Some Arithmetic Operations in Trapezoidal Fuzzy Numbers and Intuitionistic Trapezoidal Fuzzy Numbers

P. Jini Varghese [*1] and G. Michael Rosario[2]

[1]*Research Scholar, Jayaraj Annapackiam College for Women (Autonomous), Periyakulam, Theni, Tamilnadu, India*

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Department of Mathematics, Adi Shankara Institute Of Engineering And Technology, Kalady Ernakulam, Kerala, India.

[2]*Research center of Mathematics, Jayaraj Annapackiam College For Women (Autonomous), Periyakulam, Theni, Tamilnadu, India*

[1]jinijees@gmail.com

[2]tony87rio@gmail.com

Abstract--- **Comparison of the Trapezoidal Fuzzy number (TrFN) and Intuitionistic Trapezoidal Fuzzy Number (InTrFN) as in the formation of vector space. Membership function and non-membership function of intuitionistic trapezoidal fuzzy numbers are analyzed in the terms of (α,β)-cut method.**

Keywords---**Trapezoidal fuzzy numbers, trapezoidal intuitionistic fuzzy numbers, Trapezoidal Fuzzy vector space, membership and non-membership function.**

I. INTRODUCTION

The vague concept Fuzzy was introduced by Zadeh [1]. It has been researched depth in many fields and various theories and generalization have been appeared in different directions. Intuitionistic Fuzzy set (IFS) is one of the generalizations of Fuzzy Set Theory [2]. Intuitionistic Fuzzy set (IFS) is first introduced by Atanassov [3], [4]. The concept of IFS can be viewed as an appropriate approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In fuzzy sets only the degree of acceptance is considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [5]. The intuitionistic fuzzy sets are analyzed through two theorems in [6]. There are many studies about the inter-valued intuitionistic fuzzy sets and systems [7] and Correlation of intuitionistic fuzzy sets is explained in detail [8]. The relations between the fuzzy operators are defined over the intuitionistic fuzzy sets [9] and Cartesian product over intuitionistic fuzzy sets is explained in [10].Introduce positive and non-negative fuzzy numbers [11] and distance measures evaluation between intuitionistic fuzzy multisets [12]. With the help of triangular intuitionistic fuzzy numbers, the reliability evaluation calculated [13].Application of triangular fuzzy numbers are discussed [14].Arith metic operations are discussed [15].application of reliability discussed [16].

In this paper, a Trapezoidal fuzzy vector space is introduced and it is proved that set of all intuitionistic trapezoidal fuzzy numbers does not form a vector space. Also the membership and non-membership functions are analyzed.

II. SOME BASIC CONCEPTS

1) Definition: Trapeoidal Fuzzy Number: A fuzzy number is said to be trapezoidal fuzzy number with membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ 1 & for a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & for a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

And is denoted as $\tilde{A}$=(a_1, a_2, a_3, a_4).

2) Definition: Positive trapezoidal fuzzy number: A positive trapezoidal fuzzy number is a fuzzy number $\tilde{A}$ is defined as $\tilde{A} = (a_1, a_2, a_3, a_4)$,where $a_i > 0$,

$\forall\ i = 1,2,3,4.$

3) Definition: Negative trapezoidal fuzzy number: A negative trapezoidal fuzzy number is a fuzzy number $\tilde{A}$ is defined as $\tilde{A} = (a_1, a_2, a_3, a_4)$,where $a_i < 0$,

$\forall\ i = 1,2,3,4.$

4) Definition: α −cut of a trapezoidal fuzzy number: α −Cut of a trapezoidal fuzzy number is denoted as A_α and is defined as$A_\alpha = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3, (a_4 - a_3)\alpha + a_4]$.

