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AN INVENTORY MODEL FOR TWO PRODUCTION UNITS

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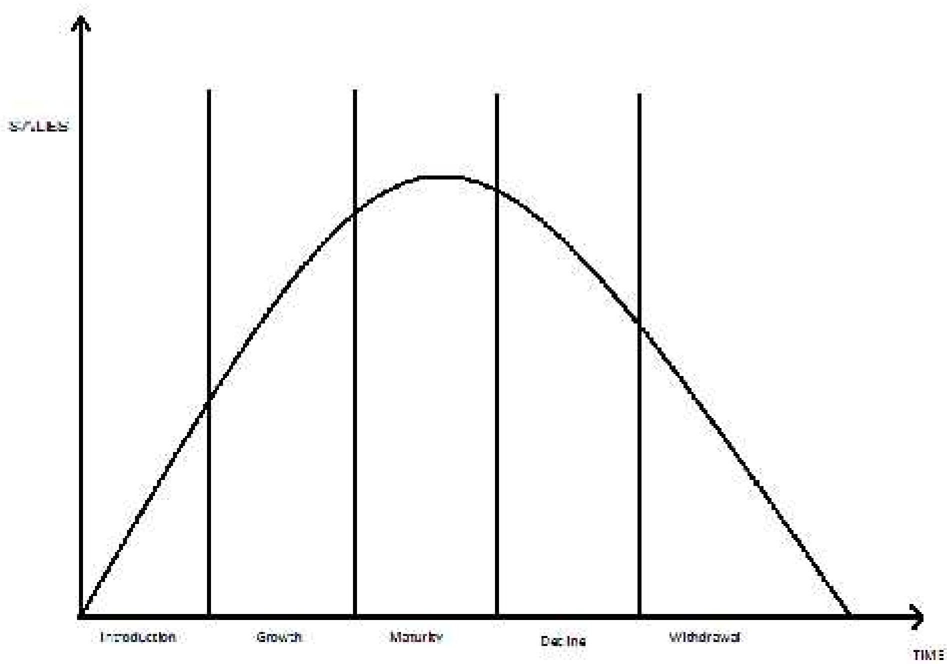
Abstract
An inventory model is developed in this paper for Product Life Cycle using two production units so as to minimize the total cost during the Product Life Cycle (PLC) as the existence of a unique machine results in limited production capacity. This paper investigates inventory control policies in manufacturing/remanufacturing system during the Product Life-Cycle, which consists of four phases: introduction, growth, maturity, and decline stages. A numerical example is presented and the results are compared with the model having single production unit.

Keywords: Imperfect production process, Inventory, Shortages, Product Life Cycle, Defective items, Production and Demand.

1. Introduction

Product Life Cycles have been widely discussed in marketing literature. The concept of the Product Life Cycle basically describes the evolution of a product, as measured by its sales over time. Product Life Cycles not only can be determined. They are particularly useful as marketing models. In an imperfect manufacturing process, a certain proportion of products become defective due to poor production quality and material defects, and subsequently defective products are scrapped if they are not re-workable or it is not cost-effective to do so. In a multi-stage production system, products move from one stage to the next stage, and every stage may yield a certain proportion of defective items. A Product Life Cycle is the cycle through which every product goes through from introduction to withdrawal from the market or eventual demise. Every product passes through a series of stages in the course of its life, with the total of the stages considered as the Product Life Cycle. At any given time, therefore, every product is located within one of four Life-Cycle stages-Introduction, Growth, Maturity and Decline which is characterized by the revenue generated by the product. Kotler [1] Present the Product Life Cycle concept as a marketing management tool for consumer branded products. Figure 1



