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DEVELOPING AND ANAYZINGOF PERISHABLE INVENTORY SYSTEM IN SUPPLY CHAINWITH DIRECT DEMNAD AND LOST SALE

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Abstract

In supply chain management, achieving effective coordination among manufacturers, distributors and retailers has become a pertinent research issue. Supply chaincoordination is a joint decision policy achieved by the manufacturer, distributor and a retailer.This work proposes ajoint two-level supply chain model with a singlemanufacturer, single distributor and a single-retailer handling two different products. The first product is a main product which is perishable in nature and the second one is complement product for the main product.The operating policy at the lower echelon for the main product is (s, S) that is whenever the inventory level drops to '**s**'on order forQ = (S-s) items is placed, the ordered items are received after a random timewhich is distributed as exponential. We assume that the demands accruing during the stock-out period are lost. The retailer replenishes the stockof main product from the supplier which adopts(0,M) policy having direct demand is assumed. The complement product is replenished instantaneously from local supplier.The joint probability disruption of the inventory levels of main product, complement item at retailer and the main product at supplier are obtained in the steady state case.Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of numerical examples, which provide insight into the behavior of the system, are presented.

Key Words: Perishable Inventory, Supply Chain, Positive lead time,Lost sale.

1. Introduction

Study on supply chain systems are much less compared to those on single location inventory systems. The determination of optimal policies and the problems related to a supply chain systems are, to some extent, dealt by Veinott and Wagner [18] and Veinott[19].Sivazlian [16] discussed the stationary characteristics of a multi commodity single period inventory system.The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain)when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw – materials, work-in process or finished goods.

The main objective for a supply chain inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire supply chain inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature.

As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies Started with the work of Harris**[9]**. Clark and Scarf **[7]** had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size. One of the oldest papers in the field of continuous review multi-echelon inventory system is written by Sherbrooke in 1968.Hadley, G and Whitin, T. M.,**[6]**, Naddor .E **[14]** analyses various inventory Systems.HP's(Hawlett Packard) Strategic Planning and Modeling(SPaM) group initiated this kind of research in 1977.

Sivazlian and Stanfel [17] analyzed a two commodity single period inventory system. Kalpakam and Arivarignan [10] analyzed a multi-item inventory model with renewal demands under a joint replenishment policy. They assumed instantaneous supply of items and obtain various operational characteristics and also an expression for the long run total expected cost rate. Krishnamoorthyet.al., [11] analyzed a two commodity continuous review inventory system with zero lead time. A two commodity problem with Markov shift in demand for the type of commodity required, is considered by Krishnamoorthy and Varghese [12]. They obtain a characterization for limiting probability distribution to be uniform. Associated optimization problems were discussed in all these cases. However in all these cases zero lead time is assumed.

Modeling of inventory for perishable items requires a characterization of the time to perishability. This time to perishability may be either deterministic or stochastic. The two most common models for perishability are outdated-ness due to reaching expiry date (e.g., food items or medicine) and sudden perishability due to disaster (e.g., spoilage because of extreme weather conditions). The perishability due to outdated-ness is typically modeled as a deterministic time to perishability and the perishability due to disaster is typically modeled as an exponential (or its discrete counterpart, geometric) time to perishability. This is because the memory-less property of these distributions often results in more tractable models. The treatment of lead time in the management of inventory for perishable items is also not trivial. One difficulty is that the items might perish during the delivery time. This might be addressed by assuming that the supplier supplies fresh items upon their delivery and changes their lead time accordingly.

Another challenge in the analysis of perishable items is that items that have not yet perished may have different remaining shelf-life. (Either because items have a unique shelf-life, or because batches with common shelf-lives arrived in different periods.) Thus, the information required to completely characterize the on-hand and on-order inventory includes not only the quantity of inventory but also the remaining shelf-lives of each unit in the inventory. This increase in information requirement makes the analysis of multi-echelon supply chains for perishable products especially challenging (because even for standard items, such an analysis often relies on dynamic programming and suffers from the curse of dimensionality). Therefore, some important theoretical contributions developed in the analysis of perishable items address the information required to characterize the inventory level process[15].

