

A FUZZY INVENTORY MODEL WITH ALLOWABLE SHORTAGE USING DIFFERENT FUZZY NUMBERS

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Abstract

In this paper, we investigate a fuzzy inventory model with allowable shortage which is completely backlogged. We fuzzify the carrying cost, backorder cost and ordering cost using triangular, trapezoidal, pentagonal fuzzy numbers to obtain the fuzzy total cost. Signed distance method is used for defuzzification to estimate the total cost. Numerical analysis is carried out and the results obtained from different fuzzy number are compared to determine the minimum total cost.

Keywords: Defuzzification, EOQ, Carrying cost, Backorder cost, Ordering cost, Fuzzy Numbers, Signed distance method

1. Introduction

In many production systems, inventory control decisions play a pivotal role in the manufacturing process. Many attempts are done to strike a balance between too much inventory which causes excessive holding costs and too little inventory, which leads to stock outs and poor customer service. As in the more commonly studied discrete production case, the goal is to minimize inventory related average costs over the long term.

The fuzzy set theory in inventory modelling is the closest possible approach to reality, as reality is not exact and can only be calculated to some extent. Same way, fuzzy theory helps one to incorporate unpredictability in the design of the model, thus bringing it closure to reality.

Kazemi et al. [1] studied an inventory with backorders with fuzzy parameters and decision variables. Gupta et al [2] introduced fuzzy Arithmetic Theory and Applications. H.J. Zimmerman[3] tried to use fuzzy sets in operational research. J.S. Yao and H.M. Lee[9] developed a fuzzy inventory model by considering backorder as a trapezoidal fuzzy number. Parvathi et al.[4] investigated a deterministic inventory model with allowable shortage. Chang [5] discussed the fuzzy production inventory model for fuzzify the product quantity as triangular fuzzy number. Wu and Yao[6] fuzzified the order quantity and shortage quantity into triangular fuzzy numbers in a inventory model with backorder and they obtained the membership function of the fuzzy cost. Chag Wang [7] applied trapezoidal fuzzy number to obtain the total cost without backorder.

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Petrovic. R., et al., [8] devolved trapezoidal fuzzy number to evaluate the total cost with backorder. Harish Nagar and Priyanka Surana [10] proposed a fuzzy inventory model for deteriorating items with fluctuating demand and using inventory parameters as pentagonal fuzzy numbers. J. K. Syed and L. A. Aziz [11] applied signed distance method to fuzzy inventory model without shortages. Urgeletti [12] referred EOQ model in fuzzy sense and used triangular fuzzy number. D. Dutta and Pavan Kumar [14] published several papers in the area of fuzzy inventory with or without shortages. D. Dutta and Pavan Kumar [13] applied an optimal policy for an inventory model without shortages considering fuzziness in demand, holding cost and ordering cost. Chang et al [15] considered the backorder inventory problem with fuzzy backorder such that the backorder quantity is a triangular fuzzy number. Jershan Chiang et al. [16] applied fuzzy Inventory with backorder and defuzzified by signed distance method. P. K. De. Apurva Rawat [17] EOQ model has been prepared without shortage cost by using triangular fuzzy number, and then the total cost has been computed by using signed-distance method.

In this paper, a fuzzy inventory model with shortage and constant demand is considered. In order to minimize the average total cost, carrying cost, backorder cost, and ordering cost are fuzzified using triangular, trapezoidal and pentagonal fuzzy numbers. For defuzzification the signed distance method is used. Further, the sensitive analysis help to find which one of the fuzzy numbers provide the minimum fuzzy total cost and numerical examples are illustrated.

In section 2 some important definitions and preliminaries are presented. In section 3 the notations and assumption for an inventory model under study are described. In section 4, we begin our formal study of crisp inventory model with shortage and constant demand. In section 5, the mathematical model for inventory in fuzzy environment is developed and numerical examples are given in section 6.