product life cycle

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The concept of a Product Life Cycle (PLC) has occupied a prominent position in the marketing literature also as a forecasting instrument by Kovac et al. [2]. Erwin Van der Laan [3] introduces the robustness of the control parameters of the PUSH&PULL disposal strategy over the different stages of a Product Life Cycle. Hsuch [4] investigates inventory control policies in a manufacturing system during the Product Life Cycle, the closed form formulas of optimal production lot size re-order point &safety stock in each phase of product life cycle are derived. When items are produced internally instead of being obtained from an outside supplier, in the manufacturing, the Economic Production Quantity (EPQ) is often employed to determine the optimal product lot size that minimizes overall production /inventory costs. Because of its assumption that the production rate must be much larger than the demand rate, this model is known as the finite production model .In classic EPQ model it is assumed that manufacturing facility functions perfect during production run. In the developing classified EPQ models it has been assumed that the product quantity& production process are perfect. Large number of research efforts has been done to extend the work of Lee [5], in the recent years. Cardenas-Barron [6] developed an EPQ type inventory model with planned backorders for determining the EPQ for a single stage manufacturing system that generates imperfect quality products & all these defective products are reworked in the same cycle & also established the range of real values of proposition of defective products for which there is an optimal solution & the close form for total cost of inventory system. Cardenas -Barron [7] presented the mathematical expressions corrected in Sarker et al. [8] are corrected and the appropriate solution to the numerical example and also establish the closed forms for the optimal total inventory cost and the mathematical expressions for determining the total additional cost for working with a non-optimal solution for both polices that were not given by Sarker et al.[8] and presented a simple approach for determining the economic production quantity for an item with imperfect quality. Krishnamoorthi[9] developed an inventory model for product life cycle with maturity stage and defective items to minimize the total net inventory cot. In this paper an inventory model is developed for Product Life Cycle using two parallel production units so as to minimize the total cost during the Product Life Cycle (PLC) which consists of introduction, growth, maturity and decline stages and to investigate the optimum production lot size. The paper is organized as follows. Section 2 presents assumptions & notations. Section 3 describes the model. Analysis on total cost and optimum production lot size is presented in Section 4 and Section 5 describes an economic production lot size model for Product Life Cycle with maturity stage and concluded in Section 6.

2. Assumptions and Notations

The assumptions and notations of an inventory model of Product Life Cycle are as follows:

a) Assumptions

- The demand rate is known, constant and continuous.
- Items are produced by two production units A and B and added to the inventory.
- Shortages are not allowed.
- Items Produced by A & B are single products, it does not interact with any other inventory items.
- The sum of production rates of A and B will be always greater than or equal to the sum of the demand rate.
- During time (t_1), inventory is built up due to demand and defective items. A product enter growth and maturity stage at time (t_2) and(t_3), demand and production increases at the rate of 'a' times of $[(P_1+P_2)-D-(d_1+d_2)]$ and 'b' times of $[(P_1+P_2)-D-(d_1+d_2)]$ where 'a' and 'b' are constant thereafter inventory level declines continuously at a rate of $[D+d_1+d_2]$ and becomes zero at time $t_1+t_2+t_3+t_4$(end of a cycle).The process is repeated.

b) Notations

- P_1-Production rate of A per unit time.
- P_2-Production rate of B per unit time.
- D-Demand rate in units per unit time.
- Q_1-On hand inventory level.
- Q*-Optimal size of Production run.
- x-Proportion of defective items from regular production unit A(x is between 0&1)
- y-Proportion of defective items from regular production unit B(y is between 0&1)
- d_1- Rate of defective items during regular production from A($d_1=P_1 x$) .
- d_2- Rate of defective items during regular production from B($d_2=P_2y$)
- C_0-Setup cost.
- C_Q-Cost of quality.
- C_h-Holding cost per unit/year.
- C_P- Production /Purchase cost per unit.
- C_g-Cost of customer return cost of disposal, shipment& penalty)