In the literature of stochastic inventory models, there are two differentassumptions about the excess demand unfilled from existing inventories:the backlog assumption and the lost sales assumption. The former ismore popular in the literature partly because historically the inventorystudies started with spare parts inventory management problems in militaryapplications, where the backlog assumption is realistic. Howeverin many other business situations, it is quite often that demand that cannot be satisfied on time is lost. This is particularly true in a competitive business environment. For example in many retail establishments, such as a supermarket or a department store, a customer chooses a competitive brand or goes to another store if his/her preferred brand is out of stock.

All these papers deal with repairable items with batch ordering. A Complete review was provided by Benito M. Beamon**[5]**. Sven Axsater**[1]** proposed an approximate model of inventory structure in SC. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Anbazhagan and Arivarignan [2,3] have analyzed two commodity inventory system undervarious ordering policies. Yadavalliet. al., [20] have analyzed a model with joint ordering policy and varying order quantities. Yadavalliet. al., [21] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

In a very recent paper, Anbazhaganet. al. [4] considered analysis of two commodity inventory system withcompliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item is also assumed when the items are out of stock. The above model is studied only at single location(Lower echelon). We extend the same in to multi-echelon structure (Supply Chain). The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 provides Numerical examples and sensitivity analysis.

2. Model

2.1.The Problem Description

The inventory control system considered in this paper is defined as follows. A finished main product perishable in nature is supplied from manufacturer to supplier which adopts (0,M) replenishment policy then the product is supplied to retailer who adopts (s,S) policy. The retailer also maintainan inventory of the complement product which has instantaneous replenishment from local supplier. The demand at retailer node follows an independent Poisson distributionwith rate λ_i (i = 1, 2) for main product and complement respectively. Demands accruing during the stock out periods of main product are assumed to be lost.The direct demand at supplier follows Poisson distribution with rate λ_D. It is assumed that the main product is perishes only at retailer with exponential rate γ>0. replacement of item in terms of product is made from supplier to retailer is administrated with exponential distribution having parameter $\mu > 0$. The maximum inventory level at retailer node for main product is S, and the recorder point is s and the ordering quantity is Q(=S-s) items. The maximum inventory at supplier inM(=nQ) .

2.2.Notations and variables

We use the following notations and variables for the analysis of the paper.

Notations /variables	Used for
$[C]_{ij}$	The element of sub matrix at (i,j)th position of ***C***
0	Zero matrix
$\lambda_1, \lambda_2, \lambda_D$	Mean arrival rate for Main&Compliment product at retailer and Distributor respectively.
μ	Mean replacement rate for main product at retailer
γ	Perishable rate for main product at retailer node
S, N	Minimum inventory level for main& Compliment product at retailer
s	reorder level for main product at retailer
M	Maximum inventory level for main product at supplier
H_m	Holding cost per item for main product at retailer
H_c	Holding cost per item for Compliment product at retailer
H_d	Holding cost per item for main product at distributor
O_r	Ordering cost per order for main product at retailer
O_c	Ordering cost per order for compliment product at retailer

O_m	Ordering cost per order for main product at retailer
I_m	Average inventory level for main product at retailer
I_c	Average inventory level for compliment product at retailer
I_d	Average inventory level for main product at retailer
R_d	Mean reorder rate for main productatsupplier.
R_c	Mean reorder rate for compliment product at retailer
R_m	Mean reorder rate for main product at retailer
S_m	Shortage rate for main product at retailer
T_m	Penalty rate for main product at retailer
$\sum_{i=Q}^{nQ} i$	Q + 2Q + 3Q + ... + nQ

3. Analysis

Let $I_m\ (t)$and $I_c\ (t)$ denote the on hand Inventory levels of Main product, Compliment product at retailer and $I_d\ (t)$denote the on hand inventory level of Main product at supplier at time t+.