5) Definition: (α, β) – cut

A set of (α, β) – cut, generated by an intuitionistic fuzzy set$\breve{A}$, where $(\alpha, \beta) \in [0,1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$\breve{A}_{\alpha,\beta} = \{(x, \mu_{\breve{A}}(x), \nu_{\breve{A}}(x)) : x \in X, \mu_{\breve{A}}(x) \ge \alpha, \nu_{\breve{A}}(x) \le \beta, \alpha, \beta \in [0,1]\}$,

denoted by$\breve{A}_{\alpha,\beta}$,as the crisp set of elements x which belongs to $\breve{A}$ at least to the degree α and which at most to the degreeβ.

6) Definition: Trapezoidal Intuitionistic Fuzzy Number

A Trapezoidal intuitionistic fuzzy number $\breve{A}$ is an intuitionistic fuzzy number in R with membership function and non-membership function as follows

$$\mu_{\breve{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ 1 & for a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & for a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

and

$$\nu_{\breve{A}}(x) = \begin{cases} \frac{a_1' - x}{a_2 - a_1'} & for\ a_1' \le x \le a_2 \\ 0 & for a_2 \le x \le a_3 \\ \frac{x - a_4'}{a_4' - a_3} & for a_3 \le x \le a_4' \\ 1 & otherwise \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$ and trapezoidal intuitionistic fuzzy number is denoted by $\breve{A}_{TrIFN} = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$.

7) Definition: Positive Trapezoidal Intuitionistic Fuzzy Number

A Positive Trapezoidal intuitionistic fuzzy number $\breve{A}$ is an intuitionistic fuzzy number with membership function and non-membership function with all a $_i$'s are positive follows

$$\mu_{\breve{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & for\ a_1 \le x \le a_2 \\ 1 & for a_2 \le x \le a_3 \\ \frac{a_4 - x}{a_4 - a_3} & for a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

And

$$\nu_{\breve{A}}(x) = \begin{cases} \frac{a_1' - x}{a_2 - a_1'} & for\ a_1' \le x \le a_2 \\ 0 & for a_2 \le x \le a_3 \\ \frac{x - a_4'}{a_4' - a_3} & for a_3 \le x \le a_4' \\ 1 & otherwise \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$ and it is denoted by $\breve{A}_{TrIFN} = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$.

8) Definition: Negative Trapezoidal Intuitionistic Fuzzy Number

A Negative Trapezoidal intuitionistic fuzzy number $\breve{A}$ is an intuitionistic fuzzy number in R with membership function and non-membership function as follows

$$\mu_{\breve{A}}(x) = \begin{cases} \frac{x + a_4}{-a_3 + a_4} & for - a_4 \le x \le -a_3 \\ 1 & for - a_3 \le x \le -a_2 \\ \frac{a_4 - x}{a_4 - a_3} & for - a_2 \le x \le -a_1 \\ 0 & otherwise \end{cases}$$

And

$$\nu_{\breve{A}}(x) = \begin{cases} \frac{a_4' - x}{a_{3+}a_4'} & for - a_4' \le x \le -a_3 \\ 0 & for - a_3 \le x \le -a_2 \\ \frac{x + a_1'}{-a_1' + a_2} & for - a_2 \le x \le -a_1' \\ 1 & otherwise \end{cases}$$

Where $-a_4' \le -a_4 \le -a_3 \le -a_2 \le -a_1 \le -a_1'$ and trapezoidal intuitionistic fuzzy number is denoted by $\breve{A}_{TrIFN} = (-a_4, -a_3, -a_2, -a_1; -a_4', -a_3, -a_2, -a_1')$.

