SOLVING FUZZY ASSIGNMENT PROBLEM USING GENETIC ALGORITHM

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Abstract: Assignment problem has various applications in the real world because of their wide applicability in industry, commerce, management science, etc. Traditional classical assignment problems cannot be successfully used for real life problem; hence the use of fuzzy assignment problem is more appropriate. Fuzzy sets were introduced by Lofti A. Zadeh in 1965 as an extension of representing imprecisenes or vagueness in day to day life.
In this paper, Triangular Fuzzy Assignment Problem has been defuzzified into crisp values using Centroid Ranking method and genetic algorithm has been applied to find an optimal solution. The proposed method is illustrated by numerical examples.

Keywords: Fuzzy Assignment Problem, Triangular Fuzzy Number, Triangular Fuzzy Assignment Problem, Genetic Algorithm, Centroid Ranking Method.

1. Introduction: The term Assignment Problem (AP) was first appeared in Votaw & Orden (1952). APs are widely applied in manufacturing and service systems. An Assignment Problem is a special type of linear programming problem where the objective is to assign n number of jobs to n number of persons at a minimum cost (time).
Zadeh (1965) has introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Since then, tremendous efforts have been spent; significant advances have been made on the development of numerous methodologies and their applications to various decision problems. Fuzzy assignment problems have received great attention in recent years. Hungarian method proposed by Kuhn (1955) is widely used for the solution to APs. Chen (1985) proposed a fuzzy assignment model that did not consider the differences of individuals and also proved some theorems. Wang (1987) solved a similar model by graph theory.
Genetic Algorithm is a powerful and broadly applicable stochastic search and optimization technique. It really works for many problems that are very difficult to solve by conventional deterministic optimization techniques. Genetic algorithms (GAs) are inspired by Darwin's Theory about evolution - "the survival of the fittest". GA was invented under developed by John Holland, his students and colleagues. This lead to Holland's book "Adaption in natural and artificial systems" [7] published in 1975.Lin and Wen (2004) investigated a fuzzy AP in which the cost depends on the quality of the job. Dubois and Fortemps (1999) proposed a flexible AP, which combines with fuzzy theory, multiple criteria decision-making and constraint-directed methodology. Huang and Xu (2005) proposed a solution procedure for the APs with restriction of qualification. Mukherjee and Basu (2010) proposed a new method for solving fuzzy APs. Kumar et al (2009) proposed a method to solve the fuzzy APs, occurring in real life situations.
Kumar and Gupta (2012) proposed two new methods for solving fuzzy APs and fuzzy travelling salesman problems. Kumar and Gupta (2011) proposed methods for solving fuzzy APs with different membership functions. K. Kalaiarasi et al (2014) proposed a fuzzy assignment model with TFN using Robust Ranking

technique. Jatinder Pal Singh et al (2014) proposed method to solve fuzzy assignment problem using RHM and Operations for Subtraction and Division on TFN proposed by Gani and Assarudeen (2012). D.Goldberg, genetic Algorithms in search, optimization and machine learning, Addison-wesley, reading, MA,USA,1989. Holland J,"Adaptation in Natural and Artificial system", university of Michigan press, Ann Arbor,MI,1975; MIT Press, Cambridge,MA 1992. In this paper the fuzzy assignment problem has been converted into crisp assignment problem using Centroid Ranking Method and Genetic assignment has been applied to find an optimal solution.

2. Preliminaries:

2.1. Fuzzy set: The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. A function μ_A such that the value assigned to the universal set X fall within a specified range i.e. $\mu_A : X \to [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{A(x)}$ is called the membership function and the set $\hat{A} = \{(x, \mu_{A(x)}); x \in X\}$ defined by $\mu_{A(x)}$ for each $x \in X$ is called a fuzzy set.

