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MATHEMATICAL MODELING ON POPULATION GROWTH RATE OF INDIA

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Abstract
The paper presents an approach for the variance of population growth in India. The Population of India is predominated in the rural with highest density. Population of India has been predicted with the help of an ordinary differential equation model known as logistic population model which is parameterized by growth rate along with carrying capacity human Population of India. Secondary data for India Population (2008-2014) from international database is collected and the model proposed in this paper to find the population growth rate is found to be a very good fit with the actual data. So the logistic model is implemented to predict the future population growth rate for India up to 2060 and the results are analyzed by using MATLAB software.

Keywords: Logistic Growth Model, Carrying Capacity, Vital Coefficients, Annual Growth Rate.

1. Introduction

The "population growth rate" is the rate at which the number of individuals in a population increases in a given time period, expressed as a fraction of the initial population. Specifically, population growth rate refers to the change in population over a unit time period, often expressed as a percentage of the number of individuals in the population at the beginning of that period. A positive growth rate indicates that the population is increasing, while a negative growth rate indicates that the population is decreasing. A growth ratio of zero indicates that there were the same number of individuals at the beginning and end of the period a growth rate may be zero even when there are significant changes in the birth rates ,death rates, immigration rates, and age distribution between the two times. Current population growth is unsustainable and technology and automation have made people less necessary, leading to high, unemployment rates. Every government and collective sectors always require accurate idea about the future size of various entities like population, resources, demands and consumptions for their planning activities. Population sizes and growth in a country directly influence the situation of economy, policy, culture, education and environment of that country and determine exploring and cost of natural resources So, the study on population growth rate of any country has becomes very essential. Mathematical modeling is a broad interdisciplinary science that uses mathematical and computational techniques to model and elucidate the phenomena arising in life sciences. A mathematical model including dynamical systems, statistical models and differential equations involves variety abstract structures. India is located in Asia. Population growth is one of the main issues in India. India is over populated country and the growth in resources has not been keeping pace with the growth in population. So the increasing trend in population is great threat to the nation. The second most populous country on the earth, India accounts for more than 17 % of the world population of the surface area with 2.4%.The use of the logistic growth model is widely established in many fields of modeling and forecasting [1]. First order differential equations govern the growth of various species. At first glance it would seem impossible to model the growth of a species by a differential equation since the population of any species always changes by integer amounts. Hence the population of any species can never be a differentiable function of time. However if a given population is very large and it is suddenly increased by one, then the change is very small compared to the given population [2]. Thus we make the assumption that large populations change continuously and even differentiable with time. The projections of future population are normally based on present population. In this paper, we will determine the carrying capacity and the vital coefficients governing the population growth of India. Further this paper gives an insight on how to determine the carrying capacity and the vital coefficients, governing population growth, by using the least square method. This paper is organized as follows In Section2 Mathematical Model of the paper is described. Section3 and Section4 give analysis on population growth rates of India and predicted population growth rates of India respectively. Conclusion of the study is given in Section5.

2. Model Description

- Let $I(t)$ indicate the population of a given species at time t
- Let 'α' be the difference between its birth rate and death rate.

If this population is isolated, then $\frac{d}{dt}I(t)$, the rate of change of the population, equals $\alpha\, I(t)$ where 'α' is a constant that does not change with either time or population. The differential equation governing population growth in this case is

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$$\frac{d}{dt}I(t) = \alpha I(t) \quad (1)$$

where, t represents the time period and *α*, referred to as the Malthusian factor, is the multiple that determines the growth rate. This mathematical model of population growth was proposed by an Englishman, Thomas R. Malthus [3] in 1798. Equation (1) is a non-homogeneous linear first order differential equation known as Malthusian law of population growth. $I(t)$ takes on only integral values and it is a discontinuous function of *t*. However, it may be approximated by a continuous and differentiable function as soon as the number of