III. Arithmetic Operations On Trapezoidal Fuzzy Numbers

Let $\breve{A} = (a_1, a_2, a_3, a_4)$ And $\breve{B} = (b_1, b_2, b_3, b_4)$ be trapezoidal fuzzy numbers then

Addition : $\breve{A} \oplus \breve{B} = (a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4)$
$= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

Subtraction : $\breve{A} \ominus \breve{B} = (a_1, a_2, a_3, a_4) - (b_1, b_2, b_3, b_4)$
$= (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$

Also, $1 - \breve{A} = 1 - (a_1, a_2, a_3, a_4)$
$= (1 - a_1, 1 - a_2, 1 - a_3, 1 - a_4)$

Multiplication : $\breve{A} \otimes \breve{B} = (a_1, a_2, a_3, a_4) \times (b_1, b_2, b_3, b_4)$
$= (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

Also, $k\breve{A} = k(a_1, a_2, a_3, a_4)$
$= (ka_1, ka_2, ka_3, ka_4)$

Here, k is any real number.

Division : $\breve{A} \oplus \breve{B} = (a_1, a_2, a_3, a_4) \div (b_1, b_2, b_3, b_4)$

$= (\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4})$ where $\forall\ b_i \neq 0$

IV. Vector Space

Let F is an arbitrary non-empty set of trapezoidal fuzzy number on which two operations addition and multiplication by scalars are defined. If the following axioms are satisfied by all numbers of F then, F is called a ***Trapezoidal fuzzy vector space***.

Take $\breve{A}$, $\breve{B}$ *and* $\breve{C}$ are trapezoidal fuzzy numbers and k, m be any real constants then,

$\breve{A} \oplus \breve{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$ is again a trapezoidal fuzzy number. So it is closed

$\breve{A} \oplus \breve{B} = (a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4)$

$= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
$= (b_1 + a_1, b_2 + a_2, b_3 + a_3, b_4 + a_4)$
$= (b_1, b_2, b_3, b_4) + (a_1, a_2, a_3, a_4)$
$= \breve{B} \oplus \breve{A}$ Commutativity is satisfied

$[\breve{A} \oplus \breve{B}] \oplus \breve{C} = [(a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4)] + (c_1, c_2, c_3, c_4)$

$= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) + (c_1, c_2, c_3, c_4)$

$= (a_1 + b_1 + c_1, a_2 + b_2 + c_2, a_3 + b_3 + c_3, a_4 + b_4 + c_4)$

$= (a_1, a_2, a_3, a_4) + (b_1 + c_1, b_2 + c_2, b_3 + c_3, b_4 + c_4)$
$= (a_1, a_2, a_3, a_4) + [(b_1, b_2, b_3, b_4)] + (c_1, c_2, c_3, c_4)]$
$= \breve{A} \oplus [\breve{B} \oplus \breve{C}]$ Associativity is satisfied

$0\oplus\breve{A} = 0+(a_1, a_2, a_3, a_4)$

$=(0 + a_1, 0 + a_2, 0 + a_3, 0 + a_4)$
$=(a_1 + 0, a_2 + 0, a_3 + 0, a_4 + 0)$
$=(a_1, a_2, a_3, a_4) + 0$
$=\breve{A}\oplus 0=\breve{A}$ 0 is called the zero vectors
$\breve{A}\oplus(-\breve{A})=(a_1, a_2, a_3, a_4) + (-a_1, -a_2, -a_3, -a_4)$

$=(a_1 - a_1, a_2 - a_2, a_3 - a_3, a_4 - a_4)$
$=(-a_1 + a_1, -a_2 + a_2, -a_3 + a_3, -a_4 + a_4)$
$=(-a_1, -a_2, -a_3, -a_4) + (a_1, a_2, a_3, a_4)$
$=(-\tilde{A})\oplus\breve{A}=0$ $(-\widetilde{A})$is called the additive inverse of $\breve{A}$

If k be any scalar then,

$$k\breve{A} = k(a_1, a_2, a_3, a_4)$$

$= (ka_1, ka_2, ka_3, ka_4)$
$k\breve{A}$, is also a trapezoidal fuzzy number

$k(\breve{A}\oplus\widetilde{B})=k[(a_1, a_2, a_3, a_4) + (b_1, b_2, b_3, b_4)]$

$=k[(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)]$

$=(ka_1 + kb_1, ka_2 + kb_2, ka_3 + kb_3, ka_4)$
$=(ka_1, ka_2, ka_3, ka_4) + (kb_1, kb_2, kb_3, kb_4)$
$=k(a_1, a_2, a_3, a_4) + k(b_1, b_2, b_3, b_4)$
$= k\breve{A}\oplus k\breve{B}$

