# PROJECTILE MOTION USING MATLAB

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**ABSTRACT**

A body in free fall experiences only to the forces of gravity and air resistance. A body when projected into air with some velocity at an angle with the horizontal is called a projectile. When the body takes the projectile motion, there may be air friction acting on the body. When there is no air friction, the projectile is moving in a typical parabolic path. The horizontal velocity remains constant and the vertical motion is influenced only by gravity. With air friction, the path is no longer parabolic. The projectile does not travel with a constant velocity, but it decelerates. The vertical motion will be under the combined influence of gravity and air friction. Gravity works only downward while air friction works against the direction of motion of the object. This paper describes the usage of **MATLAB** to trace the path of the projectile motion of any body when its velocity and angle are changing. MATLAB is a commercial software tool for the purpose of performing numerical calculations and data visualization. This can be mainly used to trace the path of the missile to hit the target by simulation process.

***Key Words: Angle, Friction, Gravity Projectile, Velocity***

## I INTRODUCTION

A body projected into air with some velocity at an angle with the horizontal is called a projectile and, which is then allowed to move under the action of gravity alone, without being propelled by any engine or fuel, is called the projectile motion. The path followed by a projectile is called its trajectory. A projectile moves at a constant speed in the horizontal direction while experiencing a constant acceleration of 9.8 m/s$^2$ downwards in the vertical direction. When the body takes the projection, the friction due to air plays a major role on the body to reach the surface. The path of the trajectory varies due to air friction. Our aim is to trace the path of the projectile motion with and without air resistance using MATLAB tools for two dimensional motions by varying the velocity and angle of projection.

## II THEORY

When the flying object moves through the air, there is a relative motion between it and the air. In our study of projectile motion, we assumed that air-resistance effects are negligibly small. But in fact air resistance often called air drag, has a major effect on the motion of many objects, including tennis balls, bicycle riders, and airplanes. When air drag is neglected, the only force acting on a projectile with mass *m* is its weight *m*g**.** The components of the projectile's acceleration are the +*x*-axis is horizontal, and the +*y*-axis is vertically upward. The air drag force is included when it acts on the projectile. The magnitude *f* of the air drag force is approximately proportional to the square of the projectile's speed relative to the air:

$$f=Dv^2 \qquad \text{-------------------------} \ (1)$$

where $v^2 = v_x^2 + v_y^2$

The constant *D* depends on the density of air, area *A* of the body and a dimensionless constant *C* called the **drag coefficient** that depends on the shape of the body.

**Time of Flight**

The time interval between the time of projection and the time when the projectile passes the same horizontal plane through the point of projection is called the time of flight.

$t_R \sim (2v_{iz}/g)$ -------------------------------(2)

## Range of projectile

The minimum horizontal distance travelled by the projectile from the point of projection during the time flight is called its flight.

Its unit is m and dimension is L.

- For Projectile motion without air resistance,

  $R= v_i^2 \sin 2\theta/g$ ----------------------------(3)

- For Projectile motion with air resistance,

  $R=(m/c)\ v_{ix}\ (1-e^{-(c/m)t}{}_{max})$ --------------(4)

## Maximum height:

The maximum height attained by the projectile motion is

$z_{max}=v^2_{iz}/2g$ -------------------(5)

## Projectile motion

A particle moves in vertical plane with some initial velocity $\mathbf{v_0}$ but its acceleration is always the free fall acceleration **g,** which is downward.

Our aim here is to analyze projectile motion using the tools for two dimensional motions and making the assumption that air has no effect on the projectile. From figure the path followed by a projectile when the air has no effect. The projectile is launched with an initial velocity $\mathbf{v_0}$ that can be written as

$\mathbf{v_0} = \mathbf{v_{0x}}\,\hat{\mathbf{i}} + \mathbf{v_{0y}}\,\hat{\mathbf{j}}$ ----------------(6)

the components $v_{0x}$ and $v_{0y}$ can then be found if we know the angle $\boldsymbol{\theta_0}$ between $\mathbf{v_0}$ and the positive x direction