We define I (t) = $\{(I_m\ (t), I_c\ (t), I_d\ (t)) : \mathrm{t}\ \geq\ 0\}$as Markov process with state space E = { (i, j,k) | i = 0, ...S, j =1, 2, …, N, k=Q,2Q,.....nQ }. Since E is finite and all its states are aperiodic, recurrent, non- null and also irreducible. That is all the states are Ergodic. Hence the limiting distribution exists and is independent of the initial state.

The infinitesimal generator matrix of this process **C** = (a ($i, j, k, :l, m, n$))$_{(i,j,k)(l,m,n)\in E}$ can be obtained from the following arguments.

- The arrival of a demand or perish of an item for main product at retailer make a state transition in the Markov process from (i, j, k) to (i-1, j-1, k) with the intensity of transition $(\lambda_1 + i\gamma) > 0$.
- The arrival of a demand for compliment product at retailer make a state transition in the Markov process from (i, j, k) to (i, j-1, k) with the intensity of transition $\lambda_2 > 0$.
- The arrival of a demand for at supplier (Distributor) make a state transition in the Markov process from (i, j, k) to (i, j, k-Q) with the intensity of transition $\lambda_D > 0$
- The replacement of inventory atretailer make a state transition in the Markov process from (i, j, k) to (i+Q, j, k-Q) or (i, j, Q) to (i+Q, j, nQ) with the intensity of transition $\mu > 0$.

The infinitesimal generator **C** is given by

$$\mathbf{C}=\begin{bmatrix} A & B & 0 & .... & 0 & 0 \\ 0 & A & B & .... & 0 & 0 \\ \vdots & \vdots & \vdots & .... & \vdots & \vdots \\ 0 & 0 & 0 & .... & A & B \\ B & 0 & 0 & .... & 0 & A \end{bmatrix}$$

The entries of C are given by

$$[C]_{pq} = \begin{cases} A & p = q; \quad q = nQ, \quad (n-1)Q, \ldots Q \\ B & p = q + Q; \quad q = (n-1)Q, \ldots . Q \\ B & p = q - (n-1)Q \quad q = nQ \\ 0 & otherwise \end{cases}$$

Where the entries of the matrices are given by

$$[A]_{ij} = \begin{cases} A_1 & i = j; \quad i = 1, 2, \ldots N \\ A_2 & i + 1 = j; \quad i = 1, 2, \ldots (N-1) \\ A_2 & i - (N-1) = j; \quad i = N \\ 0 & otherwise \end{cases}$$

$$[B]_{ij} = \begin{cases} B_1 & i = j; \mathrm{i} = 1, 2, \ldots, N \\ 0 & otherwise \end{cases}$$

The elements in the sub matrices of A and B are

$$[A_1]_{ij} = \begin{cases} -(\lambda_1 + i\gamma + \lambda_2 + \lambda_D) & if \ \ i = j \ : i = S, S-1, \ldots \ldots . . (s+1) \\ -(\lambda_1 + i\gamma + \lambda_2 + \mu + \lambda_D) & if \ \ i = j \ : i = s, s-1, \ldots \ldots . . 1 \\ -(\lambda_2 + \lambda_D + \mu) & if \ \ i = j \ : i = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$[A_2]_{ij} = \begin{cases} \lambda_2 & if \ \ i = j \quad : i = S, S-1, \ldots \ldots . . 1{,}0 \\ \lambda_1 + i\gamma & if \ \ i + 1 = j \ : i = S, S-1, \ldots \ldots . . 1 \\ 0 & \text{otherwise} \end{cases}$$

$$[B_1]_{ij} = \begin{cases} \mu & if \ \ i + Q = j \ : i = s, s-1, \ldots \ldots . . 1{,}0 \\ 0 & \text{otherwise} \end{cases}$$

3.1.Steady State Analysis

The structure of the infinitesimal matrix C, reveals that the state space E of the Markov process { I (t) : t $\geq$ 0 } is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$$\prod_{i,j}^{k} = \lim_{t \to \infty} Pr\{(I_m(t), I_c(t), I_d(t) = (i, j, k))\}$$

where$\prod_{i,j}^{k}$ is the steady state probability that the system be in state *(i, j, k*).