$(k + m)\breve{A}=(k + m)(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$

$=((k + m)a_1, (k + m)a_2, (k + m)a_3, (k + m)a_4; (k + m)a_1', (k + m)a_2, (k + m)a_3, (k + m)a_4')$

$=\big((ka_1 + ma_1), (ka_2 + ma_2), (ka_3 + ma_3), (ka_4 + ma_4); (ka_1' + ma_1'), (k + m)a_2, (ka_3 + ma_3), (ka_4' + ma_4')\big)$

$= (ka_1, ka_2, ka_3, ka_4; ka_1', ka_2, ka_3, ka_4') + (ma_1, ma_2, ma_3, ma_4; ma_1', ma_2, ma_3, ma_4')$

$= k(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4') + m(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$

$=k\breve{A} + m\breve{A}$

$k(m\breve{A}) = k[m(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')]$

$=k(ma_1, ma_2, ma_3, ma_4; ma_1', ma_2, ma_3, ma_4')$

$=(kma_1, kma_2, kma_3, kma_4; kma_1', kma_2, kma_3, kma_4')$

$=km(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$

$=km(\breve{A})$

$1.\widetilde{A}=1.(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$

$=(1.a_1, 1.a_2, 1.a_3, 1.a_4; 1.a_1', 1.a_2, 1.a_3, 1.a_4')$

$=(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$

$=\breve{A}$

So, all the conditions of a vector space are satisfied. Therefore, the set of all Trapezoidal Fuzzy numbers form a vector space called ***Trapezoidal fuzzy vector space.***

V. OPERATIONS ON INTUITIONISTIC TRAPEZOIDAL FUZZY NUMBER

Let $\check{A}$ be Intuitionistic Trapezoidal Fuzzy Number, denoted as

$\check{A}_{TrIFN} = (a_1, a_2, a_3, a_4; a'_1, a_2, a_3, a'_4)$ With $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq a'_4$ and Negative Intuitionistic Trapezoidal Fuzzy Number is defined as $\check{A}_{TrIFN} = (-a_4, -a_3, -a_2, -a_1; -a'_4, -a_3, -a_2, -a'_1)$

Here $-a'_4 \leq -a_4 \leq -a_3 \leq -a_2 \leq -a_1 \leq -a'_1$

$\check{A}$+-$\check{A}$=$(a_1 - a_4, a_2 - a_3, a_3 - a_2, a_4 - a_1; a'_1 - a'_4, a_2 - a_3, a_3 - a_2, a'_4 - a'_1) \neq 0$

Therefore we cannot find an additive inverse of$\check{A}$. Here, property 5 failed.

Therefore the set of all Intuitionistic Trapezoidal Fuzzy Number does not form a Vector Space.

VI. APPLICATION OF ARITHMETIC OPERATIONS ON INTUITIONISTIC TRAPEOIDAL FUZZY NUMBER BASED ON THE (α, β)-CUT METHOD

If $\check{A}$ is a trapezoidal intuitionistic fuzzy number and (α, β) cut is given by $\check{A}_{\alpha,\beta} = \{(x, \mu_{\check{A}}(x), \nu_{\check{A}}(x)) : x \in X, \mu_{\check{A}}(x) \geq \alpha, \nu_{\check{A}}(x) \leq \beta, \alpha, \beta \in [0,1]\}$

In other form, (α, β)-cut is written as$\check{A}_{\alpha,\beta}$={$[(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3, (a_4 - a_3)\alpha + a_4]$; $[(a_2 - a'_1)\beta + a'_1, -(a_3 - a_2)\beta + a_3, (a'_4 - a_3)\beta + a'_4]$.} , α , β∈ [0,1], where α+β≤1

