ISSN 2319 – 1953
International Journal of Scientific Research in Computer Science Applications and Management Studies

# Analysis of Two Echelon Perishable Inventory System in Supply Chain with Partial Backlogging

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***Abstract*— This paper presents a continuous review of two echelon perishable inventory systems in which two different items in stock are considered, one is main product and another is compliment item for the main product. The operating policy at the lower echelon for the main product follows (s, S) policy whenever the inventory level drops to 's' an order for Q = (S-s) items is placed, the ordered items are received after a random time which is distributed as exponential. It is also assumed that in nature the main product is perishable and it perishes with exponential rate □ > 0. We assume that the demands accruing during the stock-out period are partially backlogged. The retailer replenishes the stock for main product from the supplier follows (0, M) policy. The compliment product is replenished instantaneously from local supplier. In the steady state case, the joint probability disruption of the inventory levels of main product, compliment item at retailer and the main product at supplier are obtained. Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of numerical examples, which provide insight into the behavior of the system, are presented.**

***Keywords*— Two-echelon, Perishable Inventory system, Positive lead time, Supply chain, Partial Backlogging**

## [1].INTRODUCTION

Study on multi-echelon systems are much less compared to those on single commodity systems. The determination of optimal policies and the problems related to a multi-echelon systems are, to some extent, dealt by Veinott and Wagner [17] and Veinott[18]. Sivazlian discussed the stationary characteristics of a multi commodity single period inventory system.The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain )when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw – materials, work-in process or finished goods.

The main objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature.

As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies Started with the work of Harris[9]. Clark and Scarf [7] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size. One of the oldest papers in the field of continuous review multi-echelon inventory system is written by Sherbrooke in 1968.Hadley, G and Whitin, T. M.,[6], Naddor .E [14] analyses various inventory

Systems.HP's(Hawlett Packard) Strategic Planning and Modeling(SPaM) group initiated this kind of research in 1977.

Sivazlian and Stanfel [16] analyzed a two commodity single period inventory system. Kalpakam and Arivarignan [10] analyzed a multi-item inventory model with renewal demands under a joint replenishment policy. They assumed instantaneous supply of items and obtain various operational characteristics and also an expression for the long run total expected cost rate. Krishnamoorthyet.al., [11] analyzed a two commodity continuous review inventory system with zero lead time. A two commodity problem with Markov shift in demand for the type of commodity required, is considered by Krishnamoorthy and Varghese [12]. They obtain a characterization for limiting probability distribution to be uniform. Associated optimization problems were discussed in all these cases. However in all these cases zero lead time is assumed.

In the literature of stochastic inventory models, there are two differentassumptions about the excess demand unfilled from existing inventories:the backlog assumption and the lost sales assumption. The former ismore popular in the literature partly because historically the inventorystudies started with spare parts inventory management problems in militaryapplications, where the backlog assumption is realistic. Howeverin many other business situations, it is quite often that demand that cannot be satisfied on time is lost. This is particularly true in a competitive business environment. For example in many retail establishments, such as a supermarket or a department store, a customer chooses a competitive brand or goes to another store if his/her preferred brand is out of stock.

All these papers deal with repairable items with batch ordering. A Complete review was provided by Benito M. Beamon[5]. Sven Axsater[1] proposed an approximate model of inventory structure in SC. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Anbazhagan and Arivarignan [2,3] have analyzed two commodity inventory system undervarious ordering policies. Yadavalliet. al., have analyzed a model with joint ordering policy and varying order quantities. Yadavalliet. al., [20] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

In a very recent paper, Anbazhaganet. al. [4] considered analysis of two commodity inventory system with compliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item is also assumed when the items are out of stock. The above model is studied only at single location( Lower echelon). We extend the same in to multi-echelon structure (Supply Chain). The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 provides Numerical examples and sensitivity analysis.

