UPPER ROUGH TOPOLOGY

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Abstract

The aim of this paper is to introduce a topology called upper rough topology in terms of upper approximations. It is shown that the upper rough topology is coarser than the rough topology. An algorithm is defined in terms of upper rough topology to find the key factors or core in an information system. It is used to analyze real life problems.

Keywords: *rough sets, rough topology, upper rough topology and core*

1. Introduction

Rough set theory proposed in 1982 by Zdzislaw Pawlak, is a mathematical tool for representing, reasoning and decision making in the case of uncertain information. Rough set theory deals with the approximation of sets or concepts from the existing data by means of equivalence relations. The main advantage of rough set theory in data analysis is that, it does not require any preliminary or additional information of the data. Rough topology was introduced by Lellis Thivagar *et al.* in 2012 in terms of approximations and boundary region of a subset of an universe. The concept of upper rough topology is used to find the core factor for complete information system (briefly CIS) and incomplete information system (briefly IIS).

2. Preliminaries

Definition 2.1

Let U be a non-empty set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. The pair (U, R) is called the approximation space [5]. Let X be a subset of U.

a. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$ [5]. That is, $L_R(X)=\bigcup_{x\in U}\{R(x):R(x)\subseteq X\}$ where $R(x)$ denotes the equivalence class determined by x.

b. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$ [5]. That is, $U_R(X)=\bigcup_{x\in U}\{R(x):R(x)\cap X\neq\phi\}$.

c. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X)=U_R(X)-L_R(X)$ [5].

Definition 2.2

Let U be the universe of objects, R be an equivalence relation on U and $\tau_R=\{U,\phi,L_R(X),U_R(X),B_R(X)\}$ where $X\subseteq U$. τ_R satisfies the following axioms:

i. U and $\phi\in\tau_R$

ii. The union of the elements of any sub collection of τ_R is in τ_R

iii. The intersection of the elements of any finite sub collection of τ_R is in τ_R

τ_R forms a topology on U called as the rough topology [2] on U with respect to X. We call (U,τ_R,X) as the rough topological space.

Definition 2.3

An information system [1] is of the form $(U,A,\{V_a\},f_a)$ where U is a non empty finite set of objects, called the universe, A is a finite non empty set of attributes, V_a is the attribute value set of an attribute $a\in A$ and $f_a:U\to V_a$ is called the information function. If $f_a(x)$ is equal to a missing value for some $x\in U$ and $a\in A$, then the information system is called an incomplete information system (briefly IIS) [3]. Otherwise, it is a complete information system (briefly CIS) [3]. A missing value is

denoted by $*$. That is, an incomplete information system is of the form $(U, A, \{V_a\}, f_a)$ where $a \in A$ and $* \in \bigcup V_a$. An incomplete information system can also be denoted by (U, A).

Definition 2.4

Let U be an universe and A be a finite set of attributes. For any subset B of A, there is a binary relation on U corresponding to B given by $R(B) = \{(x, y) \in U \times U : f_a(x) = f_a(y)$ or $f_a(x) = *$ or $f_a(y) = *$ for any $a \in B\}$. Then $R(B)$ is a tolerance relation [3] on U. $S_B(x)$ denotes the maximal set of objects which are possibly indiscernible with x by the tolerance relation on U. That is, $S_B(x) = \{y \in U : (x, y) \in R(B), x \in U\}$.

Definition 2.5

If (U, A) is an incomplete information system and $B \subseteq A$, then a subset X of U is said to be the tolerance class with respect to B, if $(x, y) \in R(B)$ for any $x, y \in X$. $U / R(B)$ denotes the set of all maximal tolerance classes with respect to B and is called a full cover of U.

3. Upper rough topology

Definition 3.1

Let U be the universe of objects, R be an equivalence relation on U and $\tau_{U_R} = \{U, \phi, U_R(X)\}$ where $X \subseteq U$. τ_{U_R} satisfies the following axioms:

i. U and $\phi \in \tau_{U_R}$

ii. The union of the elements of any sub collection of τ_{U_R} is in τ_{U_R}

iii. The intersection of the elements of any finite sub collection of τ_{U_R} is in τ_{U_R}

τ_{U_R} forms a topology on U called as the upper rough topology on U with respect to X.

Example 3.2

Let $U=\{a,b,c,d,e\}$, $U/R=\{\{a,b\},\{c,d\}\{e\}\}$, the family of equivalence classes of U by the equivalence relation R and $X=\{a,c,d\}$. Then $U_R(X)=\{a,b,c,d\}$. Therefore, the upper rough topology $\tau_{U_R}=\{U,\phi,\{a,b,c,d\}\}$.