1) *Property:* If Intuitionistic Trapezoidal fuzzy number $\check{A}_{TrIFN} = (a_1, a_2, a_3, a_4; a'_1, a_2, a_3, a'_4)$ and $x = ka$ here k>0 then $\check{x} = k\check{A}$ is a Trapezoidal intuitionistic fuzzy number $(ka_1, ka_2, ka_3, ka_4; ka'_1, ka_2, ka_3, ka'_4)$
If $x = ka$ with k< 0 then $\check{x} = k\check{A}$ is a trapezoidal intuitionistic fuzzy number $(ka_4, ka_3, ka_2, ka_1; ka'_4, ka_3, ka_2, ka'_1)$

Proof:
Let k> 0 and consider the linear transformation $y = ka$. we can find the membership and non-membership function of ITrFN$\check{x} = k\check{a}$ using (α, β) −cut method.
α-cut of $\widetilde{A}$, $A_\alpha = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3, (a_4 - a_3)\alpha + a_4]$.
$\mu_{\check{A}}(x) \geq \alpha \Longrightarrow = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3, (a_4 - a_3)\alpha + a_4]$. For any α϶ [0, 1]
and x ∈ $[(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3, (a_4 - a_3)\alpha + a_4]$.
x=ka∈ $[(ka_2 - ka_1)\alpha + ka_1, -(ka_3 - ka_2)\alpha + ka_3, (ka_4 - ka_3)\alpha + ka_4]$.
Thus the membership function of $\check{x} = k\check{A}$ is given by

$$\mu_{\check{A}}(x) = \begin{cases} \dfrac{x - ka_1}{ka_2 - ka_1} & for\ ka_1 \leq x \leq ka_2 \\ 1 & for ka_2 \leq x \leq ka_3 \\ \dfrac{ka_4 - x}{ka_4 - ka_3} & for\ ka_3 \leq x \leq ka_4 \\ 0 & otherwise \end{cases}$$

Hence the rule is proved for membership function.

For, non-membership function, β-cut of $\check{A}$ is

$\nu_{\check{A}}(x) \leq \beta\ A_\alpha = [(a_2 - a_1')\beta + a_1', -(a_3 - a_2)\beta + a_3, (a_4' - a_3) + a_4']$ For any β∈ [0,1]

Ie , $x \in [(a_2 - a_1')\beta + a_1', -(a_3 - a_2)\beta + a_3, (a_4' - a_3)\beta + a_4']$.

So , $x = ka \in [(ka_2 - ka_1')\beta + ka_1', -(ka_3 - ka_2)\beta + ka_3, (ka_4' - ka_3)\beta + ka_4']$.

Thus the non-membership function of $\check{x} = k\check{A}$ is written as

$$\nu_{\check{A}}(x) = \begin{cases} \dfrac{ka_1' - x}{ka_2 - ka_1'} & for\ ka_1' \leq x \leq ka_2 \\ 0 & for ka_2 \leq x \leq ka_3 \\ \dfrac{x - ka_4'}{ka_4' - ka_3} & for ka_3 \leq x \leq ka_4' \\ 1 & otherwise \end{cases}$$

Hence the rule is proved for non-membership function.