2.2. Triangular fuzzy number (TFNs): A fuzzy number $\hat{a}$ on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\hat{a} : R \to [0,1]$ has the following characteristics

$$\mu_{a(x)} = \begin{cases} (x-a_1)/(a_2-a_1) \text{ if } & a_1 \le x \le a_2 \\ (a_3-x)/(a_3-a_2) \text{ if } & a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

We denote this triangular fuzzy number by $\hat{a} = (a_1, a_2, a_3)$. We use $F(R)$ to denote the set of all triangular fuzzy numbers.

Also if $m = a_2$, represents the modal value or midpoint, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_3 - a_2)$ represents the right spread of the triangular fuzzy number $\hat{a} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\hat{a}$ can be represented by the triple $\hat{a} = (\alpha, m, \beta)$ i.e. $\hat{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

2.3. Defuzzification: Defuzzification is the process of finding singleton value (crisp value) which represents the average value of the TFNs. Here Method of magnitude is used to defuzzify the TFNs because of its simplicity and accuracy.

2.3.1. Robust ranking technique:

For a convex fuzzy number $\tilde{a}$, the Robust's Ranking Index is defined by,

$$R(\tilde{a}) = \int_0^1 (0.5)(a^L_\alpha, a^U_\alpha)d\alpha$$

where $(a^L_\alpha, a^U_\alpha) = \{(b-a)\alpha + a, c-(c-b)\alpha\}$ which is the α -level cut of the fuzzy number $\tilde{a}$.

2.3.2. Method of magnitude:

A triangular fuzzy number $\bar{a} \in F(R)$ can also be represented as a pair $\bar{a} = (\underline{a}, \overline{a})$ of functions $(\underline{a}(r), \overline{a}(r))$ for $0 \le r \le 1$ which satisfies the following requirements:

(1) $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

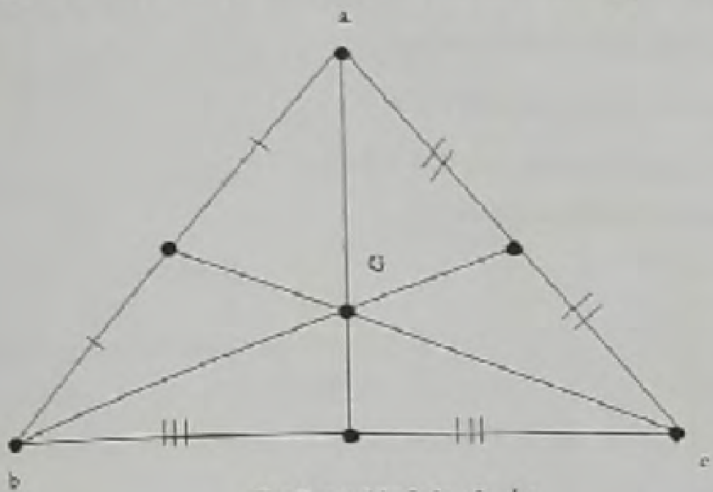
(2) $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

(3) $\underline{a}(r) \le \overline{a}(r)$ for $0 \le r \le 1$.

2.3.2.1. Definition: For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\frac{\underline{a}(1) + \overline{a}(1)}{2} \right)$ is said to be a location index number of $\tilde{a}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$ and $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index functions respectively. Hence every triangular fuzzy number $\overline{a} = (a_1, a_2, a_3)$ can also be represented by $\tilde{a} = (a_0, a_*, a^*)$.

2.4 Centroid:

The centroid of a triangle is the point where the three medians of the triangle intersect. The medians are the segments that connected a vertex to the midpoint of the opposite side.



The Centroid of triangle abc

3 Ranking Functions and Fuzzy Assignment Problem:

3.1 Ranking of triangular Fuzzy Numbers:

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers.

For an arbitrary triangular fuzzy number $\overline{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form $\tilde{a} = (\underline{a}(r), \overline{a}(r))$ we define the magnitude of the triangular fuzzy number by $\tilde{a}$ by

$$Mag(\tilde{a}) = \frac{1}{2}\left(\int_0^1 (\overline{a} + \underline{a} + a_0) f(r) dr \right)$$

$$= \frac{1}{2}\left(\int_0^1 (a^* + 3a_0 - a_*) f(r) dr \right)$$

[Since $\overline{a} + \underline{a} + a_0 = a^* + a_0 + a_0 - a_* + a_0 = a^* + 3a_0 - a_*$].