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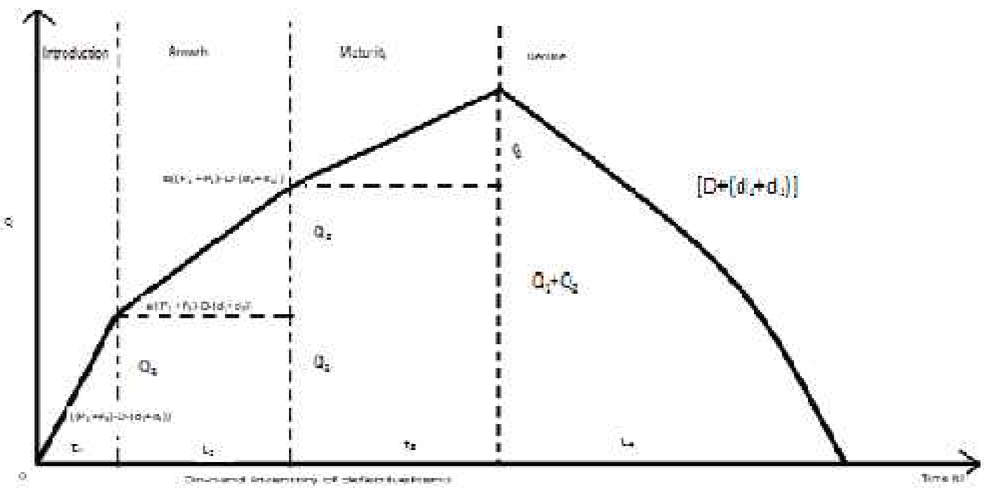
- C_d- Defective cost per item of imperfect quality from A.
- T-One cycle time.
- t_i - Unit time in periods i (i=1,2,3,...)
- T_c-Total cost.

3. Model Description

The mathematical model for optimal production lot size in this research is presented as follows. The cycle start at time t=0, in which the inventory level is zero. The production starts from two Production units and increases to the maximum inventory. During the production period t_1, inventory is increasing at the rate of (P_1+P_2) and simultaneously decreasing at the rate of $[D+d_1+d_2]$.Thus, the inventory accumulates at the rate of $[(P_1+P_2)-D-(d_1+d_2)]$ units. The Product enters growth stage at t_2. Production and increases at the rate of "a" time of $[(P_1+P_2)-D-(d_1+d_2)]$ (i.e.) a $[(P_1+P_2)-D-(d_1+d_2)]$ where "a" is a constant as more customers become aware of product and its benefits and additional market segments are targeted up to time t_2, and the product enters maturity stage at time t_3, the production and Demand increase at the rate of "b" times of $[(P_1+P_2)-D-(d_1+d_2)]$(i.e.) $b[(P_1+P_2)-D-(d_1+d_2)]$ where "b" is a constant and b>a. It is a most profitable stage. Because brand awareness is strong, advertising expenditure will be reduced. After that, the inventory level starts to decrease due to demand and defective items at a rate $[D+d_1+d_2]$ up to time t_4. The variation of the underlying inventory system for one cycle is shown in the following Figure2.



From the above fig 2, Time t_1 needed to build up Q_1 units of item.

$Q_1= [(P_1+P_2)-D-(d_1+d_2)]\, t_1$

$$t_1 = \frac{Q_1}{\left[(P_1+P_2)-D-(d_1+d_2)\right]} \qquad (1)$$

Time t_2 needed to build up Q_2 units of items

$$\therefore Q_2 = a[(P_1+P_2)-D-(d_1+d_2)]t_2$$

$$t_2 = \frac{Q_2}{a[(P_1+P_2)-D-(d_1+d_2)]} \qquad (2)$$

Time t_3 needed to build up Q_3

$$Q_3 = b[(P_1+P_2)-D-(d_1+d_2)]t_3$$

Time t_3 needed to build up Q_3,

$$t_3 = \frac{Q_3}{b[(P_1+P_2)-D-(d_1+d_2)]} \quad (3)$$

Therefore $Q_1 = \dfrac{Q[(P_1+P_2)-D-(d_1+d_2)]}{3[D+d_1+d_2]+(1+a+b)[(P_1+P_2)-D-(d_1+d_2)]}$

4. Analysis

Generally the total cost of a production system consists of three major costs. Such as setup cost, process cost, inventory holding cost, reworking cost due to reworking.

