Perishable Inventory Control in Supply Chain : A Semi-MDP Model

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Abstract

In this article , we considered a perishable inventory control system in a two stage Supply chain. A two stage Supply Chain having Distribution Center (DC) and Retail Vendor (RV) with their respective inventory system has been considered. At retailer node, each demand is served with an item from inventory at that node after a random service time. For each service, one item from Perishable inventory is used. A two dimensional MDP is framed and the optimal decision policy is obtained by minimizing the total cost rate using linear programming technique. Numerical examples are provided to illustrate the model at many instances to prove the conjective that the service rate to be employed depends only on the number of customer in the system.

Introduction:

Supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products and the distribution of these finished products to customers. Supply Chain exists in both service and manufacturing organizations, but the complexity of the chain may vary greatly from industry to industry.

Inventory decision is an important component of the supply chain management, because Inventories exist at each and every stage of the supply chain as raw material or semi-finished or finished goods.   Further maintain perishable category of inventory in a supply chain is a tedious task rather than the stock with useful life time.   .They can also be as Work-in-process between the stages or stations and perishably this time period. Since holding of inventories can cost any where between 20% to 40% of their value, their efficient management is critical in Supply Chain operations

The usual objective for a multi-echelon inventory model is to coordinate the inventories of the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. The reason is that a key concept of supply chain management is that a company should strive to develop an informal partnership relation with its suppliers and retailers that enable them jointly to maximize their total profit.

Information technology has a substantial impact on supply chains. Scanners collect sales data at the point-of-sale, and Electronic Data Interchange(EDI) allows these data to be shared immediately with all stages of the supply chain. This technology has simplified the task of maintaining perishable inventory in SC because of instantly removed of perished item from stock in portable.

Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature. As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. Even though studies related to perishable inventory in SCM is very rare[11].

The first quantitative analysis in inventory studies started with the work of Harris[9]. Clarkand Scarf[4] had put forward the multi-echelon inventory first. They analyzed N-echelon pipelining system without considering a lot size, Recent developments in two-echelon models may be found in Q.M. He and SvenAxsater[1] proposed an approximate model of inventory structure in SC. One of the oldest papers in the field of continuous review multi-echelon inventory system is a basic and seminal paper written by Sherbrooke in 1968. He assumed (S-1,S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Continuous review models of multi-echelon inventory system in 1980's concentrated more on repairable items in a Depot-Base system than as consumable items(see Graves[6,7], Moinzadeh and Lee[9]). All these papers deal with repairable

items with batch ordering. Seifbarghy and Jokar analyzed a two echelon inventory system with one warehouse and multiple retailers controlled by continuous review (R,Q) policy. A Complete review was provided by Benita M. Beamon (1998)[2]. the supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf(1960)[4]. In the case of continuous review perishable inventory models with random lifetimes for the items, most of the models assume instantaneous supply of order[8]. The assumption of positive lead times further increases the complexity of the analysis of these models and hence there are only a limited number of papers dealing with positive lead-times[n]. A continuous review perishable inventory system at Service Facilities was studied by Elango(2001)[5]. A continuous review(s,S) policy with positive lead times in two-echelon Supply Chain was considered by K.Krishnan and C.Elango[9].

In this paper we considered a inventory system maintained in a service facilities at Retailer vendor in tandem supply chain having Retailer vendor(RV) and Distribution Centre(DC). One item (perishable) from inventory at RV is used to serve the customer. (s, S) policy is adopted at Retail Vender node. Numerical examples with served instances are provided to prove as conjective that the service rate employed is depending on the number of customer in the system not on the inventory level.

2 Model Formulation:

We consider a Supply Chain system consist of Distribution Centre(DC), Retail vendor(RV) with service facility and inventory is maintained at both DC and RV nodes. For every demand at the retailer node (RV) an item is supplied only after a exponential service time with parameter $\mu_n : n = 0,1,...,K$. The waiting space in the retailer node has maximum capacity N. An arriving customer seeing N customers already in the system leaves.

Demand at RV node follows a Poisson process with parameter $\lambda(>0)$ .Inventory policy adopted at RV node is (s, S) type in which order for Q=S-s>s items are placed when the inventory level reaches the level s, and lead time is exponentially distributed

with parameter $\beta(>0)$. At DC, items are packed as Q items in one pocket with maximum inventory level nQ(n pockets). The residue policy at DC is of (0,nQ) type where the inventory level reach 0, instantaneous replenishment nQ items to attain maximum inventory level. Each item in inventory has the policy rate θ(>0), with exponentially distributed life time.