Where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0)=0, f(1)=1$ and $\int_0^1 f(r) dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$.

Hence $Mag(\tilde{a})=\left(\frac{a^{*}+3a_{0}-a_{*}}{4}\right)=\left(\frac{\bar{a}+\underline{a}+a_{0}}{4}\right)$.

The magnitude of a triangular fuzzy number $\tilde{a}$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. $Mag(\tilde{a})$ is used to rank fuzzy numbers.

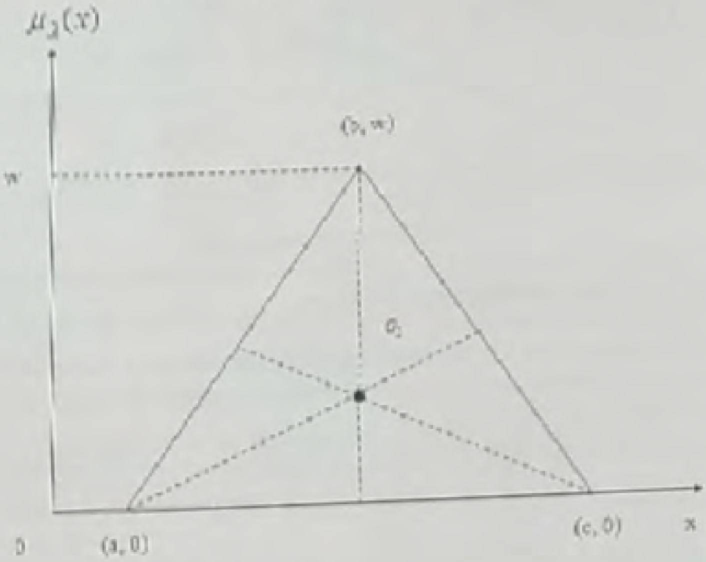
Theorem: For any two fuzzy numbers, $A=\langle s_1,l_1,r_1\rangle$ and $B=\langle s_2,l_2,r_2\rangle$ and $A\le B$ if and only if $s_1\le s_2$, $s_1-l_1\le s_2-l_2$ and $s_1+r_1\le s_2+r_2$.

Properties:

For any two triangular fuzzy number $\tilde{a}=(a_{*},a_{0},a^{*})$ and $\tilde{b}=(b_{*},b_{0},b^{*})$

1. $Mag(\tilde{a})\ge Mag(\tilde{b})$ if and only if $\tilde{a}\ge\tilde{b}$
2. $Mag(\tilde{a})\le Mag(\tilde{b})$ if and only if $\tilde{a}\le\tilde{b}$
3. $Mag(\tilde{a})=Mag(\tilde{b})$ if and only if $\tilde{a}\approx\tilde{b}$.

3.2. Centroid of a Generalized Triangular:



Centroid Ranking Function

The centroid of a triangle fuzzy number $\tilde{A}=(a,b,c;w)$ as $G_{\tilde{A}}=\left(\frac{a+b+c}{3},\frac{w}{3}\right)$.

The ranking function of the generalized triangle fuzzy number $\tilde{A}=(a,b,c;w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\tilde{A})=\left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right)$.

3.3. Assignment Problem: The assignment problem can be stated in the form of $n\times n$ cost matrix $\left[C_{ij}\right]$ of real numbers as given in the following table:

Hence $Mag(\tilde{a}) = \left(\frac{a^{*} + 3a_0 - a_*}{4}\right) = \left(\frac{\bar{a} + \underline{a} + a_0}{4}\right)$.

The magnitude of a triangular fuzzy number $\tilde{a}$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. $Mag(\tilde{a})$ is used to rank fuzzy numbers.

Theorem: For any two fuzzy numbers, $\tilde{A} = (s_1, l_1, r_1)$ and $\tilde{B} = (s_2, l_2, r_2)$ and $\tilde{A} \le \tilde{B}$ if and only if $s_1 \le s_2$, $s_1 - l_1 \le s_2 - l_2$ and $s_1 + r_1 \le s_2 + r_2$.