Rosenblatt and Lee [5] derived the total cost function as follows:
TC (t_1,t_2) = Setup cost+ Holding cost+ Rework cost

$$= \frac{DC_0}{Pt} + \frac{(P-D)tC_h}{2} + \frac{DC_R{\sim}\text{«}t}{2}$$

Chung and Hou [10]: have included the shortage cost

$$TC(t_1,t_2) = \frac{DC_0}{Pt} + \frac{(P-D)tC_hC_S}{2(C_h+C_S)} + \frac{DC_S}{Pt}E\left[D(N)\right]$$

Hou [10] has introduced the number of defectives as follows. Therefore, the total cost function is

$$TC(t_1,t_2) = \frac{DC_0}{Pt} + \frac{(P-D)tC_hC_S}{2(C_h+C_S)} + \frac{DC_R(1-e^{-\alpha t})}{Pt} + DC_{Sq2} + D(q_1-q_2)\frac{(1-e^{-\alpha t})}{\alpha t}$$

But in this research, total cost of the system TC(Q) is the accumulation of the setup cost, production cost, holding cost, cost of quality and cost of defective items. Therefore

i. Production cost

$$= \frac{1}{T}C_PQ = [D+d_1+d_2]C_P \quad (7)$$

ii. Ordering/Setup

$$\text{cost} = \frac{1}{T}C_0 = \frac{[D+d_1+d_2]}{Q}C_0 \quad (8)$$

iii. Holding cost: The holding cost is as follows (from 1 to 3)

$$HC = \frac{C_h}{T}\left(\frac{Q_1t_1}{2} + t_2Q_1 + \frac{Q_2t_2}{2} + t_3\left(Q_1+Q_2\right) + \frac{t_3Q_3}{2} + \frac{t_4\left(Q_1+Q_2+Q_3\right)}{2}\right)$$

$$HC = \frac{C_hQ[(P_1+P_2)-D-(d_1+d_2)][(5+3a+b)[D+d_1+d_2]+(1+a+b)^2[(P_1+P_2)-D-(d_1+d_2)]]}{2[3[D+d_1+d_2]+(1+a+b)[(P_1+P_2)-D-(d_1+d_2)]]} \quad (9)$$

iv. Cost of quality

$$= \frac{1}{T}(d_1+d_2)C_Q(t_1+t_2+t_3)$$

$$= \left[\frac{3\left[D+d_1+d_2\right]C_Q(P_1x+P_2y)}{\left[\left(P_1+P_2\right)-D-\left(d_1+d_2\right)\right](1+a+b)+3\left[D+d_1+d_2\right]}\right] \quad (10)$$

i. **Defective cost:** cost per defect passed forward customers (scrap & penalty cost)

DC $=\frac{1}{T}Q(x+y)C_d$

$$=\left[D+d_1+d_2\right](x+y)C_d \qquad (11)$$

Total cost=production cost +ordering cost +holding cost+ cost of quality+ defective cost.

Total cost (TC) =

$$[D+d_1+d_2]C_p+\frac{[D+d_1+d_2]C_0}{Q}$$
$$+\frac{C_hQ^2\left[(P_1+P_2)-D-(d_1+d_2)\right]\left[(5+3a+b)[D+d_1+d_2]+(1+a+b)^2\left[(P_1+P_2)-D-(d_1+d_2)\right]\right]}{2\left[3[D+d_1+d_2]+(1+a+b)\left[(P_1+P_2)-D-(d_1+d_2)\right]\right]^2} \qquad (12)$$