Let $\prod = \left\{\prod_{i,j}^{nQ}, \prod_{i,j}^{(n-1)Q}, \ldots\ldots\ldots. \prod_{i,j}^{Q}.\right\}$denote the steady state probability distribution. For each (

(i, j, k) ,$\prod_{i,j}^{k}$ *can* be obtained by solving the matrix equation $\prod$ C = 0.

The system of equations may be written as follows

i. $\prod_{j}^{nQ} A_1 + \prod_{i}^{nQ} A_2 + \prod_{j}^{Q} B_1 = 0$ *i = 1; j = S*

ii. $\prod_{j}^{nQ} B_1 + \prod_{j}^{(n-1)Q} A_1 + \prod_{i}^{(n-1)Q} A_2 = 0$ *i = 1; j = S*

iii. $\prod_{j}^{(n-1)Q} B_1 + \prod_{j}^{Q} A_1 + \prod_{i}^{Q} A_2 = 0$ *i = 1; j = S*

iv. $\prod_{j}^{nQ} A_2 + \prod_{i-1}^{nQ} A_1 + \prod_{i-1}^{Q} B_1 = 0$ *i = S, S-1,…….,2*

v. $\prod_{i-1}^{nQ} B_1 + \prod_{i}^{(n-1)Q} A_2 + \prod_{i-1}^{(n-1)Q} A_1 = 0$ *i = S, S-1,…….,2*

vi. $\prod_{i-1}^{(n-1)Q} B_1 + \prod_{i}^{Q} A_2 + \prod_{i-1}^{Q} A_1 = 0$ *i = S, S-1,…….,2*

By solving the above system of equations, together with normalizing condition $\sum_{(i,j,k)\in E} \prod_{i,j}^{k} = 1$, the steady probability of all the system states are obtained.

4. Operating characteristic

In this section we derive some important system performance measures.

4.1.Average inventory Level

The event I_m,I_c*and* I_d denote the average inventory level for main product, complement product at retailer and main product at distributor respectively,

(i) $I_m = \sum_{k=Q}^{nQ} \sum_{j=1}^{N} \sum_{i=0}^{S} i.\prod_{i,j}^{k}$

(ii) $I_c = \sum_{k=Q}^{nQ} \sum_{i=0}^{S} \sum_{j=1}^{N} j.\prod_{i,j}^{k}$

(iii) $I_d = \sum_{i=0}^{S} \sum_{j=1}^{N} \sum_{k=Q}^{nQ} k.\prod_{i,j}^{k}$

4.2Mean Reorder Rate

Let R_m, R_c, *and* $R_d$be the mean reorder rate for main product, complement product at retailer and main product at distributorrespectively,

(i) $R_m = (\lambda_1 + i\gamma) \sum_{k=Q}^{nQ} \sum_{j=1}^{N} \prod_{s+1,j}^{k}$

(ii) $R_c = (\lambda_1 + \lambda_2) \sum_{k=Q}^{nQ} \sum_{i=0}^{S} \prod_{i,1}^{k}$

(iii) $R_d = (\mu+\lambda_D)\sum_{i=0}^{S}\sum_{j=1}^{N}\prod_{i,j}^{Q}$

4.3. Shortage rate

Shortage occur at retailer only for main product. LetS_m be the shortage rate at retailer for main product, then

(i) $S_m = \lambda_1 \sum_{k=Q}^{nQ}\sum_{j=1}^{N}\prod_{0,j}^{k}$

5. Cost analysis

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time. The long run expected cost rate *TC(s, Q)*is given by

$$TC(s, Q)= I_m.H_m + I_c.H_c + I_d.H_d + R_m.\ O_m + R_c.O_c + R_d.O_d + S_m.\ T_m$$

Although we have a not proved analytically the convexity of the cost function *TC(s, Q)*our experience with considerable numberof numerical examples indicate that *TC(s, Q)*for fixed '*S'* appears to be convex in s. In some cases it turned out to be increasing function of s. For large number case of *TC(s, Q)*revealed a locally convex structure. Hence we adopted the numerical search procedure to determine the optimal value of '*s*'

6. Numerical Example and Sensitivity Analysis

6.1.Numerical Example

In this section we discuss the problem of minimizing the structure. We assume$H_c \leq H_m \leq H_d$, i.e, the holding cost for compliment product is at retailer node is less than that of main product at retailer node and the holding cost of main product is less than that of main product at distributor node. Also$O_c \leq O_m \leq O_d$ the ordering cost at retailer node for compliment product is less than that of main product. Also the ordering cost at the distributor is greater thanthat of compliment product at retailer node.