Properties:

For any two triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$

1. $Mag(\tilde{a}) \ge Mag(\tilde{b})$ if and only if $\tilde{a} \ge \tilde{b}$
2. $Mag(\tilde{a}) \le Mag(\tilde{b})$ if and only if $\tilde{a} \le \tilde{b}$
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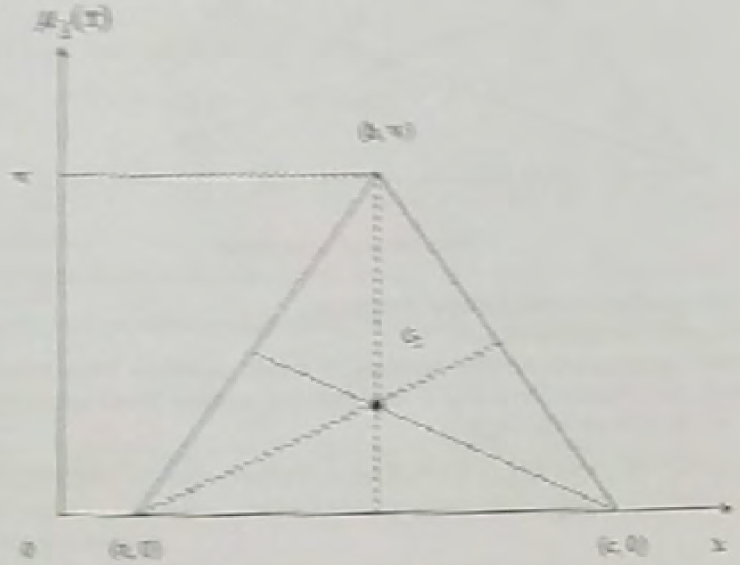

Centroid Ranking Function

The centroid of a triangle fuzzy number $\tilde{A} = (a, b, c; w)$ as $G_{\tilde{A}} = \left(\frac{a+b+c}{3}, \frac{w}{3}\right)$.

The ranking function of the generalized triangle fuzzy number $\tilde{A} = (a, b, c; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\tilde{A}) = \left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right)$.

3.3. Assignment Problem: The assignment problem can be stated in the form of $n \times n$ cost matrix $\left[C_{ij}\right]$ of real numbers as given in the following table:

Jobs → Persons	1	2	3	--j--	N
1	C_{11}	C_{12}	C_{13}	$--C_{1j}--$	C_{1n}
2	C_{21}	C_{22}	C_{23}	$--C_{2j}--$	C_{2n}
.	-	-	-	-	-
.	-	-	-	-	-
i	C_{i1}	C_{i2}	C_{i3}	$--C_{ij}--$	C_{in}
.	-	-	-	-	-
N	C_{n1}	C_{n2}	C_{n3}	$--C_{nj}--$	C_{nn}

Mathematically assignment problem can be stated as

Minimize $z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}$ where $i = 1,2,3,...n,\ j = 1,2,3,...n$

Subject to

$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1,2,3,...n$ ....(1)

$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,2,3,...n$ $\quad x_{ij} \in \{0,1\}$

where $x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person is assigned the } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases}$

is the decision variable denoting the assignment of the person i to job j, C_{ij} is the cost of assignment of j^{th} job to the i^{th} person.

The objective is to minimize the total cost of assigning all the jobs to the available persons (One job to one person). When the costs $\tilde{C}_{ij}$ are fuzzy numbers, then the fuzzy assignment problem becomes

$$Y(\tilde{z}) = \sum_{i=1}^{n}\sum_{j=1}^{n} Y(\tilde{C}_{ij})\ x_{ij} \quad ....(2)$$

Subject to the same conditions (1)

For an unbalanced problem add dummy rows / columns then follow the same procedure.