Differentiating with respect to Q

$$\frac{d(TC)}{dQ}=-\left[\frac{[D+d_1+d_2]C_0}{Q^2}\right]+\frac{C_hQ\left[(P_1+P_2)-D-(d_1+d_2)\right]\left[(5+3a+b)[D+d_1+d_2]+(1+a+b)^2\left[(P_1+P_2)-D-(d_1+d_2)\right]\right]}{2\left[3[D+d_1+d_2]+(1+a+b)\left[(P_1+P_2)-D-(d_1+d_2)\right]\right]^2}\rightarrow *$$

$$\frac{d^2(TC)}{dQ^2}=\frac{2[D+d_1+d_2]C_0}{Q^3}\succ 0$$

*FROM **

$$\frac{[D+d_1+d_2]C_0}{Q^2}=\frac{C_hQ\left[(P_1+P_2)-D-(d_1+d_2)\right]\left[(5+3a+b)[D+d_1+d_2]+(1+a+b)^2\left[(P_1+P_2)-D-(d_1+d_2)\right]\right]}{2\left[3[D+d_1+d_2]+(1+a+b)\left[(P_1+P_2)-D-(d_1+d_2)\right]\right]^2}$$

$$\therefore Q=\sqrt{\frac{2C_0[D+d_1+d_2]\left[3[D+d_1+d_2]+(1+a+b)\left[(P_1+P_2)-D-(d_1+d_2)\right]\right]^2}{C_hQ\left[(P_1+P_2)-D-(d_1+d_2)\right]\left[(5+3a+b)[D+d_1+d_2]+(1+a+b)^2\left[(P_1+P_2)-D-(d_1+d_2)\right]\right]}} \qquad (13)$$

ILLUSTRATIVE EXAMPLE :

consider the following parameter

$P_1=3000 unitA/year$

$P_2=2000 unitB/year$

$D=4500 units/year$

$C_0=Rs\,100; x=0.006; y=0.004; a=2; b=3; C_P=100$

$C_d=5; C_Q=5$

$C_h=10\,per\ units/year$

Optimum Solution

Q=800.40; Q_1=23.10; Q_2=46.2; T=0.1767;
Production cost=452600; Setup cost=565.47; Holding cost=565.46;
Defective cost=226.3; Cost of quality=107.48; Total cost=454064.98

Table 1, Variation of Defective items with inventory costs.

x	y	Q	T	Production cost	Setup cost	Holding cost	Defective cost	Quality cost	Total cost
0.006	0.004	800.40	0.1767	452600	565.47	565.46	226.3	107.48	454064.98
0.012	0.008	824.48	0.181	455200	552.11	552.10	455.2	217.24	456946.65
0.016	0.014	848.79	0.1856	457600	539.12	539.11	686.4	320.59	459685.2
0.018	0.022	872.93	0.1899	459800	526.74	526.74	919.6	417.07	461890.4
0.026	0.024	906.6	0.1958	462600	510.25	510.25	1156.5	542.31	465319.32
0.032	0.028	941.39	0.2021	465200	494.16	494.16	1395.6	661.09	467583.92
0.036	0.034	977.09	0.2087	467600	478.56	478.57	1636.6	772.89	470966.62
0.042	0.038	1020.45	0.2167	470200	460.78	460.78	1880.8	896.38	473898.74
0.055	0.035	1084.30	0.2288	473500	436.69	436.68	2130.75	1056.72	477560.84

5. An economic production lot size model for product life cycle with maturity stage Since the model is in maturity stage, defective cost is not added to the total cost. Values at maturity stage are calculated as follows:

$$Q_1 = [(P_1 + P_2) - D]t_1$$

$$\therefore t_1 = \frac{Q_1}{[(P_1 + P_2) - D]} \qquad (14)$$

$$Time\ t_2\ needed\ to\ build\ up\ Q_2\ units\ of\ items$$

$$\therefore Q_2 = a[(P_1 + P_2) - D]t_2$$

$$\therefore t_2 = \frac{Q_2}{a[(P_1 + P_2) - D]} \qquad (15)$$

$$Time\ t_3\ needed\ to\ build\ up\ Q_3,$$

$$Q_3 = b[(P_1 + P_2) - D]t_4$$

$$t_3 = \frac{Q_3}{b((P_1 + P_2 - D)} \qquad (16)$$

$$Time\ t_4\ needed\ to\ consume\ the\ \max imum\ on\ hand\ inventory\ Q_1 + Q_2 + Q_3$$