The results we obtained in the steady state case may be illustrated through the following numerical example,

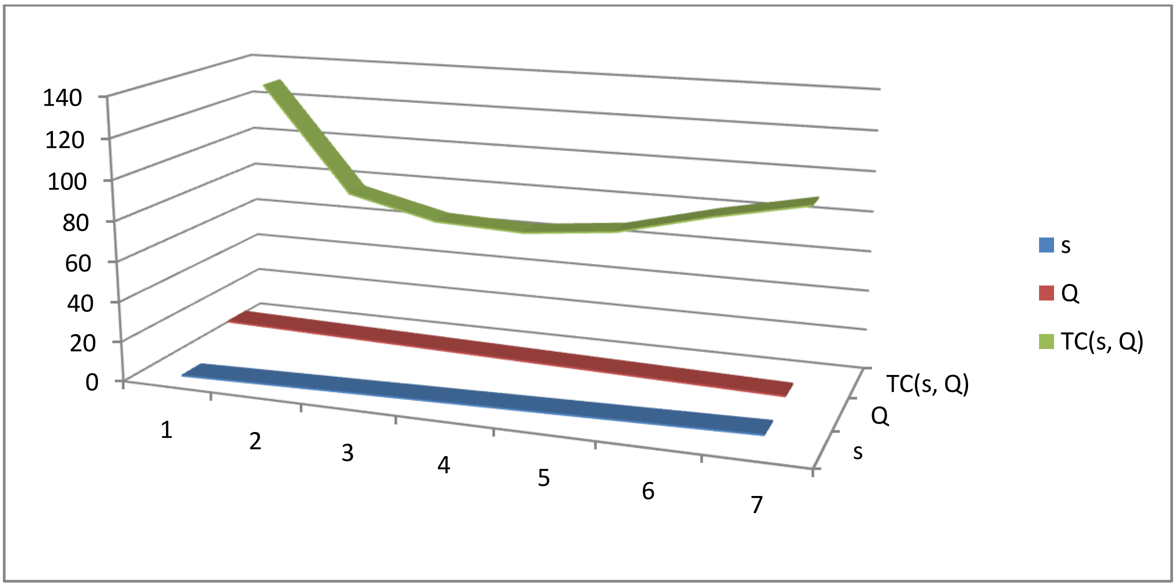
S =16, N=15, M = 80, $\lambda_1 = 3$, $\lambda_2 = 2$, $\lambda_D = 4$ $\mu = 3, \gamma = 2$ $H_c = 1.1,\ H_m = 1.2,\ H_d = 1.3$ $O_c = 2.1,\ O_m = 2.2,\ O_d = 2.3\ T_m = 3.1$

The cost for different reorder level are given by

s	1	2	3	4*	5	6	7
Q	15	14	13	12*	11	10	9
TC(s, Q)	129.919332	75.56006	64.65313	62.76118*	67.32296	78.81986	88.29787

Table:1. Total expected cost rate as a function s and Q

For the inventory capacity S, the optimal reorder level s* and optimal cost *TC(s, Q)*are indicated by the symbol *. The Convexity of the cost function is given in the graph.