Remark 3.3

The rough topology contains the upper rough topology defined on U with respect to R. Thus the upper rough topology is coarser than the rough topology.

Example 3.4

Let $U=\{a,b,c,d,e,f\}$, $U/R=\{\{a,b\},\{c,\}\{d,f\},\{e\}\}$, the family of equivalence classes of U by the equivalence relation R and $X=\{a,e,f\}$. Then $U_R(X)=\{a,b,d,e,f\}$. Therefore, the rough topology $\tau_R=\{U,\phi,\{a,b,c,d\},\{c,d\},\{a,b\}\}$ and the upper rough topology $\tau_{U_R}=\{U,\phi,\{a,b,d,e,f\}\}$. Thus $\tau_{U_R}\subset\tau_R$.

4. Algorithm

In this section an algorithm is developed to get the deciding factors or core to choose the minimum number of attributes required for the classification of objects.

Step 1: Given a finite universe U, a finite set A of attributes that is divided into two classes, C of condition attributes and D of decision attribute, an equivalence relation R on U corresponding to C and a subset X of U, represent the data as an information table, columns of which are labeled by attributes, rows by objects and entries of the table are attribute values.

Step 2: Find the upper approximation of X with respect to R

Step 3: Generate the upper rough topology $\tau_{U_{R(C)}}$ on U

Step 4: Remove an attribute x from C and find the upper approximation of X with respect to the equivalence relation on $C-(x)$

Step 5: Generate the upper rough topology $\tau_{U_{R(C-x)}}$ on U

Step 6: Repeat steps 4 and 5 for all attributes in C

Step 7: Those attributes in C for which $\tau_{U_{R\cup\{a\}}} \neq \tau_{U_{R-\{a\}}}$ form the core.

5. Upper rough topology in CIS

Example 5.1

Measles is an acute viral and infectious disease. It is spread by contact with infected person through coughing and sneezing and it is transmitted by droplet infection or air borne. The symptoms of this disease are skin rashes, fatigue, dry cough, conjunctivitis and fever. The disease can be prevented through vaccination by measles vaccine. After recovery from measles person acquires immunity against infection for his life.

Consider the following information table giving data about 8 patients.

Patients	Skin rash (S)	Conjunctivitis (C)	Dry cough (D)	Fatigue (F)	Temperature (T)	Measles
P_1	Yes	Yes	No	No	Normal	No
P_2	Yes	Yes	No	No	Very high	Yes
P_3	Yes	No	No	No	High	Yes
P_4	No	No	No	No	Very high	No
P_5	No	Yes	Yes	Yes	High	No
P_6	Yes	No	No	No	High	No
P_7	Yes	Yes	Yes	Yes	High	Yes
P_8	Yes	Yes	No	No	Very high	Yes

Table 5.1

Here $U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$. The columns represent the attributes (the symptoms for measles) and the rows represent the objects (the patients) in the above table. The entries in the table are the attribute values. Skin rash, Conjunctivitis, Dry cough, Fatigue and Temperature are the condition attributes and Measles is the decision attributes. The patient P_5 is characterized by the value set (Skin rash, No), (Conjunctivitis, Yes), (Dry cough, Yes), (Fatigue, Yes), (Temperature, High) and (Measles, No), which gives information about the patient P_5. In the table, the patients

P_1, P_2, P_3, P_6, P_7 and P_8 are indiscernible with respect to the attribute 'Skin rash'. The attribute 'Dry Cough' generates two equivalence classes, namely, $\{P_1, P_2, P_3, P_4, P_6, P_8\}$ and $\{P_5, P_7\}$, whereas the attributes 'Skin rash' and 'Dry cough' generate the equivalence classes $\{P_1, P_2, P_3, P_6, P_8\}$, $\{P_7\}$, $\{P_4\}$ and $\{P_5\}$.