2. Definition and Preliminaries

Definition 2.1: (Fuzzy point)

Let \tilde{a} be a fuzzy set on $R = (-\infty, \infty)$. It is called a fuzzy point if its membership function is

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{if } x \neq a \end{cases}$$

Definition 2.2: (Level α fuzzy interval)

Let $[a, b; \alpha]$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a level α fuzzy interval, $0 \leq \alpha \leq 1, a < b$, if its membership function is

$$\mu_{[a,b;\alpha]}(x) = \begin{cases} \alpha, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.3: (Triangular fuzzy number)

Let $\tilde{A} = (p, q, r), p < q < r$, be a fuzzy set on $R = (-\infty, \infty)$. It is called a triangular fuzzy number, if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-p}{q-p}, & \text{if } p \leq x \leq q \\ \frac{r-x}{r-q}, & \text{if } q \leq x \leq r \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.4: (Signed distance for triangular fuzzy number)[17]

Let $A = (a, b, c)$ be a triangular fuzzy number and the signed distance of A measured from \tilde{O} is defined by

$$d(A, \tilde{O}) = \frac{1}{4}(a + 2b + c)$$

Definition 2.5: (Trapezoidal fuzzy number)

A trapezoidal fuzzy number $\tilde{A} = (p, q, r, s), p < q < r < s$, is represented with membership function $\mu_{\tilde{A}}(x)$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \frac{x-p}{q-p}, & \text{if } p \leq x \leq q \\ 1, & \text{if } q \leq x \leq r \\ R(x) = \frac{s-x}{s-q}, & \text{if } r \leq x \leq s \\ 0, & \text{otherwise} \end{cases}$$

The α -cut of $\tilde{A} = (p, q, r, s), 0 \leq \alpha \leq 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

Where $A_L(\alpha) = a + (b-a)\alpha$ and $A_R(\alpha) = d - (d-c)\alpha$ are the left and right endpoints of $A(\alpha)$.

Definition 2.6: (Signed distance method for trapezoidal fuzzy number)[16]

Let \tilde{D} define the signed distance of \tilde{D} measured from \tilde{O} as

$$d(\tilde{D}, \tilde{O}) = \frac{1}{2} \int_0^1 [D_L(\alpha) + D_R(\alpha)] d\alpha$$

Definition 2.7: (Pentagonal fuzzy number)

A pentagonal fuzzy number $\tilde{A} = (a, b, c, d, e)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}} = \left\{ \begin{array}{l} L_1(x) = \frac{x-a}{b-a}, a \leq x \leq b \\ L_2(x) = \frac{x-b}{c-b}, b \leq x \leq c \\ 1, x = c \\ R_1(x) = \frac{d-x}{d-c}, c \leq x \leq d \\ R_2(x) = \frac{e-x}{e-d}, d \leq x \leq e \\ 0, otherwise \end{array} \right.$$

The α -cut of $\tilde{A} = (a, b, c, d, e)$, $0 \leq \alpha \leq 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

Where $A_{L_1}(\alpha) = a + (b-a)\alpha = L_1^{-1}(\alpha)$ and $A_{L_2}(\alpha) = b + (c-b)\alpha = L_2^{-1}(\alpha)$

And $A_{R_1}(\alpha) = d - (d-c)\alpha = R_1^{-1}(\alpha)$ and $A_{R_2}(\alpha) = e - (e-d)\alpha = R_2^{-1}(\alpha)$

Definition 2.8: (Signed distance for pentagonal fuzzy number)

Let \tilde{D} define the signed distance of \tilde{D} measured from \tilde{O} as

$$d(\tilde{D}, \tilde{O}) = \frac{1}{12}(a + 3b + 4c + 3d + e)$$

3. Model Description

3.1. Notations

For a crisp inventory model with backorder, we use the following notations and related parameters.

- T - length of plan (days).
- a - storing cost for one unit per day.
- b - backorder cost for one unit per day.
- c - cost of placing an order.
- r - total demand over the planning time period $[0, T]$.
- t_q - length of a cycle.
- q - order quantity per cycle.
- s - shortage quantity per cycle.