6.2.Sensitivity Analysis

Below tables are represented a numerical study to exhibit the sensitivity of the system on the effect of varying different parameters.

$$\lambda_1 \,\&\, \mu_, \ \lambda_2 \,\&\, \mu \ , \mathrm{H}_c \& \mathrm{H}_{m;} \mathrm{H}_m \& \mathrm{H}_d , O_m \,\&\, \mathrm{O}_c , O_m \,\&\, \mathrm{P}_{d;}$$

For the following cost structureS =16,s = 4, N=15, M = 80, $\lambda_1 = 3$, $\lambda_2 = 2$, $\lambda_D = 4$ $\mu = 3$ $H_c = 1.1$, $H_m = 1.2$, $H_d = 1.3$, $O_c = 2.1$, $O_m = 2.2$, $O_d = 2.3$ $T_m = 3.1$.

Table:2Effect on Replenishment rate& Demand rates $\mu \setminus \lambda_1$

$\mu \setminus \lambda_1$	**1**	**2**	**3**	**4**	**5**
1	44.9562	105.736	183.073	262.016	341.248
2	64.192	80.6682	132.526	204.329	281.382
3	83.6582	100.861	120.95	167.608	234.24
4	99.8989	127.775	141.663	165.368	210.307
5	114.624	151.974	171.738	187.085	213.553

Table:3 Effect on Replenishment rate& Demand rates $\mu \setminus \lambda_{2,}$

$\mu \setminus \lambda_{2,}$	**1**	**3**	**5**	**7**	**9**
1	264.4122	186.988	225.608	308.413	388.178
3	261.1814	161.193	171.85	229.599	285.558
5	260.5082	155.149	158.612	209.956	259.907
7	260.2163	152.461	152.627	201.05	248.266
9	260.0576	150.94	149.217	195.96	241.605

Table 4: Effect on Holding cost ($H_m \setminus \mathrm{H}_{d,}$)

$H_m \setminus \mathrm{H}_{d,}$	**1.1**	**2.1**	**3.1**	**4.1**	**5.1**
1.1	167.7286	169.917	171.935	173.955	175.974
2.1	171.927	173.947	175.959	177.987	180.004
3.1	175.959	177.979	179.996	182.016	184.036
4.1	179.9885	182.001	184.028	186.045	188.065

5.1	184.0205	186.038	188.058	190.077	192.097

Table 5: Effect on Ordering Cost ($H_c \setminus \mathrm{H}_{m,}$)

$H_c \setminus \mathrm{H}_{m,}$	**2.1**	**2.2**	**2.3**	**2.4**	**2.5**
2.1	167.7286	168.2995	168.778	169.306	169.83
2.2	168.2995	168.8269	169.352	159.887	170.404
2.3	168.873	169.4003	169.925	170.45	170.977
2.4	169.4464	169.4003	170.499	171.023	171.551
2.5	170.0198	170.5446	171.072	171.597	172.122

Table 6 : Effect on Penalty Cost (O_m & O_d)

$O_m \setminus \mathrm{O}_d$	**3.1**	**3.2**	**3.3**	**3.4**	**3.5**
3.1	167.7286	168.3	168.778	169.306	169.83
3.2	168.2995	168.827	169.352	159.887	170.404
3.3	168.873	169.4	169.925	170.45	170.977
3.4	169.4464	169.4	170.499	171.023	171.551
3.5	170.0198	170.545	171.072	171.597	172.122

It is observed that from the table,the total expect cost *TC(s, Q)*is increases with the cost increases.

Conclusion

In this work, a mathematical model is developed for a joint two-echelon inventory system with a single-manufacturer, single distributor and a single-retailer. In the model development, the demand distribution at retailer is assumed as poison and replenishment distribution is assumed as exponential. Inventory associated costs like ordering/setup costs, carrying costs and shortage costs are considered for model development. Based on the optimality criterion, a computer programme is written in MATLAB and the model is solved. From the findings of the research, it is concluded that the curve representing the total relevant cost of the supply chain is convex with respect to replenishment quantity and reorder level. Also, from the sensitivity analysis, it is concluded that all the model parameters have the significant influence over the optimality of replenishment quantities and total relevant costs. Research findings and the novelty of the current model can be useful in consumer goods industries. Managerial decisions like replenishment quantities and reorder levels can be decided with the help of this model. The current work can be extended to a multi- echelon supply chain with wide variations and assumptions in demand function.

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