Case 1: Patients with measles are taken as X. Here $X = \{P_2, P_3, P_7, P_8\}$. Let R be the equivalence relation on U with respect to the condition attributes $C = \{\text{Skin rash, Conjunctivitis, Dry Cough, Fatigue, Temperature}\} = \{S, C, D, F, T\}$. The equivalence classes determined by R corresponding to C is given by $U/R(C) = \{\{P_1\}, \{P_2, P_8\}, \{P_3, P_6\}, \{P_4\}, \{P_5\}, \{P_7\}\}$. The upper approximation with respect to R is given by $U_{R(C)}(X) = \{P_2, P_3, P_6, P_7, P_8\}$. The upper rough topology with respect to R is given by $\tau_{U_{R(C)}}(X) = \{\phi, U, \{P_2, P_3, P_6, P_7, P_8\}\}$. If 'Skin rash' is removed from the set of condition attributes then the equivalence classes corresponding to $C_1 = \{C, D, F, T\}$ is given by $U/R(C_1) = \{\{P_1\}, \{P_2, P_8\}, \{P_3, P_6\}, \{P_4\}, \{P_5, P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_1)}(X) = \{P_2, P_3, P_5, P_6, P_7, P_8\}$ and $\tau_{U_{R(C)}}(X) \neq \tau_{U_{R(C_1)}}(X) = \{\phi, U, \{P_2, P_3, P_5, P_6, P_7, P_8\}\}$. If 'Conjunctivitis' is removed from the set of condition attributes then the equivalence classes corresponding to $C_2 = \{S, D, F, T\}$ is given by $U/R(C_2) = \{\{P_1\}, \{P_2, P_8\}, \{P_3, P_6\}, \{P_4\}, \{P_5\}, \{P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_2)}(X) = \{P_2, P_3, P_6, P_7, P_8\}$ and $\tau_{U_{R(C)}}(X) \neq \tau_{U_{R(C_2)}}(X) = \{\phi, U, \{P_2, P_3, P_6, P_7, P_8\}\}$. If 'Dry cough' is removed from the set of condition attributes then the equivalence classes corresponding to $C_3 = \{S, C, F, T\}$ is given by $U/R(C_3) = \{\{P_1\}, \{P_2, P_8\}, \{P_3, P_6\}, \{P_4\}, \{P_5\}, \{P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_3)}(X) = \{P_2, P_3, P_6, P_7, P_8\}$ and $\tau_{U_{R(C)}}(X) = \tau_{U_{R(C_3)}}(X) = \{\phi, U, \{P_2, P_3, P_6, P_7, P_8\}\}$. If 'Fatigue' is removed from the set of condition attributes then the equivalence classes corresponding to $C_4 = \{S, C, D, T\}$ is given by $U/R(C_4) = \{\{P_1\}, \{P_2, P_8\}, \{P_3, P_6\}, \{P_4\}, \{P_5\}, \{P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_4)}(X) = \{P_2, P_3, P_6, P_7, P_8\}$

and $\tau_{U_{R(C)}}(X)=\tau_{U_{R(C_4)}}(X)=\{\phi,U,\{P_2,P_3,P_6,P_7,P_8\}\}$. If 'Temperature' is removed from the set of condition attributes then the equivalence classes corresponding to $C_5=\{S,C,D,F\}$ is given by $U/R(C_5)=\{\{P_1,P_2,P_8\},\{P_3,P_6\},\{P_4\},\{P_5\},\{P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_5)}(X)=\{P_1,P_2,P_3,P_6,P_7,P_8\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_5)}}(X)=\{\phi,U,\{P_1,P_2,P_3,P_6,P_7,P_8\}\}$. Thus $Core=\{S,T\}$.

Case 2: Let $X=\{P_1,P_4,P_5,P_6\}$, the set of patients not having measles. Let R be the equivalence relation on U with respect to the condition attributes $C=\{S,C,D,F,T\}$. The equivalence classes determined by R corresponding to C is given by $U/R(C)=\{\{P_1\},\{P_2,P_8\},\{P_3,P_6\},\{P_4\},\{P_5\},\{P_7\}\}$. The upper approximation with respect to R is given by $U_{R(C)}(X)=\{P_1,P_3,P_4,P_5,P_6\}$. The upper rough topology with respect to R is given by $\tau_{U_{R(C)}}(X)=\{\phi,U,\{P_1,P_3,P_4,P_5,P_6\}\}$. If 'Skin rash' is removed from the set of condition attributes then the equivalence classes corresponding to $C_1=\{C,D,F,T\}$ is given by $U/R(C_1)=\{\{P_1\},\{P_2,P_8\},\{P_3,P_6\},\{P_4\},\{P_5,P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_1)}(X)=\{P_1,P_3,P_4,P_5,P_6,P_7\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_1)}}(X)=\{\phi,U,\{P_1,P_3,P_4,P_5,P_6,P_7\}\}$. If 'Conjunctivitis' is removed from the set of condition attributes then the equivalence classes corresponding to $C_2=\{S,D,F,T\}$ is given by $U/R(C_2)=\{\{P_1\},\{P_2,P_8\},\{P_3,P_6\},\{P_4\},\{P_5\},\{P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_2)}(X)=\{P_1,P_3,P_4,P_5,P_6\}$ and $\tau_{U_{R(C)}}(X)=\tau_{U_{R(C_2)}}(X)=\{\phi,U,\{P_1,P_3,P_4,P_5,P_6\}\}$. If 'Dry cough' is removed from the set of condition attributes then the equivalence classes corresponding to $C_3=\{S,C,F,T\}$ is given by $U/R(C_3)=\{\{P_1\},\{P_2,P_8\},\{P_3,P_6\},\{P_4\},\{P_5\},\{P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_3)}(X)=\{P_1,P_3,P_4,P_5,P_6\}$ and $\tau_{U_{R(C)}}(X)=\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{P_1,P_3,P_4,P_5,P_6\}\}$. If 'Fatigue' is removed from the set of