Thus $\check{x} = k\check{A}$= $(ka_1, ka_2, ka_3, ka_4; ka_1', ka_2, ka_3, ka_4')$ is a TrIFN when k> 0

Similarly we can prove that $\check{x} = k\check{A}$, if $x = ka, k < 0$

Here we get the membership and non-membership function as follows

$$\mu_{\check{A}}(x) = \begin{cases} \dfrac{x - ka_4}{ka_3 - ka_4} & for\ ka_4 \leq x \leq ka_3 \\ 1 & for ka_3 \leq x \leq ka_2 \\ \dfrac{ka_1 - x}{ka_1 - ka_2} & for\ ka_2 \leq x \leq ka_1 \\ 0 & otherwise \end{cases}$$

$$\nu_{\check{A}}(x) = \begin{cases} \dfrac{ka_4' - x}{ka_3 - ka_4'} & for\ ka_4' \leq x \leq ka_3 \\ 0 & for ka_3 \leq x \leq ka_2 \\ \dfrac{x - ka_1'}{ka_1' - ka_2} & for ka_2 \leq x \leq ka_1' \\ 1 & otherwise \end{cases}$$

Thus $\check{x} = k\check{A}$=$(ka_4, ka_3, ka_2, ka_1; ka_4^1, ka_3, ka_2, ka_1')$is a TrIFN when k< 0

2) *Corollory:* If $\check{A}$ is a TrIFN , where k and m are real numbers and the linear transformation $x = (k + m)a$, then $\check{x} = (k + m)\check{A}$ is also a TrIFN

Proof:

If k and m are positive real numbers then k+m is a positive real number ,ie k> 0 (proof is same as property 1 (a))
If k and m are negative real numbers then k+m is negative real number, ie $k < 0$ (proof is same as property 1 (b))
If k and m are opposite in sign, then k+m is either positive or negative. If k+m are positive then the property (a) or property (b) can be applied.

3) *Property:* If $\widetilde{A}\ and\ \widetilde{B}$ are two trapezoidal intuitionistic fuzzy number, then $\widetilde{A} + \widetilde{B}$ is also a trapezoidal intuitionistic fuzzy number.

Proof:

Let $\check{A} = (a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$ And

$\breve{B} = (b_1, b_2, b_3, b_4; b_1', b_2, b_3, b_4')$Are two TrIFN with the transformation z=x+y

then the α-cut of $\widetilde{A}, A_\alpha = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3, (a_4 - a_3)\alpha + a_4].\forall\alpha\epsilon[0,1]$

ie, $x \in [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3, (a_4 - a_3)\alpha + a_4]$ And

$\alpha -$ cut of $\widetilde{B}, B_\alpha = [(b_2 - b_1)\alpha + b_1, -(b_3 - b_2)\alpha + b_3, (b_4 - b_3)\alpha + b_4]$.

ie, $y \in [(b_2 - b_1)\alpha + b_1, -(b_3 - b_2)\alpha + b_3, (b_4 - b_3)\alpha + b_4]$.

so $z = x + y \in [(a_2 - a_1 + b_2 - b_1)\alpha + a_1 + b_1, -(a_3 - a_2 + b_3 - b_2)\alpha + a_3 + b_3, (a_4 - a_3 + b_4 - b_3)\alpha + a_4 + b_4]$.

ie,

$\alpha - cut\ of\ \widetilde{A} + \widetilde{B} = [(a_2 - a_1 + b_2 - b_1)\alpha + a_1 + b_1, -(a_3 - a_2 + b_3 - b_2)\alpha + a_3 + b_3, (a_4 - a_3 + b_4 - b_3)\alpha + a_4 + b_4]$.

So the membership function of $\breve{Z} = \breve{A} + \breve{B}$ is given by

$$\mu_{\breve{A}+\breve{B}}(z) = \begin{cases} \dfrac{z - (a_1 + b_1)}{a_2 - a_1 + b_2 - b_1} & for\ a_1 + b_1 \le x \le a_2 + b_2 \\ 1 & for a_2 + b_2 \le x \le a_3 + b_3 \\ \dfrac{a_4 + b_4 - z}{a_4 - a_3 + b_4 - b_3} & for a_3 + b_3 \le x \le a_4 + b_4 \\ 0 & otherwise \end{cases}$$

Hence $\breve{A} + \breve{B}$ is a TrIFN for membership function.