*MODEL*

*The Problem Description*

The inventory control system considered in this paper is defined as follows. A finished perishable product is supplied from manufacturer to supplier which adopts (0, M) replenishment policy then the product is supplied to retailer who adopts (s, S) policy. The retailer also maintain inventory of the compliment product which has instantaneous replenishment from local supplier. The demand at retailer node follows an independent Poisson distribution with rate $\lambda_i$ (i = 1, 2) for main product and compliment respectively. Demands accruing during the stock out periods of main product are backlogged up to some finite number b. The main product perishes with exponential rate □ > 0. The replenishment of items in terms of product is made from supplier to retailer is administrated with exponential distribution having parameterThe maximum inventory level at retailer node for main product is 'S', and the reorder point is 's' and the ordering quantity is Q (=S-s) items. The maximum inventory at supplier is M (=nQ) .

*1.1. Notations and Variables*

We use the following notations and variables for the analysis of the paper.

| Notations /variables | Used for |
|---|---|
| $[C]_{ij}$ | *The element of sub matrix at (i,j)$^{th}$ position of C* |
| 0 | *Zero matrix* |
| $\lambda_1, \lambda_2$ | *Mean arrival rate for Main& Compliment product at retailer* |
| $\mu$ | *Mean replacement rate for main product at retailer* |
| $\gamma$ | *Perishable rate for main product* |
| S, N | *Minimum inventory level for main& Compliment product at retailer* |
| s | *reorder level for main product at retailer* |
| b | *Backlog capacity* |
| M | *Maximum inventory level for main product at supplier* |
| $H_m$ | *Holding cost per item for main product at retailer* |
| $H_c$ | *Holding cost per item for Compliment product at retailer* |
| $H_d$ | *Holding cost per item for main product at distributor* |
| $O_m$ | *Ordering cost per order for main product at retailer* |
| $O_c$ | *Ordering cost per order for compliment product at retailer* |
| $O_d$ | *Ordering cost per order for main product at retailer* |
| $I_m$ | *Average inventory level for main product at retailer* |
| $I_c$ | *Average inventory level for compliment product at retailer* |
| $I_m$ | *Average inventory level for main product at retailer* |
| $R_d$ | *Mean reorder rate for main product at supplier.* |
| $R_c$ | *Mean reorder rate for compliment product at retailer* |
| $R_m$ | *Mean reorder rate for main product at retailer* |
| $S_m$ | *Shortage rate for main product at retailer* |
| $T_m$ | *Penalty rate for main product at retailer* |
| $\sum_{i=Q}^{nQ} i$ | $Q + 2Q + 3Q + ........ + nQ$ |

*2. Analysis*

Let Im(t) and Ic(t) denote the on hand Inventory levels of Main product, Compliment product at retailer and

Id(t) denote the on hand inventory level of Main product at supplier at time t+.

We define I(t) = {(Im(t), Ic(t), Id(t)):t ≥ 0} as Markov process with state space E{(i, j, k) | i = S, S-1, ....s,..1, 0, -1, -2, ... –b, j=1, 2, ....., N, k=Q, 2Q, ......nQ}.

Since E is finite and all its states are aperiodic, recurrent, non-null and also irreducible. That is all the states are Ergodic. Hence the limiting distribution exists and is independent of the initial state.

The infinitesimal generator matrix of this process C = ( a ( i, j, k, :l, m, n)) ( i , j , k )( l , m , n )□E can be obtained from the following arguments.

- The arrival of a demand for main product at retailer make a state transition in the Markov process from
  (i, j, k ) to ( i-1, j-1, k ) with the intensity of transition $\lambda_1 > 0$.
- The perish or an item for main product at retailer make a state transition in the Markov process from (i, j, k ) to ( i-1, j, k ) with the intensity of transition i γ.
- The arrival of a demand for compliment product at retailer make a state transition in the Markov process from (i, j, k ) to ( i, j-1, k ) with the intensity of transition $\lambda_2 > 0$.
- The replacement of inventory at retailer make a state transition in the Markov process from (i, j, k ) to ( i+Q, j, k-Q ) or (i, j, Q ) to ( i+Q, j, nQ ) with the intensity of transition $\mu > 0$

