful in fixing the Mark-up rate; (2)When the customer's fraction of the total amount due at the time of placing an order to the retailer is increasing, the retailer will order a smaller quantity. The retailer can save a larger amount of cost when customer's fraction of the total amount due at the time of placing an order within the delay period offered by retailer is small; (3). When a longer trade credit period offered to customer, the retailer will order a larger quantity to save interest payments paid to suppliers to compensate the loss of interest earned paid by his/her customers; and (4). When the unit purchasing price is increasing, retailer will order a smaller quantity to enjoy the benefits of the trade credit more frequently; (5). When the annual demand is increased, the optimal cycle time decreases so that the retailer can enjoy the interest earned in sales revenue until the next replenishment time.

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