Individuals is large enough [4]. The solution of equation (1) is

$$I(t) = I_0 e^{\alpha t} \quad (2)$$

Hence any species satisfying the Malthusian law of population growth grows exponentially with time. This model is often referred to as *The Exponential Law* and is widely regarded in the field of population ecology as the first principle of population Dynamics. At best, it can be described as an approximate physical law as it is generally acknowledged that nothing can grow at a constant rate indefinitely. As population increases in size, the environment's ability to support the population decreases. As the population increases per capita food availability decreases, waste products may accumulate and birth rates tend to decline while death rates tend to increase. Thus it seems reasonable to consider a mathematical model which explicitly incorporates the idea of carrying capacity (limiting value). A Belgian Mathematician Verhulst [5], showed that the population growth not only depends on the population size but also on how far this size is from its upper limit i.e. its carrying capacity (maximum supportable population). He modified Malthus's Model to make the population size proportional to both the previous population and a new term

$$\frac{\alpha - \beta I[t]}{\alpha} \quad (3)$$

Where *α* and are called the vital coefficients of the population. This term Reflects how far the population is from its maximum limit. However, as the population value grows and gets closer to $\frac{\alpha}{\beta}$ this new term will become very small and tend to zero, providing the right feedback to limit the population growth. Thus the second term models the competition for available resources, which tends to limit the population growth. So the modified equation using this new term is:

$$\frac{d}{dt}I(t) = \frac{\alpha I[t](\alpha - \beta I(t)]}{\alpha} \quad (4)$$

This is a nonlinear differential equation unlike equation (1) in the sense that one cannot simply multiply the previous population by a factor. In this case the population $I(t)$ on the right of equation (4) is being multiplied by itself. This equation is known as the logistic law of population growth. Putting $I = I_0$ for $t = 0$, where I_0 represents the population at some specified time, $t = 0$ equation (4) becomes1

$$\frac{d}{dt}I = \alpha I - \beta I^2 \quad (5)$$

Separating the variables in equation (5)

$$\frac{1}{\alpha}\left(\frac{1}{I} + \frac{\beta}{\alpha - \beta I}\right) dI = dt \quad (6)$$

Integrating equation (6), we obtain $\int \frac{1}{\alpha}\left(\frac{1}{I} + \frac{\beta}{\alpha - \beta I}\right) dI = t + c$, so that

$$\frac{1}{\alpha}(\log I - \log(\alpha - \beta I)) = t + c \quad (7)$$

Using $t = 0$ and $I = I_0$ we see that $c = \frac{1}{\alpha}(\log I_0 - \log(\alpha - \beta I_0))$.

Equation (7) become

$\frac{1}{\alpha}(\log I - \log(\alpha - \beta I)) = t + \frac{1}{\alpha}(\log I_0 - \log(\alpha - \beta I_0))$. Solving for I yields

$$I = \frac{\frac{\alpha}{\beta}}{1 + \left(\frac{\frac{\alpha}{\beta}}{I_0} - 1\right)e^{-\alpha t}} \quad (8)$$

If we take the limit of equation (8) as $t \to \infty$, we get (since $\alpha > 0$)

$$I_{max} = \lim_{t \to \infty} I = \frac{\alpha}{\beta} \quad (9)$$

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Next, we determine the values of α, β and I_{max} by using the least square method. Differentiating *equation (8), twice with respect to t gives*

$$\frac{d^2 I}{dt^2} = \frac{C\alpha^2 e^{\alpha t}(C - e^{\alpha t}]}{\beta(C + e^{\alpha t}]^3} \qquad (10)$$

Where $C = \frac{\frac{\alpha}{\beta}}{I_0} - 1$

At the point of inflection this second derivative of I must be equal to zero. This will be so, when

$$C = e^{\alpha t} \qquad (11)$$

Let the time when the point of inflexion occurs be $t = t_k = y$. Then $C = e^{\alpha t}$ becomes $C = e^{\alpha t_k}$. Using this new value of C and replacing $\frac{\alpha}{\beta}$ by K equation (8) becomes

$$I = \frac{K}{1 + e^{-\alpha(t - t_k)}} \qquad (12)$$

Let the coordinates of the actual population values be (t, i) and the coordinates of the predicted population values with the same abscissa on the fitted curve be (t, I). Then the error in this case is given by $(I - i)$. Since some of the actual population data points lie below the curve of predicted values while others lie above it, we square $(I - i)$ to ensure that the error is positive. Thus, the total squared error, e, in fitting the curve is given by