The infinitesimal generator **C** is given by

$$C=\begin{bmatrix} A & B & 0 & .... & 0 & 0 \\ 0 & A & B & .... & 0 & 0 \\ & & & .... & & \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & \ldots & A & B \\ B & 0 & 0 & \ldots & 0 & A \end{bmatrix}$$

The entries of C are given by

$$[A_2]_{ij} = \begin{cases} \lambda_2 & i = j; i = S, S-1, \ldots\ldots, 1 \\ \lambda_1 + i\gamma & i+1 = j; s, s+1, \ldots\ldots 1, 0, -1, \ldots. -(b+1) \\ 0 & otherwise \end{cases}$$

$$[B_1]_{ij} = \begin{cases} \mu & i + Q = j;\ s, s-1, \ldots\ldots 1, 0, -1, -2, \ldots., -b \\ 0 & otherwise \end{cases}$$

*2.1. Steady State Analysis*

$$[C]_{pq} = \begin{cases} A & p = q; & q = nQ, (n-1)Q, \ldots.Q \\ B & p = q + Q & q = (n-1)Q, \ldots.Q \\ B & p = q - (n-1)Q & q = nQ \\ 0 & otherwise \end{cases}$$

Where the entries of the matrices are given by

$$[A]_{ij} = \begin{cases} A_1 & i = j; & i = 1, 2, \ldots N \\ A_2 & i+1 = j; & i = 1, 2, \ldots (N-1) \\ A_2 & i - (N-1) = j; & i = N \\ 0 & otherwise \end{cases}$$

$$[B]_{ij} = \begin{cases} B_1 & i = j;\ i = 1, 2, \ldots, N \\ 0 & otherwise \end{cases}$$

The elements in the sub matrices of A and B are

$$[A_1]_{ij} = \begin{cases} -(\lambda_1 + i\gamma + \lambda_2) & i = j;\ i = S, S-1, \ldots\ldots (s+1) \\ -(\lambda_1 + i\gamma + \lambda_2 + \mu) & i = j;\ s, s+1, \ldots\ldots 1, 0, -1, \ldots. -(b+1) \\ (\lambda_2 + \mu) & i = j;\ i = 0 \\ 0 & otherwise \end{cases}$$

vi. $\prod_{i-1}^{(n-1)Q} B_1 + \prod_i^{Q} A_2 + \prod_{i-1}^{Q} A_1 = 0$

i = S, S-1,…….,2,1,0,-1,-2…-(b+2)

By solving the above system of equations, together with normalizing condition $\sum_{(i,j,k)\in E} \prod_{i,j}^{k} = 1$, the steady probability of all the system states are obtained.

*1. Operating characteristic*

In this section we derive some important system performance measures.

The structure of the infinitesimal matrix C, reveals that the state space E of the Markov process {I (t): t ≥ 0 } is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$$\prod_{i,j}^{k} = \lim_{t\to\infty} Pr\{(I_m(t), I_c(t), I_d(t) = (i, j, k))\}$$

$\Pi_{i,j}^{k}$ is where the steady state probability that the system be in state *(i, j, k)*.

Let $\Pi = \{\Pi_{i,j}^{nQ}, \Pi_{i,j}^{(n-1)Q}, \ldots\ldots\ldots. \Pi_{i,j}^{Q}\}$ denote the steady state probability distribution. For each $((i, j, k), \Pi_{i,j}^{k})$ $((i, j, k),$ be obtained by solving the matrix equation $\prod C = 0$.