$v_{0x}=v_0 \cos\theta_0$ and $v_{0y}=v_0\sin\theta_0$ ------------(7)

during its two dimensional motion, the projectile's position vector **r** and velocity vector **v** change continuously, but its acceleration vector **a** is constant and always directed vertically downward. The projectile has no horizontal acceleration. In Projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

The path of a projectile that is launched at $x_0 = 0$ and $y_0 = 0$, with an initial velocity $\mathbf{v_0}$ **.**The initial velocity and the velocities at various points along its path, along with their components. Note that the horizontal velocity components remain constant but the vertical velocity component changes continuously. The range R is the horizontal distance the projectile has travelled when it returns to its launch height.

**The vertical motion**

The vertical motion is the motion for a particle in free fall, where the acceleration is constant.

$$y - y_0 = v_{0y}t - 1/2gt^2 \quad \text{----------------------(8)}$$

$$= (v_0 \sin \theta_0)t - 1/2gt^2 \quad \text{----------------------(9)}$$

Where the initial vertical velocity component $v_{0y}$ is replaced with the equivalent $v_0 \sin\theta_0$.

$$V_y = v_0 \sin \theta_0 - gt \quad \text{----------------------------(10)}$$

$$V^2_y = (v_0 \sin \theta_0)^2 - 2g(y - y_0). \quad \text{-----------------(11)}$$

The vertical velocity component behaves just as for a ball thrown vertically upward. It is directed upward initially, and its magnitude steadily decreases to zero, which marks the maximum height of the path. The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

**The horizontal motion**

Since there is no acceleration in the horizontal direction, the horizontal component $v_x$ of the projectile's velocity remains unchanged from its initial value $v_{0x}$ throughout the motion. At any time t, the projectile's horizontal displacement $x - x_0$ from an initial position $x_0$, is given by the equation

$$x - x_0 = (v_0 \cos\theta_0)t \text{---------------------------------(12)}$$

## The equation of the path

We can find the equation of the projectile's path (its trajectory) is

$$Y = (\tan\theta_0)x - gx^2/2(v_0\cos\theta_0)^2 \text{ ----------------(13)}$$

As g, $\theta_0$, and $v_0$ are constants, equation (13) is of the form $y = ax + bx^2$, in which a & b are constants. This is the equation of a parabola, so the path is parabolic.

The horizontal range R of the projectile is the horizontal distance the projectile has travelled when it returns to its initial (launch) height. To find range R, let us put $x - x_0 = R$ in equation (12) and $y - y_0 = 0$ in equation (8), the equation becomes

$$R = (v_0\cos\theta_0)t \quad \text{------------------------------(14)}$$

$$0 = (v_0\sin\theta_0)t - 1/2gt^2 \text{ ------------------------(15)}$$

Eliminating t between these two equations yields,

$$R = 2v_0^2/g \ \sin\theta_0 \cos\theta_0 \quad \text{--------------------(16)}$$

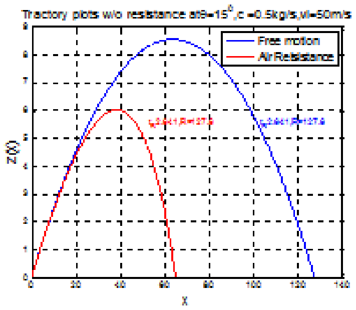
Using identity $\sin2\theta_0 = 2\sin\theta_0 \cos\theta_0$. We obtain

$$R = v_0^2/g \ \sin2\theta_0. \text{ -----------------------------(17)}$$

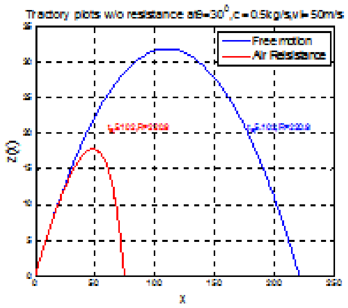
This equation does not give the horizontal distance travelled by a projectile when the final height not the launch height. Note that R in eqn(17) has its maximum value when $\sin2\theta_0 = 1$, which corresponds to $2\theta_0 = 90^o$ or $\theta_0 = 45^o$.The horizontal range R is maximum for a launch angle of $45^o$.However, when the launch and landing heights differ, as in shot put, hammer throw, and basketball, a launch angle of $45^o$ does not yield the maximum horizontal distance.We have assumed that the air through which the projectile moves has no effect on its motion. However, in many situations, the disagreement between our calculations and the actual motion of the projectile can be large because the air resists (oppose) the motion.