condition attributes then the equivalence classes corresponding to $C_4=\{S,C,D,T\}$ is given by $U/R(C_4)=\{\{P_1\},\{P_2,P_8\},\{P_3,P_6\},\{P_4\},\{P_5\},\{P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_4)}(X)=\{P_1,P_3,P_4,P_5,P_6\}$ and $\tau_{U_{R(C_4)}}(X)=\tau_{U_{R(C_4)}}(X)=\{\phi,U,\{P_1,P_3,P_4,P_5,P_6\}\}$. If 'Temperature' is removed from the set of condition attributes then the equivalence classes corresponding to $C_5=\{S,C,D,F\}$ is given by $U/R(C_5)=\{\{P_1,P_2,P_8\},\{P_3,P_6\},\{P_4\},\{P_5\},\{P_7\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_5)}(X)=\{P_1,P_2,P_3,P_4,P_5,P_6,P_8\}$ and $\tau_{U_{R(C_5)}}(X)\neq\tau_{U_{R(C_5)}}(X)=\{\phi,U,\{P_1,P_2,P_3,P_4,P_5,P_6,P_8\}\}$. Thus $Core=\{S,T\}$.

Observation

From both cases it is found that Skin rash and Temperature are the symptoms which are closely connected with the disease Measles.

Example 5.2

The following table gives information about 6 patients suffering from Flu.

Patients	Headache	Myalgia	Temperature	Flu
P_1	Yes	Yes	Normal	No
P_2	Yes	Yes	High	Yes
P_3	Yes	Yes	Very high	Yes
P_4	No	Yes	Normal	No
P_5	No	No	High	No
P_6	No	Yes	Very high	Yes

Table 5.2

Here $U=\{P_1,P_2,P_3,P_4,P_5,P_6\}$. The columns represent the attributes (the symptoms for flu) and the rows represent the objects (the patients) in the above table. The entries in the table are the attribute values. Headache, Myalgia and Temperature are the condition attributes and Flu is the decision attributes. The patient P_5 is characterized by the value

set (Headache, No), (Myalgia, No), (Temperature, High) and (Flu, No), which gives information about the patient P_5.

Case 1: Patients with flu are taken as X. Here $X=\{P_2,P_3,P_6\}$. Let R be the equivalence relation on U with respect to the condition attributes $C=\{\text{Headache, Myalgia, Temperature}\}=\{H,M,T\}$. The equivalence classes determined by R corresponding to C is given by $U/R(C)=\{\{P_1\},\{P_2\},\{P_3\},\{P_4\},\{P_5\},\{P_6\}\}$. The upper approximation with respect to R is given by $U_{R(C)}(X)=\{P_2,P_3,P_6\}$. The upper rough topology with respect to R is given by $\tau_{U_{R(C)}}(X)=\{\phi,U,\{P_2,P_3,P_6\}\}$. If 'Headache' is removed from the set of condition attributes then the equivalence classes corresponding to $C_1=\{M,T\}$ is given by $U/R(C_1)=\{\{P_1,P_4\},\{P_2\},\{P_3,P_6\},\{P_5\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_1)}(X)=\{P_2,P_3,P_6\}$ and $\tau_{U_{R(C_1)}}(X)=\tau_{U_{R(C_1)}}(X)=\{\phi,U,\{P_2,P_3,P_6\}\}$. If 'Myalgia' is removed from the set of condition attributes then the equivalence classes corresponding to $C_2=\{H,T\}$ is given by $U/R(C_2)=\{\{P_1\},\{P_2\},\{P_3\},\{P_4\},\{P_5\},\{P_6\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_2)}(X)=\{P_2,P_3,P_6\}$ and $\tau_{U_{R(C_2)}}(X)=\tau_{U_{R(C_2)}}(X)=\{\phi,U,\{P_2,P_3,P_6\}\}$. If 'Temperature' is removed from the set of condition attributes then the equivalence classes corresponding to $C_3=\{H,M\}$ is given by $U/R(C_3)=\{\{P_1,P_2,P_3\},\{P_4,P_6\},\{P_5\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_3)}(X)=\{P_1,P_2,P_3,P_4,P_6\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{P_1,P_2,P_3,P_4,P_6\}\}$. Thus $Core=\{T\}$.