$$e = \sum_{j=1}^{n} \left(I_j - i_j\right)^2 \qquad (13)$$

Equation (13) contains three parameters K, α and t_k. To eliminate K we let

$$I = Kh \qquad (14)$$

Where

$$h = \frac{1}{1 + e^{-\alpha(t - t_k)}} \qquad (15)$$

Using the value of I in equation (14) and algebraic properties of inner product equation (13), We have

$$\begin{aligned}
e &= \sum_{j=1}^{n}(I_j - i_j)^2 \\
&= (I_1 - i_1)^2 + (I_2 - i_2)^2 + \ldots\ldots\ldots + (I_n - i_n)^2 \\
&= (Kh_1 - i_1)^2 + \ldots\ldots\ldots\ldots + (Kh_n - i_n)^2 \\
&= |(Kh1 - i_1 \ldots\ldots\ldots \; Kh_n - i_n)| \\
&= |(Kh_1 \ldots\ldots\ldots Kh_n) - (i_1, \ldots\ldots\ldots\ldots, i_n)|^2 \\
&= |KH - W|^2 \\
&= \langle KH - W, \; KH - W \rangle \\
&= K^2 \langle H, H \rangle - 2K \langle H, W \rangle + \langle W, W \rangle
\end{aligned}$$

Where, $H = (h_1, \ldots\ldots, h_n)$ and $W = (i_1, \ldots\ldots, i_n)$. Thus,

$$e = K^2 \langle H, H \rangle - 2K \langle H, W \rangle + \langle W, W \rangle \qquad (16)$$

Taking partial derivative of equation (16) with respect to K and equating it to zero, we obtain $2K \langle H, H \rangle - 2 \langle H, W \rangle = 0$. This gives

$$K = \frac{\langle H, W \rangle}{\langle H, H \rangle} \qquad (17)$$

Substituting this value of K into equation (16), we get

$$e = \langle W, W \rangle - \frac{\langle H, W \rangle^2}{\langle H, H \rangle} \qquad (18)$$

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This equation is converted into an error function substitute the approximate values of α and *y in* equation (17).Then find the value of K by using MATLAB program.

3. Analysis of growth rate of India

3.1. Annual percentage growth rate

Annual percentage growth rate can be calculated from the formula [10]

$$APG = \left(\frac{V_{present} - V_{past}}{N} \right) \times 100$$

Example

In 2007 the population in India was 1124134801. This grew to 1140566211 in 2008. Then we can calculate annual percentage growth rate

N = 2008 − 2007 = 1

$$APG = \frac{\left(\frac{1140566211-1124134801}{1124134801}\right)\times 100}{1} = 1.46\%$$

So in 2008 APG is 1.46%.

3.2. Predicted growth rate of India in 2060

Actual values of Population of India, collected from International Data Base is given in Table 1

Table 1

NO	Years	Actual population	Annual growth rate
1	2008	1140566211	1.46
2	2009	1156897766	1.43
3	2010	1173108018	1.40
4	2011	1189172906	1.36
5	2012	1205073612	1.33
6	2013	1220800359	1.30
7	2014	1236344631	1.27

$$\alpha = \frac{\text{Average of APG}}{100} = \frac{1.364}{100} = 0.01364 = r$$

The Population growth rate of India is approximately 1.364% per annum .We find that the values α and t_k (or y) are 0.01364 and 2060 respectively. Substituting the values of α and t_k in MATLAB program using equation (17) we get

$$I_{max} = K = 3504086441 \qquad (19)$$

This is the predicted carrying capacity or limiting value of the population of India. Using equation (9), we find that

$$\beta = \frac{0.01364}{3504086441} = 3.8925 \times 10^{-12} \qquad (20)$$

This is another vital coefficient of the population. Let t=0 correspond to the base year 2008, then the initial population will be denoted by I_0. Where I_0 = x = 1140566211.