For non-membership function

β -cut of $\widetilde{A}, A_\beta = [(a_2 - a_1')\beta + a_1', -(a_3 - a_2)\beta + a_3, (a_4' - a_3)\beta + a_4'].\forall\beta \in [0,1]$

Ie, $x \in [(a_2 - a_1')\beta + a_1', -(a_3 - a_2)\beta + a_3, (a_4' - a_3)\beta + a_4']$

$\beta -$ cut of $\widetilde{B}, B_\beta = [(b_2 - b_1')\beta + b_1', -(b_3 - b_2)\beta + b_3, (b_4' - b_3)\beta + b_4'].\ \forall\beta \in [0,1]$

ie, $y \in [(b_2 - b_1')\beta + b_1', -(b_3 - b_2)\beta + b_3, (b_4' - b_3)\beta + b_4']$.

So $z = x + y \in (a_2 - a_1' + b_2 - b_1')\beta + a_1' + b_1', -(a_3 - a_2 + b_3 - b_2)\beta + a_3 + b_3, (a_4' - a_3 + a_4' - b_3)\beta + a_4' + b_4']$.

ie,$\beta - cut\ of\ \widetilde{A} + \widetilde{B} = [(a_2 - a_1' + b_2 - b_1')\beta + a_1' + b_1', -(a_3 - a_2 + b_3 - b_2)\beta + a_3 + b_3, (a_4' - a_3 + a_4' - b_3)\beta + a_4' + b_4']$.

So the non- membership function of $\breve{Z} = \breve{A} + \breve{B}$ is

$$\nu_{\breve{A}+\breve{B}}(z) = \begin{cases} \dfrac{a_1' + b_1' - z}{a_2 - a_1' + b_2 - a_1'} & for\ a_1' + b_1' \le x \le a_2 + b_2 \\ 0 & for a_2 + b_2 \le x \le a_3 + b_3 \\ \dfrac{z - (a_4' + b_4')}{a_4' - a_3 + b_4' - b_3)} & for a_3 + b_3 \le x \le a_4' + b_4' \\ 1 & otherwise \end{cases}$$

Hence the addition rule is proved for non-membership function.

Thus $\breve{A}\oplus\breve{B}$=$(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; a_1' + b_1', a_2 + b_2, a_3 + b_3, a_4' + b_4')$ is a TrIFN

VII. Numerical Evaluation

Let $\breve{A} = (1,2,3,4; 0.5,2,3,5)$ and k$\breve{A}$=$(k, 2k, 3k, 4k; 05k, 2k, 3k, 5k)$ where k is a real number then the membership and non-membership function are as follows:

$$\mu_{\breve{A}+\breve{B}}(x) = \begin{cases} \dfrac{x-k}{k} & for\ k \le x \le 2k \\ 1 & for 2k \le x \le 3k \\ \dfrac{4k-x}{k} & for 3k \le x \le 4k \\ 0 & otherwise \end{cases}$$

$$\nu_{\breve{A}+\breve{B}}(x) = \begin{cases} \dfrac{1.5-x}{3.5} & for\ 1.5 \le x \le 5 \\ 0 & for 5 \le x \le 7 \\ \dfrac{x-11}{4} & for 7 \le x \le 11 \\ 1 & otherwise \end{cases}$$

Let $\breve{A}$=(1,2,3,4; 0.5,2,3,5) and $\breve{B}$=(2,3,4,5; 1,2,3,6) are two TrIFN then their sum $\breve{A}+\breve{B}$=(3,5,7,9; 1.5,4,6,11) with the membership and non-membership function are as follows:

$$\mu_{\breve{A}}(x) = \begin{cases} \dfrac{x-3}{2} & for\ 3 \le x \le 5 \\ 1 & for 5 \le x \le 7 \\ \dfrac{9-x}{2} & for 7 \le x \le 9 \\ 0 & otherwise \end{cases}$$

$$\nu_{\breve{A}+\breve{B}}(x) = \begin{cases} \dfrac{0.5k-x}{1.5k} & for\ 0.5k \le x \le 2k \\ 0 & for 2k \le x \le 3k \\ \dfrac{x-5k}{2k} & for 3k \le x \le 5k \\ 1 & otherwise \end{cases}$$