3.2. Assumptions

In this paper, the following assumptions are considered

- Total demand is considered as a constant
- Time plan is constant
- Shortage cost is allowed
- To fuzzify holding cost, ordering cost and shortage cost only

4. Proposed Inventory Model in Crisp Sense

The crisp total cost on the planning period $[0, T]$ is given by

$$F(q, s) = \left[at_1 \frac{q-s}{2} + bt_2 \frac{s}{2} + c \right] \frac{r}{q}$$

$$= \frac{a(q-s)^2 T}{2q} + \frac{bs^2 T}{2q} + \frac{cr}{q}, (0 < s < q).$$

Here t_1 denotes ordering time of length

And t_2 denotes shortage time of length so that

$$\frac{q-s}{t_1} = \frac{q}{t_q} = \frac{s}{t_2} = \frac{r}{T}$$

The crisp total cost on the planning period $[0, T]$ is given by

$$F(q, s) = \left[at_1 \frac{q-s}{t_q} + bt_2 \frac{s}{2} + c \right] \frac{r}{q}$$

$$= \frac{a(q-s)^2 T}{2q} + \frac{bs^2 T}{2q} + \frac{cr}{q}, (0 < s < q). \dots\dots\dots (4.1)$$

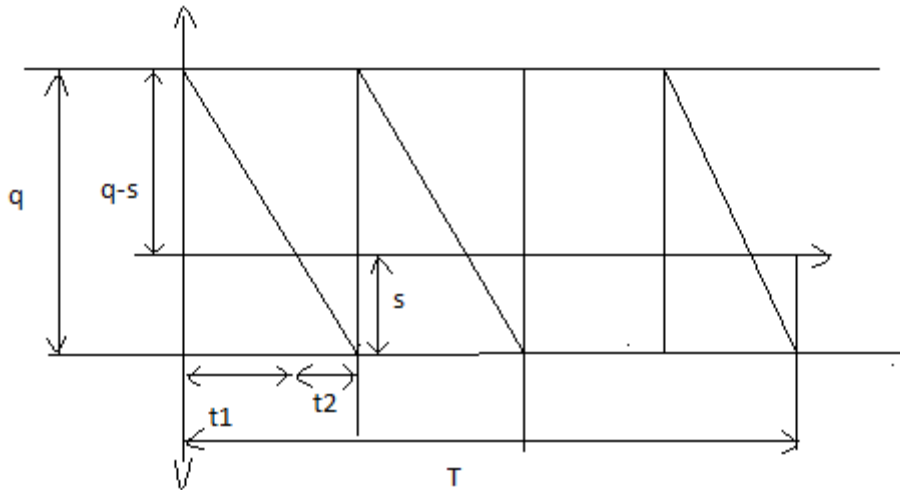
The crisp optimal solutions are [9]

$$\text{Optimal order quantity } q_* = \sqrt{\frac{2(a+b)cr}{abT}} \dots\dots\dots (4.2)$$

$$\text{Optimal backorder quantity } s_* = \sqrt{\frac{2acr}{b(a+b)T}} \dots\dots\dots (4.3)$$

$$\text{Minimal total cost } F(q_*, s_*) = \sqrt{\frac{2abcrT}{a+b}} \dots\dots\dots (4.4)$$

Diagrammatic representation



5. Proposed Inventory Model in Fuzzy Environment

5.1. Triangular fuzzy total cost

In this model, we consider the ordering cost, holding cost and shortage cost as imprecise and are represented as triangular fuzzy numbers.

Let us denote the above mentioned costs respectively using LR-form as:

$$\tilde{a} = (a_1, a_2, a_3), \tilde{b} = (b_1, b_2, b_3), \tilde{c} = (c_1, c_2, c_3)$$

$$FTC = \frac{\tilde{a}(q-s)^2 T}{2q} + \frac{\tilde{b}s^2 T}{2q} + \frac{\tilde{c}r}{q}$$

$$FTC = \frac{(a_1, a_2, a_3) \otimes (q-s)^2 \otimes T}{2q} \oplus \frac{(b_1, b_2, b_3) \otimes s^2 \otimes T}{2q} \oplus \frac{(c_1, c_2, c_3) \otimes r}{q}$$