- [t=0(2008),t=1(2009),……………………………..,t=52(2060)]
- (2008 is base year and 2060 is end year)

Substituting the values of I_0 , $\frac{\alpha}{\beta}$ (or K) and α in the equation (8), we obtain

$$I = \frac{3504086441}{1+\left(\left(\frac{3504086441}{1140566211}\right)-1\right)*(0.986)^t} \qquad (21)$$

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The equation (21) is used to compute the predicted values of the population.

t = end year − base year

t = 2060 - 2008 = 52 (22)

Substituting the values t into the equation (21), we get

$$\frac{\alpha}{2\beta} = 1752043221 \quad (23)$$

Thus, the population of India is predicted to be 1752043221 in the year 2060. This predicted population value is half of its carrying capacity.

Similarly predicted population values from 2009 to 2014 is calculated having 2008 as base year and is given in the following Table 2

Table 2

No	Years	Actual population	Predicted population
1	2008	1140566211	1140566211
2	2009	1156897766	1151439316
3	2010	1173108018	1162365099
4	2011	1189172906	1173342850
5	2012	1205073612	1184437184
6	2013	1220800359	1195451329
7	2014	1236344631	1206580552

The graph of actual and predicted population values against time is given in fig 1.

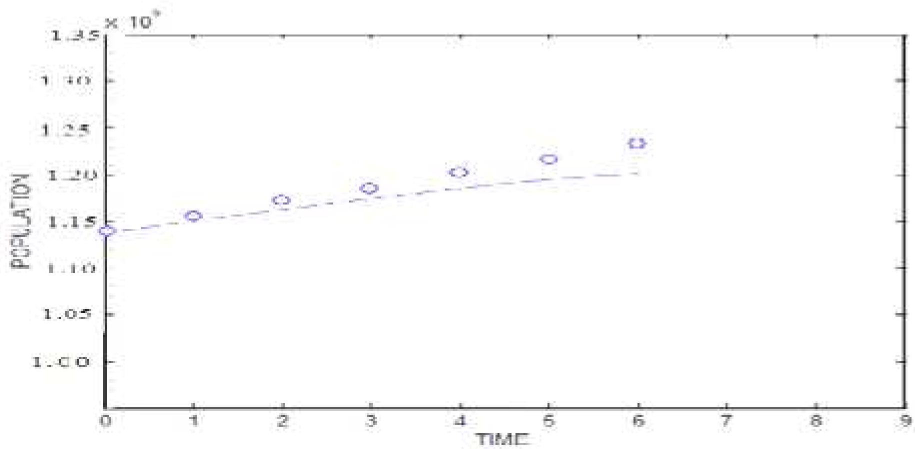

Figure 1

The graph of predicted population values against time is given in fig 2. The values are computed using equation (21).

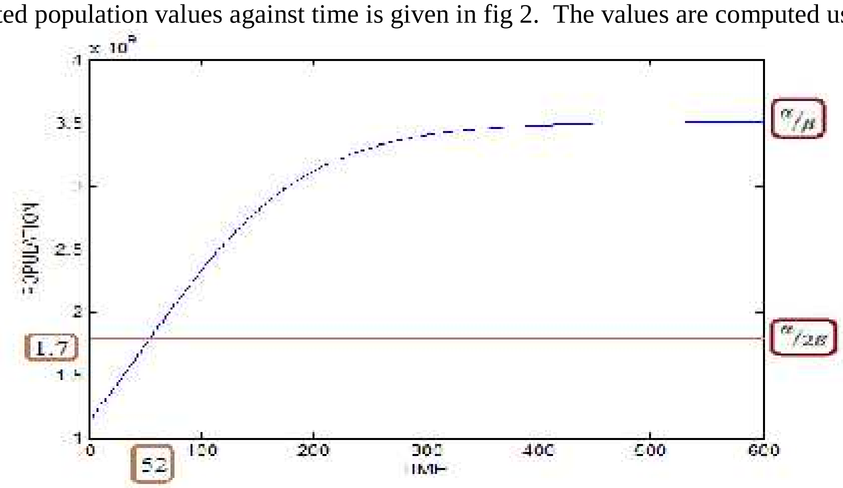

Figure 2

From fig 2, we understand that the population of India will reach the carrying capacity after 600 years**.**

4. Analysis of population growth rate of India from 2017 -2040

The value of the population growth rate of India is 1.34% approximately per annum. In January 2016 India's population is 1304162999 and it is collected from country meters. From this data the population and global rank of India is calculated and presented in Table 3.