3.4 Fuzzy Assignment Problem:

The generalized fuzzy assignment problem can be represented in the form of $n \times n$ fuzzy cost matrix $[\tilde{C}_{ij}]$ as given below:

Jobs → Persons ↓	1	2	3	---j---	N
1	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{13}$	$---\tilde{C}_{1j}---$	$\tilde{C}_{1n}$
2	$\tilde{C}_{21}$	$\tilde{C}_{22}$	$\tilde{C}_{23}$	$---\tilde{C}_{2j}---$	$\tilde{C}_{2n}$
.	-	-	-	-	-
.	-	-	-	-	-
i	$\tilde{C}_{i1}$	$\tilde{C}_{i2}$	$\tilde{C}_{i3}$	$---\tilde{C}_{ij}---$	$\tilde{C}_{in}$
.	-	-	-	-	-
N	$\tilde{C}_{n1}$	$\tilde{C}_{n2}$	$\tilde{C}_{n3}$	$---\tilde{C}_{nj}---$	$\tilde{C}_{nn}$

The cost or time $\left[\hat{C}_{ij}\right]$ are generalized trapezoidal fuzzy numbers $\hat{C}_{ij}=\left[C_{ij}^{(1)},C_{ij}^{(2)},C_{ij}^{(3)},C_{ij}^{(4)};w_{ij}\right]$. The goal is, effective way of assigning the j^{th} job to the i^{th} resource (all jobs to available resources) by minimizing the total cost with minimum time.

4. Algorithms:

4.1. Genetic Algorithm

Genetic Algorithm is a biological inspired method for function optimization. GAs work with a population of "individuals", each representing a possible solution to a given problem. Each individual is assigned a "fitness score" according to how good the solution is to the problem. The highly- fit individuals are given opportunities to "reproduce", by "cross breeding" with other individuals in the population. This produce new individuals as "offspring", which share some features taken from each "parent". The least fit members of the populations are less likely to get selected for reproduction, and so "die out". A whole new population of possible solutions is the produced by selecting the best individuals from the current "generation", and mating them to produce a new set of individuals. This new generation contains a higher proportion of the characteristic possessed by the good members of the previous generation. In this way, over many generations, good characteristics are spread throughout the population

(i.e.) the algorithm converges to the best chromosome.

Initial conditions:

The required initial information to start the GA are:

1. Population size: It is the number of the chromosomes or scenarios that we will keep in each generation and we denote it by M.
2. Crossover rate: This is the probability of performing a crossover in the GA methods, denoted by p_c.
3. Mutation rate: This is the probability of performing mutation in the GA method denoted by p_m.

Chromosome: In classical genetic algorithm Chromosome is represented by binary number 0 and 1. In this paper we represent a chromosome by permutation of order n, it is denoted by $(i_1, i_2, \ldots\ldots i_n)$. That is first machine is assigned to job i_1, second machine is assigned to job i_2 and so on.

Selections:

In this paper we use Tournament selection method. This method randomly chooses a set of chromosomes and picks out the best chromosome for reproduction. The number of chromosome in the set is called the tournament size. The process continues until the population is full.

The proposed genetic algorithm for FAP:

The outline of the proposed algorithm is as follows:

Step 1: Initial population of M permutation of order n is created randomly.

Step 2: The fitness value $\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}x_{ij}$ of all the members of the current population are evaluated.

Step 3: The M members of the next generation are generated as follows:

(i) M_1 elite members of the next generation are retained. That is M_1 members with best M_1 fitness values are selected.

(ii) The crossover percentage p_c is fixed. Then $M_2=p_c(M-M_1)$ members are generated using the crossover operation. For this purpose M_2 pairs of members are selected from the current population. From each pair of parent solution one offspring solution is created using a crossover operator.

(iii) $M_3=(1-p_c)(M-M_1)=p_m(M-M_1)$ members are created using mutation. For this purpose M_3 members of the current population are selected. From each of these M_3 members a new solution is created by applying a mutation operator.

Hence altogether $M_1+M_2+M_3=M$ members of the next generation (or the new population) are generated.

Step 4: Step 2 and step 3 are repeated until the algorithm converges to the best chromosome.