The system of equations may be written as follows

i. $\prod_j^{nQ} A_1 + \prod_i^{nQ} A_2 + \prod_j^{Q} B_1 = 0$

i = 1; j = S

ii. $\prod_j^{nQ} B_1 + \prod_j^{(n-1)Q} A_1 + \prod_i^{(n-1)Q} A_2 = 0$

i = 1; j = S

iii. $\prod_j^{(n-1)Q} B_1 + \prod_j^{Q} A_1 + \prod_i^{Q} A_2 = 0$

i = 1; j = S

iv. $\prod_j^{nQ} A_2 + \prod_{i-1}^{nQ} A_1 + \prod_{i-1}^{Q} B_1 = 0$

i = S, S-1,…….,2,1,0,-1,-2…-(b+2)

$\prod_{i-1}^{nQ} B_1 + \prod_i^{(n-1)Q} A_2 + \prod_{i-1}^{(n-1)Q} A_1 = 0$

i = S, S-1,…….,2,1,0,-1,-2…-(b+2)

*1.1. Average inventory Level*

The event $I_m$, $I_c$ *and* $I_d$ denote the average inventory level for main product, complement product at retailer and main product at distributor respectively,

$$(i)\ I_m = \sum_{k=Q}^{nQ}\sum_{j=1}^{N}\sum_{i=0}^{S} i \prod_{i,j}^{k}$$

$$(ii)\ I_c = \sum_{k=Q}^{nQ}\sum_{i=0}^{S}\sum_{j=1}^{N} j \prod_{i,j}^{k}$$

$$(iii)\ I_d = \sum_{i=0}^{S}\sum_{j=1}^{N}\sum_{k=Q}^{nQ} k \prod_{i,j}^{k}$$

*1.2Mean Reorder Rate*

Let $R_m$ , $R_c$ , *and* $R_d$ be the mean reorder rate for main product, complement product at retailer and main product at distributor respectively,

$$(i)\ R_m = \left(\lambda_1 + \left(S+1\right)\gamma\right) \sum_{k=Q}^{nQ}\sum_{j=1}^{N} \prod\nolimits_{S+1,j}^{k}$$

$$(ii)\ R_c = \left(\lambda_1 + \lambda_2\right) \sum_{k=Q}^{nQ}\sum_{i=0}^{S} \prod\nolimits_{i,1}^{k}$$

$$(iii)\ R_d = \mu \sum_{i=0}^{S}\sum_{j=1}^{N} \prod\nolimits_{i,j}^{Q}$$

*1.3. Shortage rate*

Shortage occur at retailer only for main product. Let $S_m$ be the shortage rate at retailer for main product, then

$$(i)\ S_m = \lambda \sum_{1}^{nQ}\sum^{N} \prod\nolimits_{-b,j}^{k}$$

2. *Cost analysis*

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time. The long run expected cost rate TC(s, Q)is given by

$$TC(s,Q) = I_m.H_m + I_c H_c + I_d H_d + R_m.O_m + R_c.O_c + R_d.O_d + S_m.T_m$$

Although we have a not proved analytically the convexity of the cost function *TC(s, Q)* our experience with considerable number of numerical examples indicate that *TC(s, Q)* for fixed '***S***' appears to be convex in s. In some cases it turned out to be increasing function of s. For large number case of *TC(s, Q)* revealed a locally convex structure. Hence we adopted the numerical search procedure to determine the optimal value of '***s***'

*3. Numerical Example and Sensitivity Analysis*

3.*1. Numerical Example*

In this section we discuss the problem of minimizing the structure. We assume $H_c \leq H_m \leq H_d$, i.e, the holding cost for compliment product is at retailer node is less than that of main product at retailer node and the holding cost of main product is less than that of main product at distributor node. Also $O_c \leq O_m \leq O_d$ the ordering cost at retailer node for compliment product is less than that of main product. Also the ordering cost at the distributor is greater than that of compliment product at retailer node.

The results we obtained in the steady state case may be illustrated through the following numerical example,

S =16, N=15, M = 80, $\lambda_1 = 3$ , $\lambda_2 = 2$ , $\gamma = 2$, b = 3, $\mu = 3$ ,

$H_c = 1.1,\ H_m = 1.2,\ H_d = 1.3$

$O_c = 2.1,\ O_m = 2.2,\ O_d = 2.3\ T_m = 3.1$

The cost for different reorder level are given by

Table:1. Total expected cost rate as a function s and **Q**

| ***S*** | 1 | 2 | 3 | 4* | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| ***Q*** | 15 | 14 | 13 | 12* | 11 | 10 | 9 |
| ***TC(s, Q)*** | 155.9 | 90.7 | 77.6 | 75.3* | 80.8 | 94.6 | 105.9 |

For the inventory capacity S, the optimal reorder level s* and optimal cost *TC(s, Q)*are indicated by the symbol *. The Convexity of the cost function is given in the graph.