$$= \left[\frac{a_1(q-s)^2 T}{2q}, \frac{a_2(q-s)^2 T}{2q}, \frac{a_3(q-s)^2 T}{2q} \right] + \left[\frac{b_1 s^2 T}{2q}, \frac{b_2 s^2 T}{2q}, \frac{b_3 s^2 T}{2q} \right] + \left[\frac{c_1 r}{q}, \frac{c_2 r}{q}, \frac{c_3 r}{q} \right]$$

$$= \left[\frac{a_1(q-s)^2 T}{2q} + \frac{b_1 s^2 T}{2q} + \frac{c_1 r}{q} \right], \left[\frac{a_2(q-s)^2 T}{2q} + \frac{b_2 s^2 T}{2q} + \frac{c_2 r}{q} \right], \left[\frac{a_3(q-s)^2 T}{2q} + \frac{b_3 s^2 T}{2q} + \frac{c_3 r}{q} \right]$$

The signed distance method for defuzzification is given by the following equation,

$$d(F_{(q,s)}(\tilde{a}, \tilde{b}, \tilde{c}), 0) = \frac{1}{2}(a + 2b + c)$$

Applying defuzzification

$$\begin{aligned}
 d(F_{(q,s)}(\tilde{a}, \tilde{b}, \tilde{c}), 0) &= \frac{1}{2} \left[\frac{a_1(q-s)^2 T}{2q} + \frac{b_1 s^2 T}{2q} + \frac{c_1 r}{q} + 2 \left(\frac{a_2(q-s)^2 T}{2q} + \frac{b_2 s^2 T}{2q} + \frac{c_2 r}{q} \right) \right. \\
 &\quad \left. + \frac{a_3(q-s)^2 T}{2q} + \frac{b_3 s^2 T}{2q} + \frac{c_3 r}{q} \right] \\
 &= \frac{1}{2} \left[\frac{(q-s)^2 T}{2q} (a_1 + 2a_2 + a_3) + \frac{s^2 T}{2q} (b_1 + 2b_2 + b_3) + \frac{r(c_1 + 2c_2 + c_3)}{q} \right] \\
 &= F_d(q, s) \quad \text{----- (5.1.1)}
 \end{aligned}$$

Computation of minimum value of q^* and s^* ,

$F_d(q, s)$ is minimum at $F_d'(q, s) = 0$, where $F_d'' = 0$ is positive

$$\begin{aligned}
 \Rightarrow F_d'(q, s) &= \frac{1}{2} \left[\frac{T}{2} (a_1 + 2a_2 + a_3) \left(1 - \frac{s^2}{q^2}\right) - \frac{s^2 T}{2q^2} (b_1 + 2b_2 + b_3) - \frac{r}{q^2} (c_1 + 2c_2 + c_3) \right] = 0 \\
 \frac{T}{2} (a_1 + 2a_2 + a_3) \left(1 - \frac{s^2}{q^2}\right) - \frac{s^2 T}{2q^2} (b_1 + 2b_2 + b_3) - \frac{r}{q^2} (c_1 + 2c_2 + c_3) &= 0
 \end{aligned}$$

After simplification we get,

$$q^* = \sqrt{\frac{2r(c_1 + c_2 + c_3)(a_1 + 2a_2 + a_3 + b_1 + 2b_2 + b_3)}{T(a_1 + 2a_2 + a_3)(b_1 + 2b_2 + b_3)}} \quad \text{----- (5.1.2)}$$

and

$$s^* = \sqrt{\frac{2r(c_1 + c_2 + c_3)(a_1 + 2a_2 + a_3)}{T(b_1 + 2b_2 + b_3)(a_1 + 2a_2 + a_3 + b_1 + 2b_2 + b_3)}} \quad \text{----- (5.1.3)}$$

This shows that $F_d(q, s)$ is minimum at q_d^* and s_d^*

5.2. Trapezoidal fuzzy total cost

In this model, we consider the ordering cost, holding cost and shortage cost as imprecise and are represented as Trapezoidal fuzzy numbers.

