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Cost and Reliability Optimization in Lost Sales Inventory Systems

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ABSTRACT

This article deals with cost and reliability optimization of a continuous review inventory system with (Q,r) policy with reorder level $0 \leq r < Q$. The demand time points are assumed to form a Poisson process and the lead time is constant L, with possible lost sales during stock out period. Explicit expressions for the various operating characteristics of the system are obtained. The cost optimization and reliability optimization are done subject to other constraints. Numerical examples are provided to illustrate the results.

Keywords

Continuous review, Lost sales inventory, Reliability, Cost optimization.

1. Introduction

Only a limited literature is available for lost sale inventory system because of its intractable nature during stock out period. Models with full/partial backlogging of demands are considerably easy and smooth for maintaining and continuous review[3]. A continuous review (s,S) policy inventory problem with lost sales was studied by Blyth C. Archbald [1], Pressman [7] and Elango [3]. The content of the article is organized in the following manner: section 1 gives a brief introduction and review of literature, section 2 deals with the model formulation. Section 3 depicts the derivations for system performance measures and section 4 deals with the cost consideration using the optimization criteria namely 'average annual cost'. The last section is devoted to present numerical examples to illustrate the results obtained.

2. Problem formulation

Consider lost sales inventory system with stochastic demand and constant lead time. The inventory control policy implemented is (Q,r) type with continuous review. The policy implementation rule is described as follows: Whenever the inventory level (on hand or position) reaches the level r an order for Q items is placed to pull back the level to Q+r at the maximum. The lead time for each other initiated at level r is assumed to be a fixed constant L(>0). As with the backorder case, we assume that the stochastic process generating the demands is such that the system can be described by a Markov process for which there exists steady state. The procurement lead time is a constant L and that a Poisson process generates demands with units being demanded one at a time. We desire to find optimal values of Q and r when the system is operated using a lot size reorder point control policy.

Even though the distribution of on hand inventory from inventory position as in the case of backorder is not possible here, it is still true that everything on order at time t- ج will arrive in the system by time t and nothing not on order at time t- ج can arrive by t.

The probability that $x \geq 0$ units are on hand at time t is the probability that r+j-x units were demanded in the time ج. Those demands occurring when the system is out of stock are lost and hence x units can be on hand at time t even if more than r+j-x demands occurred in the lead time.

3.System Performance Measures

3.1 Expected length of out of stock period.

The expected length of time, the system is out stock per cycle is given by

$$\check{T} = \int_0^L \mu\,(L-t)\,(\mu t)^{r-1}\, e^{-\mu t}\,)\,/\,((r-1)!)\,dt$$

$= L\,P(r, \mu L) - (r/\mu)\, P(r+1, \mu L)$

(Formula-16: Appendix 3: [5])

Here the service completion time for each customer is exponentially distributed with mean $1/\mu$, consequently the service completion epochs follow a Poisson process with parameter $\mu(>0)$.

3.2 Number of lost sales / cycle.

The expected number of lost sales occurred in the system per cycle is given by

$$\hat{S} = \sum_{x=r+1}^{\infty} (x - r)\, p(x : \mu r)$$

$$= \mu r\ P(r-1, \mu L) - r\ P(r, \mu L),$$

where x denotes the demand in lead time L, x > r, and the number of lost sales is x- r . (Formula-10: Appendix 3: [5])

3.3 Expected unit years of stock held / cycle.

Suppose y is the on hand inventory of the system after the arrival of procurement, then y is a random variable with probability distribution,

$$\Psi(y) = \begin{cases} P(r ; \mu L) & \text{if } y = Q \\ p(Q+r-y ; \mu L) & \text{if } Q < y \le Q+r \end{cases}$$

We spilt this in to two parts:

(i) Given that y is the on-hand inventory after the procurement, the expected number of unit years of stock held until the reorder point r reached is:

$$H_1 = (1/\mu)\ \{ y + (y-1) + \ldots\ldots + [y - (y - r - 1)]\}$$

$$= (1/2\mu)\{y(y+1) - r(r-1)\}.$$

The expected value of H_1 is

$$\hat{H}_1 = 1/(2\mu) \sum_{y=Q}^{Q+r} [y(y+1) - r(r+1)]\psi(y).$$

(ii) Given that x is the demand during lead time, the expected number of unit years of stock held from reorder point to next replenishment is given by

$$\hat{H}_2 = \int_0^L \sum_{x=0}^{r-1} (r-x)\, p(x ; \mu t)\, dt.$$

Combining the above two expectations (taking sum), we get the expected unit years of stock held per cycle:

$$\hat{H} = (1/2\mu)\ Q(Q+1) + (Q/\mu)(r - \mu L) + (Q/\mu)[\mu L\, P(r-1, \mu L) - r\ P(r, \mu L)].$$