*1.1. Sensitivity Analysis*

Below tables are represented a numerical study to exhibit the sensitivity of the system on the effect of varying different parameters.

$\lambda_1$ & $\mu$, $\lambda_2$ & $\mu$ , $H_c$ & $H_m$; $H_m$ & $H_d$ , $O_m$ & $O_c$ , $O_m$ & $P_d$ ;

For the following cost structure S =16, s = 4, N=15, M = 80, $\lambda_1 = 3$ , $\lambda_2 = 2$ , $\gamma$ = 2, $\mu$ $= 3\ H_c = 1.1,\ H_m = 1.2,\ H_d = 1.3$, $O_c = 2.1,\ O_m = 2.2,\ O_d = 2.3\ T_m = 3.1$.

Table: 2 Effect on Replenishment rate& Demand rates $\mu \backslash \lambda_1$

| $\mu \backslash \lambda_1$ | *1* | *2* | *3* | *4* | *5* |
|---|---|---|---|---|---|
| *1* | *42.9562* | *104.736* | *182.073* | *261.016* | *340.248* |
| *2* | *62.192* | *79.6682* | *131.526* | *203.329* | *282.382* |
| *3* | *82.6582* | *99.861* | *119.95* | *166.608* | *233.24* |
| *4* | *98.8989* | *126.775* | *140.663* | *164.368* | *209.307* |
| *5* | *113.624* | *150.974* | *170.738* | *186.085* | *212.553* |

Table:3 Effect on Replenishment rate& Demand rates $\mu \backslash \lambda_2,$

| $\mu \backslash \lambda_2,$ | *1* | *3* | *5* | *7* | *9* |
|---|---|---|---|---|---|
| *1* | *263.4122* | *185.988* | *224.608* | *307.413* | *387.178* |
| *3* | *260.1814* | *160.193* | *170.85* | *228.599* | *287.558* |
| *5* | *259.5082* | *154.149* | *157.612* | *208.956* | *258.907* |
| *7* | *259.2163* | *151.461* | *151.627* | *200.05* | *247.266* |
| *9* | *260.0576* | *150.94* | *148.217* | *194.96* | *240.605* |

Table 4: Effect on Holding cost ( $H_m \backslash H_d,$ )

| $H_m \backslash H_d$ | *1.1* | *2.1* | *3.1* | *4.1* | *5.1* |
|---|---|---|---|---|---|
| *1.1* | *167.728* | *168.91* | *170.93* | *172.95* | *174.9* |
| *2.1* | *170.92* | *173.94* | *174.95* | *176.98* | *180.0* |
| *3.1* | *174.959* | *176.979* | *178.996* | *181.016* | *183.0* |
| *4.1* | *178.9885* | *181.001* | *183.028* | *185.045* | *187.0* |
| *5.1* | *183.0205* | *186.038* | *187.058* | *190.077* | *192.0* |

Table 5: Effect on Ordering Cost ( $H_c \backslash H_m,$ )

| $H_c \backslash H_m$ | *2.1* | *2.2* | *2.3* | *2.4* | *2.5* |
|---|---|---|---|---|---|
| *2.1* | *166.7286* | *167.299* | *167.77* | *168.30* | *168.83* |
| *2.2* | *167.2995* | *167.826* | *168.35* | *158.88* | *170.40* |
| *2.3* | *167.873* | *168.400* | *168.92* | *170.45* | *170.97* |
| *2.4* | *168.4464* | *168.400* | *169.49* | *170.02* | *171.55* |
| *2.5* | *170.0198* | *170.544* | *170.07* | *171.59* | *171.12* |

It is observed that from the table, the total expect cost *TC(s, Q)* is increases with the cost increases.