**MATLAB**

MATRIX LABORATORY is a numerical computing environment and fourth generation programming language developed by math works. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces and interfacing with programs written in other languages, including C++. Various applications of MATLAB are algorithm development, computation, Data acquisition, Modelling, simulation, Data analysis, exploration, and visualization. We have used the MATLAB version 7.9.0.529(R2009b).With the developed algorithm, a program is written to trace the path of the projectile by varying angle, velocity and drag coefficient. Fig. 1(a) – Fig.1(e) depicts the path of the trajectory when angle is varied and keeping velocity and drag coefficient constant. Fig2. shows the trajectory path for various angles. The program can be used effectively to trace the path of a missile.



1(a)



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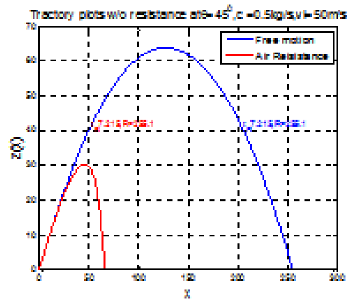

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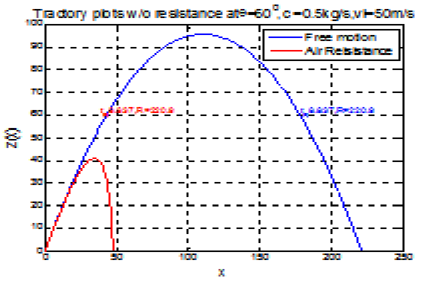

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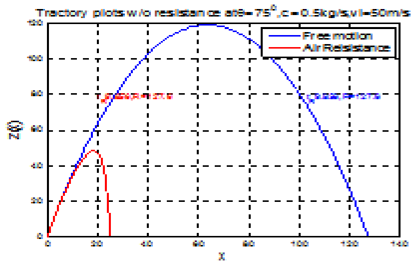

1(e)

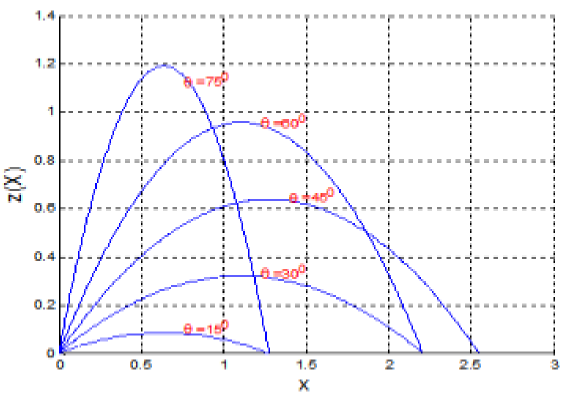


**Fig.2 Trajectory path for various angles**

## IV CONCLUSION

While projectile motion functions well under the circumstances for which it was designed, many measurements of time an object remains in the air and the distance travelled will deviate from the predictions of projectile motion. This deviation is largely due to air resistance. Air Resistance is the force that exerts on any object moving through it due to collisions with the object and surrounding air pressure. The movement of air varies at a steady rate governed by the forces acting on it. When the projectile is launched at a steep angle, it spends more time in the air than it does when launched at a shallow angle. When the projectile is launched at a shallow angle, it goes faster in the horizontal direction than if it is launched at a steep angle. Projectiles are widely used by sportsmen, especially the javelin throw; shot put, discus and hammer throw archery and shooting. The main application of it is used in missile launching.

## REFERENCES

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