Let us denote the above mentioned costs respectively using LR-form as:

$$\tilde{a} = (a_1, a_2, a_3, a_4), \quad \tilde{b} = (b_1, b_2, b_3, b_4) \quad \text{and} \quad \tilde{c} = (c_1, c_2, c_3, c_4)$$

$$FTC = \frac{\tilde{a}(q-s)^2 T}{2q} + \frac{\tilde{b}s^2 T}{2q} + \frac{\tilde{c}r}{q}$$

$$\begin{aligned}
 FTC &= \left[\frac{(a_1, a_2, a_3, a_4) \otimes (q-s)^2 \otimes T}{2 \otimes q}, \frac{(b_1, b_2, b_3, b_4) \otimes s^2 \otimes T}{2 \otimes q}, \right. \\
 &\quad \left. \frac{(c_1, c_2, c_3, c_4) \otimes r}{q} \right] \\
 &= \left[\frac{a_1 \otimes (q-s)^2 \otimes T}{2 \otimes q}, \frac{a_2 \otimes (q-s)^2 \otimes T}{2 \otimes q}, \right. \\
 &\quad \left. \frac{a_3 \otimes (q-s)^2 \otimes T}{2 \otimes q}, \frac{a_4 \otimes (q-s)^2 \otimes T}{2 \otimes q} \right] \\
 &\quad \oplus \left[\frac{b_1 \otimes s^2 \otimes T}{2 \otimes q}, \frac{b_2 \otimes s^2 \otimes T}{2 \otimes q}, \frac{b_3 \otimes s^2 \otimes T}{2 \otimes q}, \frac{b_4 \otimes s^2 \otimes T}{2 \otimes q} \right] \\
 &\quad \oplus \left[\frac{c_1 \otimes r}{q}, \frac{c_2 \otimes r}{q}, \frac{c_3 \otimes r}{q}, \frac{c_4 \otimes r}{q} \right] \\
 &= \left[\frac{a_1(q-s)^2 T}{2q} + \frac{b_1 s^2 T}{2q} + \frac{c_1 r}{q}, \frac{a_2(q-s)^2 T}{2q} + \frac{b_2 s^2 T}{2q} + \frac{c_2 r}{q}, \right. \\
 &\quad \left. \frac{a_3(q-s)^2 T}{2q} + \frac{b_3 s^2 T}{2q} + \frac{c_3 r}{q}, \frac{a_4(q-s)^2 T}{2q} + \frac{b_4 s^2 T}{2q} + \frac{c_4 r}{q} \right]
 \end{aligned}$$

The signed distance method for defuzzification is given by the following equation,

$$d(F_{(q,s)}(\tilde{a}, \tilde{b}, \tilde{c}), 0) = \frac{1}{2} \int_0^1 [F_{(q,s)_L}(\alpha) + F_{(q,s)_R}(\alpha)] d\alpha$$

Applying defuzzification

$$\begin{aligned}
 d(F_{(q,s)}(\tilde{a}, \tilde{b}, \tilde{c}), 0) &= \frac{1}{4} \left[\frac{(q-s)^2 T}{2q} (a_1 + a_2 + a_3 + a_4) + \frac{s^2 T}{2q} (b_1 + b_2 + b_3 + b_4) \right. \\
 &\quad \left. + \frac{r}{q} (c_1 + c_2 + c_3 + c_4) \right] \\
 &= F_d(q, s) \quad \text{----- (5.2.1)}
 \end{aligned}$$

$$F_{(q,s)} 'd = \frac{1}{4} \left[\frac{T}{2} (a_1 + a_2 + a_3 + a_4) \left(1 - \frac{s^2}{q^2}\right) - \frac{s^2 T}{2q^2} (b_1 + b_2 + b_3 + b_4) - \frac{r}{q^2} (c_1 + c_2 + c_3 + c_4) \right] = 0$$