The average annual cost incurred by the service facility system is given by

$$\acute{K} = (\mu/[Q + \mu T])\ \{ A + h[(1/2\mu)\, Q(Q+1) + (Q/\mu)(r - \mu L)] + [(hQ/\mu) + \pi][\mu L\, P(r-1; \mu L) - (r/\mu)\ P(r, \mu L)]\},$$

where π is the cost of lost sales inventory including the lost profit and $\check{T}$ is the expected out of stock period per cycle. The minimization of this average annual cost will yield the same result as the maximization of the average annual profit. The time averages, we computed above are all based on the general time average formulae (see Appendix).

When the number of customers waiting in the system is zero (v = 0), and the inventory position is > r, the service facility become idle. This case deserves to be discussed as a special case. In this case the system reaches the absolute failure state, to study its behavior we need the marginal distribution of the position

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inventory from the joint distribution of inventory level and queue size. This remains still an open problem for lost sales inventory system.

4. Reliability and Hazard Rate Functions

Consider a lost sale inventory system with capacity Q. The mean time between failures is a Gamma distribution with parameters Q and μ. If X_i denote the random variable representing the time taken for the i^{th} demand, then $S_n = \sum_{i=1}^{n} X_i$ is the time upto the n^{th} demand occur. Since X_i are all independent and identically distributed exponential random variables, S_n follows a Gamma distribution with parameters Q and μ.

Thus the mean time between failures MTBF = Q/μ .

The reliability of the system is given by

$$R(t) = 1 - F(t)$$
$$= 1 - \Pr\{Sn < t\}$$
$$= 1 - \int_0^t (\mu e^{-\mu y} (\mu y)^{Q-1}) / (\Gamma(Q))\, dy .$$

The hazard rate function is of the form

$$h(t) = f(t) / R(t).$$
$$= (\mu e^{-\mu t} (\mu t)^{Q-1}) \,/\, (\Gamma(Q) [1 - \mu / \Gamma(Q) \int_0^t e^{-\mu y} (\mu y)^{Q-1} dy]).$$

5. Redundant Components Optimization

5.1 Number of components Vs Cost

Let C_1 be the cost of maintenance of each of the n identical inventory component put in parallel and R(t) be its reliability.

Let C_2 be the cost of failure before the specified time t. The total expected cost for the system is given by

$$\textbf{TEC}\,(n) = C_1 n + C_2 (1 - R(t))^n .$$

The optimal number of components for smooth operation is given by

$$n^* = \frac{\log_e C_2 + \log_e [\log_e (1/(1 - R(t))] - \log_e C_1}{\log_e (1/(1-R(t))}.$$

5.2 Number of components Vs Reliability

Suppose each component has different reliability function R_i (t) and the cost of maintenance of the system per unit time is C_i , i= 1,2,….n, then the optimization problem becomes a non-linear programming problem:

Maximize $R(n) = [1 - (1-R_1)] \ldots [1-(1-R_n)]$ ***subject to***

$\sum_{i=1}^{n} C_i \le B$, $n \ge 1$ where B is the total budget for the system in operation.

6. Numerical Examples

6.1 Example 1

Consider a service facility system with the following parameters:

$\lambda = 3$, $\mu = 2$, $C_1 = 20$, $C_2 = 5000$, $R = 0.9$.

The optimal value of n is given by n* = 4.

6.2 Example 2

Consider a service facility system with the following parameters:

$C_1 = 20$, $C_2 = 30$, $C_3 = 15$, $C_4 = 25$, $C_5 = 50$, $R_i = 0.6$ for i=1,2,… n

and the total budget for the system is B = 1500. Now the problem becomes

Maximize* R(n) = [1- (1-R_1)]. . . [1- (1-R_n)] = $(0.6)^n$, *subject to
140n ≤ 1500. The optimal value of n is given by n* = 1.