Case 2: Let $X=\{P_1,P_4,P_5\}$, the set of patients not having flu. Let R be the equivalence relation on U with respect to the condition attributes $C=\{H,M,T\}$. The equivalence classes determined by R corresponding to C is given by $U/R(C)=\{\{P_1\},\{P_2\},\{P_3\},\{P_4\},\{P_5\},\{P_6\}\}$. The upper approximation with respect to R is given by $U_{R(C)}(X)=\{P_1,P_4,P_5\}$. The upper

rough topology with respect to R is given by $\tau_{U_{R(C_1)}}(X)=\{\phi,U,\{P_1,P_4,P_5\}\}$. If 'Headache' is removed from the set of condition attributes then the equivalence classes corresponding to $C_1=\{M,T\}$ is given by $U/R(C_1)=\{\{P_1,P_4\},\{P_2\},\{P_3,P_6\},\{P_5\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_1)}(X)=\{P_1,P_4,P_5\}$ and $\tau_{U_{R(C_1)}}(X)=\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{P_1,P_4,P_5\}\}$. If 'Myalgia' is removed from the set of condition attributes then the equivalence classes corresponding to $C_2=\{H,T\}$ is given by $U/R(C_2)=\{\{P_1\},\{P_2\},\{P_3\},\{P_4\},\{P_5\},\{P_6\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_2)}(X)=\{P_1,P_4,P_5\}$ and $\tau_{U_{R(C_2)}}(X)=\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{P_1,P_4,P_5\}\}$. If 'Temperature' is removed from the set of condition attributes then the equivalence classes corresponding to $C_3=\{H,M\}$ is given by $U/R(C_3)=\{\{P_1,P_2,P_3\},\{P_4,P_6\},\{P_5\}\}$. The upper approximation with respect to this equivalence relation is given by $U_{R(C_3)}(X)=\{P_1,P_2,P_3,P_4,P_5,P_6\}=U$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{P_1,P_2,P_3,P_4,P_5,P_6\}\}$. Thus $Core=\{T\}$.

Observation

Since the core has T as its only element, Temperature is the key attribute that has connection to the disease Flu.

6. Upper rough topology in IIS

Example 6.1

Consider the following table giving information about the selection list for recruitment in a software company. Qualification, Experience, Technical Skill, Communication Skill and Salary Expectation are the conditional attributes of the system, whereas Decision is the decision attribute.

Candidates	Qualification (Q)	Experience (E)	Technical Skill (T)	Communication Skill (Cs)	Salary (S) Expectation	Decision (D)
X_1	P.G	Medium	Good	*	Very High	Accept
X_2	U.G	*	Bad	Good	Under norms	Reject
X_3	P.G	High	Good	Bad	*	Reject
X_4	U.G	High	*	Very Good	Under norms	Accept
X_5	P.G	*	Very Good	*	Above norms	Accept
X_6	P.G	Medium	*	Very Good	*	Accept
X_7	U.G	Low	Bad	*	Under norms	Reject

Table 6.1

The columns of the table represent the factors evaluated in the interview and the rows represent the individual ability of the candidates, who attended the interview. The entries in the table are the attribute values. The given information system is incomplete and is given by (U, A) where $U = \{X_1, X_2, X_3, X_4, X_5, X_6, X_7\}$ and $A = \{Q, E, T, C, S, D\}$ which is divided into two subsets namely, the condition attributes $C = \{Q, E, T, Cs, S\}$ and the decision attribute D. The attribute 'Experience' generates the tolerance classes $\{X_1, X_2, X_5, X_6\}$, $\{X_2, X_3, X_4, X_5\}$ and $\{X_2, X_5, X_7\}$, since the missing attribute value for X_2 and X_5 can be 'Low' or 'Medium' or 'High'. Similarly, the maximal tolerance classes for other combination of attributes can be formed. Considering all the condition attributes together, the maximal tolerance classes are $\{X_1, X_6\}, \{X_2, X_7\}, \{X_3\}, \{X_4\}, \{X_5, X_6\}$ and $U / R(C) = \{\{X_1, X_6\}, \{X_2, X_7\}, \{X_3\}, \{X_4\}, \{X_5, X_6\}\}$.