After simplification we get,

$$q^* = \sqrt{\frac{2r(a_1 + a_2 + a_3 + a_4 + b_1 + b_2 + b_3 + b_4)(c_1 + c_2 + c_3 + c_4)}{T(a_1 + a_2 + a_3 + a_4)(b_1 + b_2 + b_3 + b_4)}} \text{----- (5.2.2)}$$

and

$$s^* = \sqrt{\frac{2r(c_1 + c_2 + c_3 + c_4)(a_1 + a_2 + a_3 + a_4)}{T(b_1 + b_2 + b_3 + b_4)(a_1 + a_2 + a_3 + a_4 + b_1 + b_2 + b_3 + b_4)}} \text{----- (5.2.3)}$$

This shows that $F_d(q, s)$ is minimum at q_d^* and s_d^*

5.3. Pentagonal fuzzy total cost

In this model, we consider the ordering cost, holding cost and shortage cost as imprecise and are represented as Pentagonal fuzzy numbers.

Let us denote the above mentioned costs respectively using LR-form as:

$$\tilde{a} = (a_1, a_2, a_3, a_4, a_5), \tilde{b} = (b_1, b_2, b_3, b_4, b_5) \text{ and } \tilde{c} = (c_1, c_2, c_3, c_4, c_5)$$

$$FTC = \frac{\tilde{a}(q-s)^2 T}{2q} + \frac{\tilde{b}s^2 T}{2q} + \frac{\tilde{c}r}{q}$$

$$FTC = \left[\frac{(a_1, a_2, a_3, a_4, a_5) \otimes (q-s)^2 \otimes T}{2 \otimes q}, \frac{(b_1, b_2, b_3, b_4, b_5) \otimes s^2 \otimes T}{2 \otimes q}, \frac{(c_1, c_2, c_3, c_4, c_5) \otimes r}{q} \right]$$

$$= \left[\frac{a_1(q-s)^2 T}{2q} + \frac{b_1 s^2 T}{2q} + \frac{c_1 r}{q}, \frac{a_2(q-s)^2 T}{2q} + \frac{b_2 s^2 T}{2q} + \frac{c_2 r}{q}, \frac{a_3(q-s)^2 T}{2q} + \frac{b_3 s^2 T}{2q} + \frac{c_3 r}{q}, \frac{a_4(q-s)^2 T}{2q} + \frac{b_4 s^2 T}{2q} + \frac{c_4 r}{q}, \frac{a_5(q-s)^2 T}{2q} + \frac{b_5 s^2 T}{2q} + \frac{c_5 r}{q} \right]$$

The signed distance method for defuzzification is given by the following equation

$$d(F_{(q,s)}(\alpha)) = \frac{a + 3b + 4c + 3d + e}{12}$$

Applying defuzzification,

$$d(F_{(q,s)}(\tilde{a}, \tilde{b}, \tilde{c}), 0) = \frac{1}{12} \left[\begin{array}{l} \frac{(q-s)^2 T}{2q} (a_1 + 3a_2 + 4a_3 + 3a_4 + a_5) + \\ \frac{s^2 T}{2q} (b_1 + 3b_2 + 4b_3 + 3b_4 + b_5) + \\ \frac{r}{q} (c_1 + 3c_2 + 4c_3 + 3c_4 + c_5) \end{array} \right]$$

$$= F_q(q, s) \quad \text{----- (5.3.1)}$$

$$F_{(q,s)}' d = \frac{1}{2} \left[\begin{array}{l} \frac{T}{2} (a_1 + 3a_2 + 4a_3 + 3a_4 + a_5) \left(1 - \frac{s^2}{q^2}\right) \\ - \frac{s^2 T}{2q^2} (b_1 + 3b_2 + 4b_3 + 3b_4 + b_5) \\ - \frac{r}{q^2} (c_1 + 3c_2 + 4c_3 + 3c_4 + c_5) \end{array} \right] = 0$$