7.References

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Appendix : A

We select arbitrary values for Q and r and start our inventory control operation. Starting observation at any time and ζ be the present time, the system may go through n complete cycles and perhaps part of another one. Here we consider lost sales inventory control system with demand rate λ in which the demands occur during the stock-out period are lost. With this general setup for inventory control system, we prove the following theorem :

A.1 Theorem:

Consider a inventory control system with (Q, r) policy, which operates in long run, having constant demand rate λ and lead time L(>0) do not change with time. If the units are demanded one at a time, Ω_i denote the unit years of inventory held, Δ_i be the unit years of shortages and ξ_i denote the number of lost sales, during cycle i, n+ε (ε = 0 or1) is the number of orders placed in the time duration ζ , then the probability that the system is out of stock is

$$P_{out} = \lim_{\zeta\to\infty} \sum_{i=1}^{n} T_i'' / \zeta.$$

Also

$$D = \lim_{\zeta\to\infty} \sum_{i=1}^{n} \Omega i / \zeta \quad ; \quad B = \lim_{\zeta\to\infty} \sum_{i=1}^{n} \Delta_i / \zeta \quad ; \quad E = \lim_{\zeta\to\infty} \sum_{i=1}^{n} \xi_i / \zeta \quad ,$$

are the average unit years of stock, average unit years of shortages and average numbers of lost sales respectively incurred per year.

Proof: Let T_i be the length of the cycle i, T_i' be the length of time during which the cycle has positive stock on hand in cycle i, T_i'' be the length of time for which the system is out of stock in cycle i.. $T_i = T_i' + T_i''$

Let Ωi be the unit years of inventory held during cycle i, Δ_i be the unit years of shortages (lost sales) incurred during the cycle i and ξ_i be the number of lost sales during cycle i. Now the number of orders placed during the period of length ζ is n+ ε , where ε = 0 or1, depending on whether or not an order placed in the fraction of cycle which may be included. The average number of orders placed per year over the time space ζ is (n+ε)/ζ . As ζ → ∞ , it must be true that n → ∞ . Thus the average number of orders placed per year approaches λ / Q. That is

lim n = 1 lim nQ = λ .

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$\zeta\to\infty\ \zeta$ Q $\zeta\to\infty\ \zeta$ Q

In the lost sales case the average number of orders placed per year is λ / Q. This is independent of specific nature of the demand or lead time distribution provided that an order is placed after every Q demands. The fraction of time the system is out of stock will be given by

$$P_{out} = \lim_{\zeta\to\infty} \sum_{i=1}^{n} T_i'' / \zeta.$$

This gives the probability that the system is out of stock if we observe the system at a random point in time.

Also $D = \lim_{\zeta\to\infty} \sum_{i=1}^{n} \Omega i / \zeta$; $B = \lim_{\zeta\to\infty} \sum_{i=1}^{n} \Delta_i / \zeta$; $E = \lim_{\zeta\to\infty} \sum_{i=1}^{n} \xi_i / \zeta$,

are the average unit years of stock held per year, average unit years of shortages incurred per year and average numbers of lost sales per year respectively . The expression for E can also be rewritten using the fact that limit of a product is the product of the limit as:

$$E = \lim_{\zeta\to\infty} \sum_{i=1}^{n} \xi_i / \zeta = \lim_{\zeta\to\infty} \sum_{i=1}^{n} T_i'' \sum_{i=1}^{n} \xi_i / (\zeta \sum_{i=1}^{n} T_i'') = \lambda P_{out} .*$$

A 1.1 Remark:

(i) The values of D, B, E and P_{out} will depend on the nature of the stochastic processes generating demand and lead times. They can be computed first computing the corresponding expected values per cycle and then multiplying by λ / Q.

(ii) If we can determine the steady state probabilities $\psi(x)$ that on-hand (net) inventory has the value x then the values of D, B, E and P_{out} may be computed directly :

$$D = \sum_{x=0}^{\infty} x\,\psi(x)\ ;\ B = - \sum_{-\infty}^{-1} x\,\psi(x)\ ;\ P_{out} = \sum_{-\infty}^{-1} \psi(x) .$$

In particular for the lost sales case, the average number of lost sales per year is λP_{out} and the average unit of shortage per year is given by

$$D = \sum_{x=0}^{\max x} x\,\psi(x).$$

(iii) The analysis of lost sales case is same as backorder case if the same definitions are used. We get the average number of lost sales per year as λP_{out}. The average number of cycles per year in this case is given by

$$\lim_{\zeta\to\infty} n/\zeta = \lambda / Q (1- P_{out}) .$$

It is also true that $\lim_{\zeta\to\infty} n/\zeta = \lambda/(Q+\lambda T)$ ■

$$$