Case 1: Let $X = \{X_1, X_4, X_5, X_6\}$ be the set of candidates those who are selected for the software concern. The equivalence classes corresponding to $C = \{Q, E, T, Cs, S\}$ is given by $U / R(C) = \{\{X_1, X_6\}, \{X_2, X_7\}, \{X_3\}, \{X_4\}, \{X_5, X_6\}\}$. The upper approximation with respect to R is given by $U_{R(C)}(X) = \{X_1, X_4, X_5, X_6\}$. The upper rough topology with respect to R is

given by $\tau_{U_{R(C)}}(X)=\{\phi,U,\{X_1,X_4,X_5,X_6\}\}$. If we remove the attribute 'Qualification' from the set of condition attributes, then the equivalence classes corresponding to $C_1=\{E,T,Cs,S\}$ is given by $U/R(C_1)=\{\{X_1,X_6\},\{X_2,X_7\},\{X_3\},\{X_4\},\{X_5,X_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_1)}(X)=\{X_1,X_4,X_5,X_6\}$ and the upper rough topology is given by $\tau_{U_{R(C_1)}}(X)=\{\phi,U,\{X_1,X_4,X_5,X_6\}\}=\tau_{U_{R(C)}}(X)$. If 'Experience' is removed from the condition attributes, then the equivalence class corresponding to $C_2=\{Q,T,Cs,S\}$ is given by $U/R(C_2)=\{\{X_1,X_6\},\{X_1,X_3\},\{X_2,X_7\},\{X_4,X_7\},\{X_5,X_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_2)}(X)=\{X_1,X_3,X_4,X_5,X_6,X_7\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_2)}}(X)=\{\phi,U,\{X_1,X_3,X_4,X_5,X_6,X_7\}\}$. If 'Technical Skill' is removed from the set of condition attributes, then the equivalence class corresponding to $C_3=\{Q,E,Cs,S\}$ is given by $U/R(C_3)=\{\{X_1,X_6\},\{X_2,X_7\},\{X_3,X_5\},\{X_4\},\{X_5,X_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_3)}(X)=\{X_1,X_3,X_4,X_5,X_6\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{X_1,X_3,X_4,X_5,X_6\}\}$. If 'Communication Skill' is removed from the set of condition attributes, then the equivalence class corresponding to $C_4=\{Q,E,T,S\}$ is given by $U/R(C_4)=\{\{X_1,X_6\},\{X_2,X_4\},\{X_2,X_7\},\{X_3\},\{X_5,X_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_4)}(X)=\{X_1,X_2,X_4,X_5,X_6\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_4)}}(X)=\{\phi,U,\{X_1,X_2,X_4,X_5,X_6\}\}$. If 'Salary Expectation' is removed from the set of condition attributes, then the equivalence class corresponding to $C_5=\{Q,E,T,Cs\}$ is given by $U/R(C_5)=\{\{X_1,X_6\},\{X_2,X_7\},\{X_3\},\{X_4\},\{X_5,X_6\}\}$ which is the same as $U/R(C)$ and hence $\tau_{U_{R(C_5)}}(X)=\{\phi,U,\{X_1,X_4,X_5,X_6\}\}=\tau_{U_{R(C)}}(X)$. Therefore, $Core=\{E,T,Cs\}$.