$$\therefore Q_1 + Q_2 + Q_3 = Dt_4$$

$$\therefore t_4 = \frac{Q_1 + Q_2 + Q_3}{D} \qquad (17)$$

$$Time\ t\ needed\ to\ consume\ all\ units\ Q\ at\ demand\ rate\ plus\ defects$$

$$Q = Dt$$

$$therefore\ t = \frac{Q}{D} \qquad (18)$$

$$From\ the\ triangular\ identies(\ from\ 14\ to\ 17)$$

$$\frac{t_1}{t_2} = \frac{Q_1}{Q_2} = \frac{[(P_1 + P_2) - D]}{a[(P_1 + P_2) - D]}$$

$$(i.e)Q_2 = aQ_1$$

$$\frac{t_1}{t_3} = \frac{Q_1}{Q_2} = \frac{[(P_1 + P_2) - D]}{a[(P_1 + P_2) - D]}$$

$$(i.e)Q_3 = bQ_1$$

$$Inventory\ level\ during\ production\ cycle$$

$$t = t_1 + t_2 + t_3 + t_4 \qquad (19)$$

$$\frac{Q}{D} = \frac{Q_1}{[(P_1 + P_2) - D]} + \frac{Q_2}{a[(P_1 + P_2) - D]} + \frac{Q_3}{b[(P_1 + P_2) - D]} + \frac{Q_1 + Q_2 + Q_3}{D}$$

$$\therefore Q_1 = \frac{Q[(P_1+P_2)-D]}{3D+(1+a+b)[(P_1+P_2)-D]}$$

Table 2,Effects of Demand and Defective Parameters on Optimal Polices

Parameters		Optimal values					production cost	Setup cost	Holding cost	Cost of quality	Defective cost	Total cost
		Q	Q_1	Q_2	t_1	T						
C_0	80	715.73	20.66	41.32	0.0436	0.1582	452600	505.89	505.89	107.48	226.3	453945.56
	90	759.15	21.91	43.82	0.0462	0.1676	452600	536.57	536.58	107.48	226.3	454006.93
	100	800.40	23.10	46.2	0.0487	0.1768	452600	565.47	565.73	107.48	226.3	454064.98
	110	839.27	24.22	48.45	0.0511	0.1854	452600	593.21	593.21	107.48	226.3	454120.20
	120	816.59	25.30	50.60	0.1068	0.1936	452600	619.58	619.58	107.48	226.3	454172.94
C_h	8	894.67	25.82	51.65	0.0545	0.1977	452600	505.88	505.89	107.48	226.3	453945.56
	9	843.50	24.35	48.69	0.0514	0.1865	452600	536.57	536.58	107.48	226.3	454006.93
	10	800.50	23.10	46.20	0.0487	0.1768	452600	565.47	565.73	107.48	226.3	454064.98
	11	762.97	22.02	44.04	0.0465	0.1687	452600	593.21	593.20	107.48	226.3	454120.19
	12	730.49	21.08	42.17	0.0445	0.1614	452600	619.58	619.58	107.48	226.3	454172.94
C_P	80	800.40	23.10	46.20	0.0487	0.1768	362080	565.47	565.47	107.48	226.3	363544.98
	90	800.40	23.10	46.20	0.0487	0.1768	407340	565.47	565.47	107.48	226.3	408804.98
	100	800.40	23.10	46.20	0.0487	0.1768	452600	565.47	565.47	107.48	226.3	454064.98
	110	800.40	23.10	46.20	0.0487	0.1768	497860	565.47	565.47	107.48	226.3	499324.98
	120	800.40	23.10	46.20	0.0487	0.1768	543120	565.47	565.47	107.48	226.3	544584.98
a	1	887.71	26.38	52.76	0.0557	0.1962	452600	509.85	509.85	110.68	226.3	453956.68
	2	800.40	23.10	46.20	0.0487	0.1768	452600	565.47	565.73	107.48	226.3	454064.98
	3	737.74	20.70	41.40	0.0431	0.1631	452600	613.50	613.50	104.47	226.3	454157.77
	4	690.48	18.84	37.68	0.0398	0.1527	452600	655.49	655.49	101.62	226.3	454238.90
	5	653.25	17.35	34.70	0.0366	0.1443	452600	692.84	692.84	98.92	226.3	454310.90
b	3	800.40	23.10	46.20	0.0487	0.1768	452600	565.47	565.73	107.48	226.3	454064.98
	4	773.52	21.70	43.40	0.0458	0.1710	452600	585.12	585.12	104.47	226.3	454101.01
	5	748.84	20.43	40.86	0.0431	0.1654	452600	604.40	604.40	101.62	226.3	454136.72
	6	726.09	19.29	38.58	0.0407	0.1604	452600	623.34	623.33	98.62	226.3	454171.89
	7	705.16	18.25	36.50	0.0385	0.1558	452600	641.84	641.84	96.36	226.3	454206.34