After simplification we get,

$$q^* = \sqrt{\frac{2r(c_1 + 3c_2 + 4c_3 + 3c_4 + c_5)(a_1 + 3a_2 + 4a_3 + 3a_4 + a_5 + b_1 + 3b_2 + 4b_3 + 3b_4 + b_5)}{T(a_1 + 3a_2 + 4a_3 + 3a_4 + a_5)(b_1 + 3b_2 + 4b_3 + 3b_4 + b_5)}} \quad \text{----- (5.3.2)}$$

and

$$s^* = \sqrt{\frac{2r(c_1 + 3c_2 + 4c_3 + 3c_4 + c_5)(a_1 + 3a_2 + 4a_3 + 3a_4 + a_5)}{T(a_1 + 3a_2 + 4a_3 + 3a_4 + a_5 + b_1 + 3b_2 + 4b_3 + 3b_4 + b_5)(b_1 + 3b_2 + 4b_3 + 3b_4 + b_5)}} \quad \text{----- (5.3.3)}$$

This shows that $F_d(q, s)$ is minimum at q_d^* and s_d^*

6. Numerical Analysis

6.1. Crisp model:

Let $T = 6$, $a = 4$, $b = 10$, $c = 20$

Using Equations (4.1), (4.2) & (4.3) and we get the order quantity q^* , shortage quantity s^* and minimum total cost (TC).

Table 1

Demand	q^*	s^*	TC
1000	48.3046	13.8013	828.0787
1100	50.6623	14.4749	868.4962
1200	52.9150	15.1186	907.1147
1300	55.6787	15.7359	944.1550
1400	57.1548	16.3299	979.7959
1500	59.1608	16.9031	1014.2

6.2. Triangular fuzzy model

Let $T = 6$ $\tilde{a} = (1, 4, 7)$ $\tilde{b} = (8, 10, 12)$ $\tilde{c} = (15, 20, 25)$

Using Equations (5.1.1), (5.1.2) & (5.1.3) and we get the order quantity q^* , shortage quantity s^* and minimum fuzzy total cost (FTC).

Table 2

Demand(r)	q^*	s^*	FTC
1000	49.4413	13.4840	1618.1
1100	51.8545	14.1421	1697.1
1200	54.1603	14.7710	1772.5
1300	56.3718	15.3741	1844.9
1400	58.4998	15.9545	1914.5
1500	60.5530	16.5145	1981.7

6.3. Trapezoidal fuzzy model

Let $T = 6$ $\tilde{a} = (1, 3, 5, 6)$ $\tilde{b} = (8, 9, 11, 12)$ $\tilde{c} = (15, 18, 22, 25)$

Using Equations (5.2.1), (5.2.2) & (5.2.3) and we get the order quantity q^* , shortage quantity s^* and minimum fuzzy total cost.

Table 3

Demand(r)	q^*	s^*	FTC
1000	49.4413	13.4840	809.0398
1100	51.8545	14.1421	848.5281
1200	54.1603	14.7710	886.2587
1300	56.3718	15.3741	922.4473
1400	58.4998	15.9545	957.2688
1500	60.5530	16.5145	990.8674

6.4. Pentagonal fuzzy model

Let $T = 6$ $\tilde{a} = (1, 3, 4, 5, 6)$ $\tilde{b} = (8, 9, 10, 11, 12)$ $\tilde{c} = (15, 18, 20, 22, 25)$

Using Equations (5.3.1), (5.3.2) & (5.3.3) and we get the order quantity q^* , shortage quantity s^* and minimum fuzzy total cost.

Table 4

Demand (r)	q^*	s^*	FTC
1000	48.6703	13.6976	821.8571
1100	51.0458	14.3662	861.9710
1200	53.3156	15.0050	900.2994
1300	55.4926	15.6177	937.0613
1400	57.5874	16.2074	972.4344
1500	59.6087	16.7761	1006.6

7. Conclusion

In this paper, a fuzzy inventory model with shortage and constant demand is considered and carrying cost, backorder cost, and ordering cost are fuzzified using triangular, trapezoidal and pentagonal fuzzy numbers. By comparing the results of fuzzy models and crisp model, it is shown triangular Fuzzy Number is not suitable to find the

minimum total cost. Trapezoidal and Pentagonal Fuzzy numbers provide minimum total cost when compared to the crisp model. But still using Trapezoidal Fuzzy number we get the better result. Numerical examples illustrates the analysis.

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