Case 2: Let $X=\{X_2,X_3,X_7\}$ be the set of candidates those who are rejected for the software concern. The equivalence classes corresponding to $C=\{Q,E,T,Cs,S\}$ is given by

$U/R(C)=\{\{X_1,X_6\},\{X_2,X_7\},\{X_3\},\{X_4\},\{X_5,X_6\}\}$. The upper approximation with respect to R is given by $U_{R(C)}(X)=\{X_2,X_3,X_7\}$. The upper rough topology with respect to R is given by $\tau_{U_{R(C)}}(X)=\{\phi,U,\{X_2,X_3,X_7\}\}$. If we remove the attribute 'Qualification' from the set of condition attributes, then the equivalence classes corresponding to $C_1=\{E,T,Cs,S\}$ is given by $U/R(C_1)=\{\{X_1,X_6\},\{X_2,X_7\},\{X_3\},\{X_4\},\{X_5,X_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_1)}(X)=\{X_2,X_3,X_7\}$ and the upper rough topology is given by $\tau_{U_{R(C_1)}}(X)=\{\phi,U,\{X_2,X_3,X_7\}\}=\tau_{U_{R(C)}}(X)$. If 'Experience' is removed from the condition attributes, then the equivalence class corresponding to $C_2=\{Q,T,Cs,S\}$ is given by $U/R(C_2)=\{\{X_1,X_6\},\{X_1,X_3\},\{X_2,X_7\},\{X_4,X_7\},\{X_5,X_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_2)}(X)=\{X_1,X_2,X_3,X_4,X_7\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_2)}}(X)=\{\phi,U,\{X_1,X_2,X_3,X_4,X_7\}\}$. If 'Technical Skill' is removed from the set of condition attributes, then the equivalence class corresponding to $C_3=\{Q,E,Cs,S\}$ is given by $U/R(C_3)=\{\{X_1,X_6\},\{X_2,X_7\},\{X_3,X_5\},\{X_4\},\{X_5,X_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_3)}(X)=\{X_2,X_3,X_5,X_7\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{X_2,X_3,X_5,X_7\}\}$. If 'Communication Skill' is removed from the set of condition attributes, then the equivalence class corresponding to $C_4=\{Q,E,T,S\}$ is given by $U/R(C_4)=\{\{X_1,X_6\},\{X_2,X_4\},\{X_2,X_7\},\{X_3\},\{X_5,X_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_4)}(X)=\{X_2,X_3,X_4,X_7\}$ and $\tau_{U_{R(C)}}(X)\neq\tau_{U_{R(C_4)}}(X)=\{\phi,U,\{X_2,X_3,X_4,X_7\}\}$. If 'Salary Expectation' is removed from the set of condition attributes, then the equivalence class corresponding to $C_5=\{Q,E,T,Cs\}$ is given by $U/R(C_5)=\{\{X_1,X_6\},\{X_2,X_7\},\{X_3\},\{X_4\},\{X_5,X_6\}\}$ which is the same as $U/R(C)$ and hence $\tau_{U_{R(C_5)}}(X)=\{\phi,U,\{X_2,X_3,X_7\}\}=\tau_{U_{R(C)}}(X)$. Therefore, $Core=\{E,T,Cs\}$.

Observation

From both cases the core is found to be Experience, Technical Skill and Communication Skill which have close connection to the selection of a candidate for the software concern.

Example 6.2

Consider the following table giving information about six patients. Blood Pressure, Cholesterol and Chest Pain are the conditional attributes of the system, whereas Heart Problem is the decision attribute.

Patients	Blood Pressure	Cholesterol	Chest Pain	Heart Problem
P_1	High	High	*	Yes
P_2	*	High	Yes	Yes
P_3	Normal	*	No	No
P_4	High	High	*	Yes
P_5	*	Low	Yes	No
P_6	Normal	Normal	No	No

Table 6.2

The columns of the table represent the symptoms for heart problem and the rows represent the patients. The entries in the table are the attribute values. The incomplete information system is given by by (U, A) where $U = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ and $A = \{BP, Ch, CP, D\}$ which is divided into two subsets namely, the condition attributes $C = \{BP, Ch, CP\}$ and the decision attribute D.

Case 1: Let $X = \{P_1, P_2, P_4\}$ be the set of patients having heart problem. The equivalence classes corresponding to $C = \{BP, Ch, CP\}$ is given by $U / R(C) = \{\{P_1, P_2, P_4\}, \{P_3, P_6\}, \{P_5\}\}$. The upper approximation with respect to R is given by $U_{R(C)}(X) = \{P_1, P_2, P_4\}$. The upper rough topology with respect to R is given by $\tau_{U_{R(C)}}(X) = \{\phi, U, \{P_1, P_2, P_4\}\}$. If we remove the attribute 'Blood Pressure' from the set of condition attributes, then the equivalence