Production cost $= \frac{1}{T} C_P Q = DC_P$

1) Ordering/setup cost

$$= \frac{1}{T} C_0 = \frac{D}{Q} C_0 \qquad (21)$$

3) Holding cost: the holding cost is as follows

$$HC = \frac{C_h}{T}\left[\frac{Q_1 t_1}{2} + t_2 Q_1 + \frac{Q_2 t_2}{2} + t_3(Q_1+Q_2) + \frac{t_3 Q_3}{2} + \frac{t_4(Q_1+Q_2+Q_3)}{2}\right]$$

$$= \frac{C_h Q[(P_1+P_2)-D][(5+a+b)D+(1+a+b)^2[(P_1+P_2)-D]]}{2\left[3D+(1+a+b)[(P_1+P_2)-D]\right]} \qquad (22)$$

Total cost=Production cost+ Setup cost+ Holding cost

$$TC= DC_p + \frac{D}{Q}C_0 + \frac{C_h Q[(P_1+P_2)-D][(5+3a+b)D+(1+a+b)^2[(P_1+P_2)-D]]}{2[3D+(1+a+b)[(P_1+P_2)-D]]^2}$$

Diff. with respect to Q

$$\frac{d}{dc(TC)} = -\frac{D}{Q^2}C_0 + \frac{C_h Q[(P_1+P_2)-D][(5+3a+b)D+(1+a+b)^2[(P_1+P_2)-D]}{2[3D+(1+a+b)[(P_1+P_2)-D]]^2}$$

$$\frac{d^2}{dQ^2}(TC) = \frac{2D}{Q^3}C_0 > 0$$

$$\frac{D}{Q^2}C_0 = \frac{C_h Q[(P_1+P_2)-D][(5+3a+b)D+(1+a+b)^2[(P_1+P_2)-D]}{2[3D+(1+a+b)[(P_1+P_2)-D]]^2}$$

Therefore

$$Q = \sqrt{\frac{2DC_0[3D+(1+a+b)[(P_1+P_2)-D]^2}{C_h[(P_1+P_2)-D][(5+3a+b)D+(1+a+b)^2[(P_1+P_2)-D]}} \qquad (23)$$

ILLUSTRATIVE EXAMPLE :

consider the following parameter

$P_1 = 3000 unitA/year$

$P_2 = 2000 unitB/year$

$D = 4500 units/year$

$C_0 = Rs\,100; x = 0.006; y = 0.004; a = 2; b = 3; C_P = 100$

$C_d = 5; C_Q = 5$

$C_h = 10\, per\ units/year$

Optimum Solution

Q=777.82; Q_1=23.75; Q_2=47.14; T=0.1727;
Production cost=450000; Setup cost=578.54; Holding cost=578.54;
Total cost=451157.08;

6. Conclusion

In this model an optimal production lot size and total cost are derived using two production units and the results are compared with the model having single production unit [9]. From the comparative study it is concluded that if we receive item for inventory from two production units total cost will be reduced and shortages will be avoided. The proposed model can be used in inventory control of certain items such as fashionable commodities, electronic components, super market etc. The constraint is demand must be less than the production. Using this we have accurately determined the optimal quantity, cycle time and annual total cost.

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