VIII. Conclusion

In this paper, definition of positive trapezoidal intuitionistic fuzzy number and negative trapezoidal intuitionistic fuzzy number are introduced. The set of all trapezoidal fuzzy numbers form a vector space, but the set of all trapezoidal intuitionistic fuzzy numbers does not form a vector space are proved. Membership and non-membership function are analyzed using the (α,β)- cut method. In future, will try to calculate the reliability evaluation using trapezoidal intuitionistic fuzzy numbers with the help of trapezoidal fuzzy vector space.

References

[1] L A Zadeh, "*Fuzzy sets ,information and control"* , 8, pp. 338-353, 1965.
[2] K T Atanassov, *"Intuitionistic Fuzzy sets", VII ITKR's session, Sofia(deposed in central science-Technical library of Bulgarian Academy of Science*,1697/84), *(in Bulgarian*, 1983*).*
[3] K T Atanassov, *"Intuitionistic Fuzzy sets, Fuzzy sets and Systems"* 20, pp. 87-96, 1986.
[4] K T Atanassov, *"more Intuitionistic Fuzzy sets, Fuzzy sets and Systems"*, 33(1), pp. 37-46, 1989.
[5] K T Atanassov, "*Intuitionistic Fuzzy sets", Physica-Verlag, Heidelberg, New York*, 1999.
[6] K T Atanassov, "*Two theorems for Intuitionistic Fuzzy sets, Fuzzy sets and Systems"*,110, pp. 267-269, 2000.
[7] K T Atanassov, and G Gargov, "*Inter-valued Intuitionistic Fuzzy sets, Fuzzy sets and Systems"*,31(3), pp. 343-349, 1989.
[8] T Gerstenkorn, and J Manko, "*Correlation of intuitionistic fuzzy sets, Fuzzy sets and systems"*,44, pp. 39-43, 1991.
[9] D.Stoyanova, and K T Atanassov, "*Relation between operators defined over intuitionistic fuzzy sets", IM-MFAIS, Sofia, Bulgaria*. 1, pp. 46-49, 1990.
[10] D. Stoyanova, "*More on Cartesian product over intuitionistic fuzzy sets", BUSEFAL* 54, pp. 9-13, 1993.

[11] Hadi Nasseri, "*Fuzzy Numbers: Positive and Non-negative", International Mathematical Forum*, 3, no.36, pp. 1777-1780, 2008.
[12] P A Ejegwa, O P Edibo, "*Some distance measures between intuitionistic Fuzzy multisets" (IFMSS), 'International journal of scientific and Technology',Research volume* 3, *issue* 4, *April*, 2014.
[13] G.S. Mahapatra, and T.K. Roy, "*Reliability Evaluation using Triangular Intuitionistic Fuzzy Numbers Arithmetic Operations*",'*Proceedings of World Academy of Science, Engineering and Technology, Malaysia'*, 38, pp. 587-585, 2009.
[14] A K Shaw, and T K Roy, "*Some arithmetic operations on Triangular Intuitionistic Fuzzy Number and its application on Reliability Evaluation",'International Journal of Fuzzy mathematics and Systems', Volume* 2, *Number* 4, pp. 363-382, 2012.
[15] J. Vahidi, and S Rezvani, "*Arithmetic operations on Trapezoidal Fuzzy numbers", 'Journal nonlinear Analysis and application' volume*: 3, *Article jnna-*001, 2013.
[16] G. Michael Rosario, "*Intuitionistic Fuzzy Equations and its Application on Reliability Evaluation", 'International Journal of Advanced Technology Engineering and Science', Volume* 03, *Number*03, *March*, 2015.