classes corresponding to $C_1=\{Ch,CP\}$ is given by $U/R(C_1)=\{\{P_1,P_2,P_4\},\{P_1,P_3,P_4\},\{P_3,P_6\},\{P_5\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_1)}(X)=\{P_1,P_2,P_3,P_4\}$ and the upper rough topology is given by $\tau_{U_{R(C_1)}}(X)=\{\phi,U,\{P_1,P_2,P_3,P_4\}\}\neq\tau_{U_{R(C)}}(X)$. If 'Cholesterol' is removed from the set of condition attributes, then the equivalence classes corresponding to $C_2=\{BP,CP\}$ is given by $U/R(C_2)=\{\{P_1,P_2,P_4,P_5\},\{P_3,P_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_2)}(X)=\{P_1,P_2,P_4,P_5\}$ and the upper rough topology is given by $\tau_{U_{R(C_2)}}(X)=\{\phi,U,\{P_1,P_2,P_4,P_5\}\}\neq\tau_{U_{R(C)}}(X)$. If 'Chest Pain' is removed from the set of condition attributes, then the equivalence classes corresponding to $C_3=\{BP,Ch\}$ is given by $U/R(C_3)=\{\{P_1,P_2,P_4\},\{P_2,P_3\},\{P_3,P_5\},\{P_3,P_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_3)}(X)=\{P_1,P_2,P_3,P_4\}$ and the upper rough topology is given by $\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{P_1,P_2,P_3,P_4\}\}\neq\tau_{U_{R(C)}}(X)$. Therefore, $Core=\{BP,CP,Ch\}$.

Case 2: Let $X=\{P_3,P_5,P_6\}$ be the set of patients not having heart problem. The equivalence classes corresponding to $C=\{BP,Ch,CP\}$ is given by $U/R(C)=\{\{P_1,P_2,P_4\},\{P_3,P_6\},\{P_5\}\}$. The upper approximation with respect to R is given by $U_{R(C)}(X)=\{P_3,P_5,P_6\}$. The upper rough topology with respect to R is given by $\tau_{U_{R(C)}}(X)=\{\phi,U,\{P_3,P_5,P_6\}\}$. If we remove the attribute 'Blood Pressure' from the set of condition attributes, then the equivalence classes corresponding to $C_1=\{Ch,CP\}$ is given by $U/R(C_1)=\{\{P_1,P_2,P_4\},\{P_1,P_3,P_4\},\{P_3,P_6\},\{P_5\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_1)}(X)=\{P_1,P_3,P_4,P_5,P_6\}$ and the upper rough topology is given by $\tau_{U_{R(C_1)}}(X)=\{\phi,U,\{P_1,P_3,P_4,P_5,P_6\}\}\neq\tau_{U_{R(C)}}(X)$. If 'Cholesterol' is removed from the set of condition attributes, then the equivalence classes corresponding to $C_2=\{BP,CP\}$ is given by $U/R(C_2)=\{\{P_1,P_2,P_4,P_5\},\{P_3,P_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_2)}(X)=\{P_1,P_2,P_3,P_4,P_5,P_6\}$ and the upper rough

topology is given by $\tau_{U_{R(C_2)}}(X)=\{\phi,U\}\neq\tau_{U_{R(C_1)}}(X)$. If 'Chest Pain' is removed from the set of condition attributes, then the equivalence classes corresponding to $C_3=\{BP,Ch\}$ is given by $U/R(C_3)=\{\{P_1,P_2,P_4\},\{P_2,P_5\},\{P_3,P_5\},\{P_5,P_6\}\}$. The upper approximation corresponding to this equivalence relation is given by $U_{R(C_3)}(X)=\{P_2,P_3,P_5,P_6\}$ and the upper rough topology is given by $\tau_{U_{R(C_3)}}(X)=\{\phi,U,\{P_2,P_3,P_5,P_6\}\}\neq\tau_{U_{R(C_1)}}(X)$. Therefore, $Core=\{BP,CP,Ch\}$.

Observation

From both cases we conclude that Blood Pressure, Cholesterol and Chest Pain are the symptoms which are closely connected with heart problem.

7. Conclusion

In this paper we have shown that the concept of upper rough topology has been applied to find the key factors of Measles, Flu and Heart Problem using the algorithm. We could find that Skin rash and Temperature are the deciding symptoms for Measles and Temperature is the key factor for Flu. Also Blood Pressure, Chest Pain and Cholesterol are the key symptoms which are closely connected to the Heart Problem. The key factors for the recruitment process of a software concern are found using the proposed algorithm.

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