Table 3

No	Years	Predicted population of India	Global rank of India
1	2017	1313	2
2	2018	1321.9	2
3	2019	1330.9	2
4	2020	1340.1	2
5	2021	1349.3	2
6	2022	1358.8	2
7	2023	1368.3	2
8	2024	1378.0	2
9	2025	1387.7	2
10	2026	1397.7	2
11	2027	1407.7	2
12	2028	1417.9	2
13	2029	1428.2	1
14	2030	1438.7	1
15	2031	1449.3	1
16	2032	1460.1	1
17	2033	1471	1
18	2034	1482	1
19	2035	1493.2	1
20	2036	1504.6	1
21	2037	1516.1	1
22	2038	1527.7	1
23	2039	1539.5	1
24	2040	1551.5	1

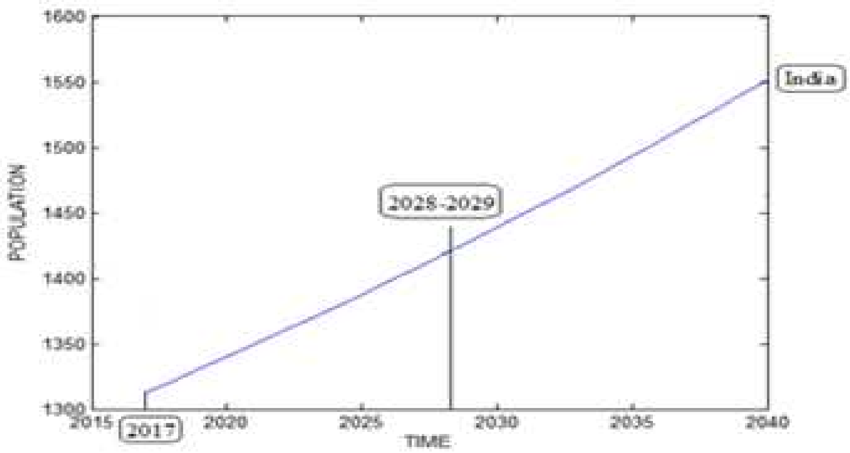

Figure 3: Graph of future population of India

5. Conclusion

From the graph given in fig 3 we can understand that the population of India in 2028-2029 will reach its global rank as **1**. If the Population continues to grow without bound, then India will continue to remain in its global rank as 1 and it is an alarming situation. So the government and people of India must take necessary measures to control increasing Annual growth rate of India.

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Matlab Program

```
 y=2060;
r=0.01356;
x=1178.131;
 t=[2009,2010,2011,2012,2013,2014,20];
i=[1178.131,1194.623,1210.98,1227.193,1243.337,1259.695];
 h=1./(1+exp(-r*(t-y)))
Columns 1 through 6
h  =  0.333618100891924   0.336640830534295   0.339676987253564   0.342726379485801   0.345788811547068
0.348864083666449
t=t'
t  = 2009  2010 2011  2012  2013  2014
h=h'
h  =  0.333618100891924   0.336640830534295   0.339676987253564   0.342726379485801   0.345788811547068
0.348864083666449
 i=i'
i =   1.0e+03 *
i=    1.178131000000000    1.194623000000000    1.210980000000000    1.227193000000000    1.243337000000000
1.259695000000000
 e=(i'*i)-((h'*i)^2/(h'*h))
e =     5.098165196385235e+02
K=(h'*i)/(h'*h)
K =     3.572875768599328e+03
P=K./2
P=1.786437884 +03
>> t=0:5;
 format long
 I=K./(1+((K/x)-1)*(0.986).^t)
I  =1.178131000000000    1.189290825965392    1.200503369350978    1.211767882338157    1.223083599782736
1.234449739344948
 t=0:600;
format long
 I=K./(1+((K/x)-1)*(0.986).^t)
plot(t,I)
xlabel('TIME')
ylabel('POPULATION')
```