## II. Conclusion

This paper deals with a two echelon perishable inventory system with two products namely maim and compliment product. The demand at retailer node follows independent Poisson with rate $\lambda_1$ for main product and $\lambda_2$ for compliment product. If the demand occur for the main product then it is also the demand for the compliment product. But the compliment product demand donot disturb the main product. The structure of the chain allows vertical movement of goods from to supplier to retailer. If there is no stock for main product at retailer the demand is backlogged. The model is analyzed within the framework of Markov processes. Joint probability distribution of inventory levels at DC and Retailer for both products are computed in the steady state. Various system performance measures are derived and the long-run expected cost is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values on the total expected cost rate. It would be interesting to analyze the problem discussed in this paper by relaxing the assumption of exponentially distributed lead-times to a class of arbitrarily distributed lead-times using techniques from renewal theory and semi-regenerative processes. Once this is done, the general model can be used to generate various special eases.

## References


[1]. Axsater, S. (1993). Exact and approximate evaluation of batch ordering policies for two level inventory systems. Oper. Res. 41. 777-785.

[2]. Anbazhagan N, Arivarignan G. Two-commodity continuous review inventory system withCoordinated reorder policy. International Journal of Information and Management Sciences. 2000; 11(3): 19 - 30.

[3]. Anbazhagan N, Arivarignan G. Analysis of two-commodity Markovian inventory system with lead time. The Korean Journal of Computational and Applied Mathematics. 2001; 8(2): 427-438.

[4]. Anbazhagan N, Jeganathan. K. Two-Commodity Markovian Inventory System with Compliment and Retrial Demand: British Journal of Mathematics & Computer Science: 3(2), 115-134, 2013.

[5]. Benita M. Beamon. (1998). Supply Chain Design and Analysis: Models and Methods. International Journal of Production Economics.Vol.55, No.3, pp.281-294.

[6]. Cinlar .E, Introduction to Stochastic 'Processes, Prentice Hall, Engle-wood Cliffs, NJ, 1975.

[7]. Clark, A. J. and H. Scarf, (1960). Optimal Policies for a Multi-Echelon Inventory Problem.Management Science, 6(4): 475-490.

[8]. Hadley, G and Whitin, T. M., (1963), Analysis of inventory systems, Prentice- Hall.

[9]. Englewood Cliff, Harris, F., 1915, Operations and costs, Factory management series, A.W. Shah Co., Chicago,48 - 52.

[10]. Kalpakam S, Arivarigan G. A coordinated multicommodity (s, S) inventory system. Mathematical and Computer Modelling. 1993;18:69-73.

[11]. Krishnamoorthy A, IqbalBasha R, Lakshmy B. Analysis of a two commodity problem. International Journal of Information and Management Sciences. 1994;5(1):127-136.

[12]. Krishnamoorthy A, Varghese TV. A two commodity inventory problem. International Journal of Information and Management Sciences. 1994;5(3):55-70.

[13]. Medhi .J,(2009) Stochastic processes, Third edition, New Age International Publishers, New Delhi.

[14]. Naddor.E (1966), Inventory System, John Wiley and Sons, New York.

[15]. Sivazlian BD. Stationary analysis of a multicommodity inventory system with interacting set-up costs. SIAM Journal of Applied Mathematics. 1975;20(2):264-278.

[16]. Sivazlian BD, Stanfel LE. Analysis of systems in Operations Research. First edition. Prentice Hall; 1974.

[17]. Veinott AF, Wagner H.M. Computing optimal (s, S) inventory policies. Management Science. 1965;11:525-552.

[18]. Veinott AF. The status of mathematical inventory theory. Management Science 1966;12:745-777.

[19]. Yadavalli VSS, Anbazhagan N, Arivarignan G. A two commodity continuous review inventory system with lost sales. Stochastic Analysis and Applications. 2004;22:479-497.

[20]. Yadavalli VSS, De WC, Udayabaskaran S. A substitutable two-product inventory system with joint ordering policy and common demand. Applied Mathematics and Computation. 2006;172(2):1257-1271.