4.2. Algorithm to solve fuzzy assignment problem:

Step 1: First test whether the given fuzzy cost matrix of a fuzzy assignment problem is a balanced one or not. If not change this unbalanced assignment problem into balanced one by adding the number of dummy row(s) / column(s) and the values for the entries are zero. If it is a balanced one (i.e. number of persons are equal to the number of works) then go to step 2.

Step 2: Defuzzify the fuzzy cost by using Centroid ranking method.

Step 3: Apply Genetic Algorithm to determine the best combination to produce the lowest total costs, where each machine should be assigned to only one job and each job requires only one machine.

5. Numerical Example:

Example 5.1:

Here we are going to solve fuzzy Assignment problem using Centroid Ranking Technique:

To allocate 4 jobs to 4 different machines, the fuzzy assignment cost C_{ij} is given below:

$$\begin{bmatrix} (1,5,9) & (3,7,11) & (7,11,15) & (2,6,10) \\ (4,8,12) & (1,5,9) & (4,9,13) & (2,6,10) \\ (0,4,8) & (3,7,11) & (6,10,14) & (3,7,11) \\ (6,10,14) & (0,4,8) & (4,8,12) & (-1,3,7) \end{bmatrix}$$

Solution: The given Triangular fuzzy cost matrix is balanced one. Now we calculate $R(1,5,9)$ by applying Centroid Ranking Technique of a Triangular fuzzy number $R(\tilde{A}) = \left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right)$, where $w = \min(w_1, w_2, \ldots, w_n)$ and the problem is done by taking the value of w as 1.

$$R(1,5,9) = \left(\frac{1+5+9}{3}\right)\left(\frac{1}{3}\right)$$
$$= 1.67$$
$$R(3,7,11) = \left(\frac{3+7+11}{3}\right)\left(\frac{1}{3}\right)$$
$$= 2.33$$

Similarly,

$$R(7,11,15) = 3.67$$
$$R(2,6,10) = 2$$
$$R(4,8,12) = 2.67$$
$$R(1,5,9) = 1.67$$
$$R(4,9,13) = 2.89$$
$$R(2,6,10) = 2$$
$$R(0,4,8) = 1.33$$
$$R(3,7,11) = 2.33$$
$$R(6,10,14) = 3.33$$
$$R(3,7,11) = 2.33$$
$$R(6,10,14) = 3.33$$
$$R(0,4,8) = 1.33$$
$$R(4,8,12) = 2.67$$
$$R(-1,3,7) = 1$$

The rank of triangular fuzzy cost table is:

	I	II	III	IV
A	1.67	2.33	3.67	2
B	2.67	1.67	2.89	2
C	1.33	2.33	3.33	2.33
D	3.33	1.33	2.67	1

Proceeding by Genetic Algorithm, the optimal allocation is
A→I, B→II, C→III, D→IV.
The fuzzy optimal total cost

$$\tilde{a}_{11} + \tilde{a}_{22} + \tilde{a}_{33} + \tilde{a}_{44} = (1,5,9) + (1,5,9) + (6,10,14) + (-1,3,7)$$
$$= (7,23,39).$$

Now, by using Centroid Ranking Technique of a Triangular fuzzy number then

$$R(7,23,39) = 7.67.$$

Result: We will compare the assignment cost which has been found out in example 5.1 with the assignment cost calculated by existing methods (kalaiarasi et al 2014, Jatinder pal singh et al 2015, Selvi et al 2017)

	Existing Methods			Proposed Method
	Kalaiarasi et al 2014	Jatinder Pal Singh et al 2015	Selvi et al 2017	
Example 5.1	Assignment cost = 23	Assignment cost = 22.75	Assignment cost = 8.5	Assignment cost = 7.67

6. Conclusion: In this paper, the fuzzy costs of a Triangular Fuzzy Assignment Problem has been defuzzified into crisp values by using Centroid Ranking method and solved by using Genetic Algorithm. By comparing the results of the proposed method and existing methods, it is shown that the proposed method has given better results than the existing methods.

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