Let $I_0(t)$ and $I_1(t)$ denote the on-hand inventory levels at retailer node and Distribution Centre respectively at time t with maximum inventory level S and nQ respectively and L(t) denote the number of customers in the waiting space at time with maximum capacity N. From the assumptions on the input and output processes,

$I(t) = \{ (I_0(t), L(t)) : t \geq 0 \}$ is a Markov process with state space

$E = \{0, 1, 2, ..., S\} \, x \, \{0, 1, 2, ..., N\}$.

Here we ignore the inventory level $I_1(t)$ at DC, because it has no influence on the service process at retailer node, moreover its replenishment policy is instantaneous.

Now the process $I(t) = \{(I_0(t), L(t)) : t \geq 0\}$ is a two dimensional Markov Process with state space. $E = \{(i, j) : i = 0, 1, 2, ..., S; j = 0, 1, ..., N\}$. Since E is finite and all its states are recurrent non-null, $\{I(t), t \geq 0\}$ is an irreducible Markov process with state space E and it is also an Ergodic process. Hence, the limiting distribution exists and is independent of the initial state. In this article our objective is to find a policy that specifies the expected service rate adopted so as to minimize the long run expected cost rate.

The cost associated with the system operation have the following components:

h: inventory carrying cost per unit time.

c_1: cost per order.

c_2: waiting time cost per unit time

g: cost per customer lost.

η_n : cost associated with using rate μ_n, $\eta_0 = 0$.

p: perishing cost per unit.

3 Analysis

3.1 System Analysis

Let $(I_0^R, L^R) = \{I_0^R(t); L^R(t) \geq 0\}$ denote the Markov process, when a stationary policy R is adopted. From our assumptions, it can be seen that the controlled process (I_0^R, L^R) is a finite state semi-Markov decision process. A policy R is called a stationary policy if it is randomized, time invariant and Markovian. Further, a process is said to be completely Ergodic if every stationary policy give rise to an irreducible Markov chain. From our assumptions it can be seen that for every stationary policy f, (I_0^f, L^f) is completely Ergodic. Since the action space is also finite, a stationary optimal policy exists. Hence we consider the class $\mathfrak{I}$ of all stationary policies.

Denote by (k) the action of choosing rate $\mu_k (k = 0, 1, 2, ..., N)$. Whenever $L(t) = 0$ or $I_0(t) = 0$, we must choose the rate μ_0. Based on the choice of actions, the state space E can be partitioned as follows:

$E_1 = \{(i,q) : 0 \leq i \leq S, q = 0; i = 0, 0 \leq q \leq N\}, E_2 = \{(i,q) : 1 \leq i \leq S; 1 \leq q \leq N\}$,

$E = E_1 \cup E_2$.

Let A_j (j=1,2) represent the set of all possible actions of the system when it belongs to the set E_j.

Then we have $A_1=\{0\}$, $A_2=\{(k):1\le k\le N\}$ and $A=A_1\cup A_2$

A decision rule from the class f is equivalent to a function f:E$\rightarrow$ A and is given by

$f(i,q)=\{(k);(i,q)\in E,(k)\in A_j, j=1,2\}$.

For any fixed $f\in F$ and $(i,q),(j,r)\in E,$ define

$$P_{iq}^f(j,r,t)=\Pr\{I_0^{\ f}(t)=j,\mathrm{L}^f(t)=r\mid I_0^{\ f}(0)=i,\mathrm{L}^f(0)=q\}, (i,q),(j,r)\in E. \tag{1}$$

Then $P_{iq}(j,r,t)$ satisfies the Kolmogorov forward differential equations. As each policy, f, results in an irreducible Markov chain and action spaces are finite,

$P^f(j,r)=\lim_{t\to\infty}P_{iq}(j,r,t)$ exists and is independent of the initial conditions.

Hence we have the balance equation (2)-(13) given below. The balance equations can also be obtained by using the fact that transition out of a state is equal to transition into a state. For example, let us consider a typical state is equal to transition into a state. For example, let us consider a typical state (j,r) that lies in the range $s+1\le j\le S-1; 1\le r\le N-1.$ This state is represented in Eq.(12) below. when (j, r) is in this range, there is no order pending, and hence transition out of this state can be only due to either a demand or a service completion. This fact is reflected on the left-hand side of Eq.(12) A service completion in state $(j+1,r+1)$ will decrease both the inventory level and the number of customers by one unit, thus bringing it to state (j,r). State (j,r) can be reached from state $(j+1,r+1)$ when a customer arrives. These

are the only two possible ways of reaching state (j, r) and are reflected on the right-hand side of Eq.(12). The state (i,j) will transmit to (i-1,j) with state of transition i_θ where $0 \leq \theta \leq 1$.
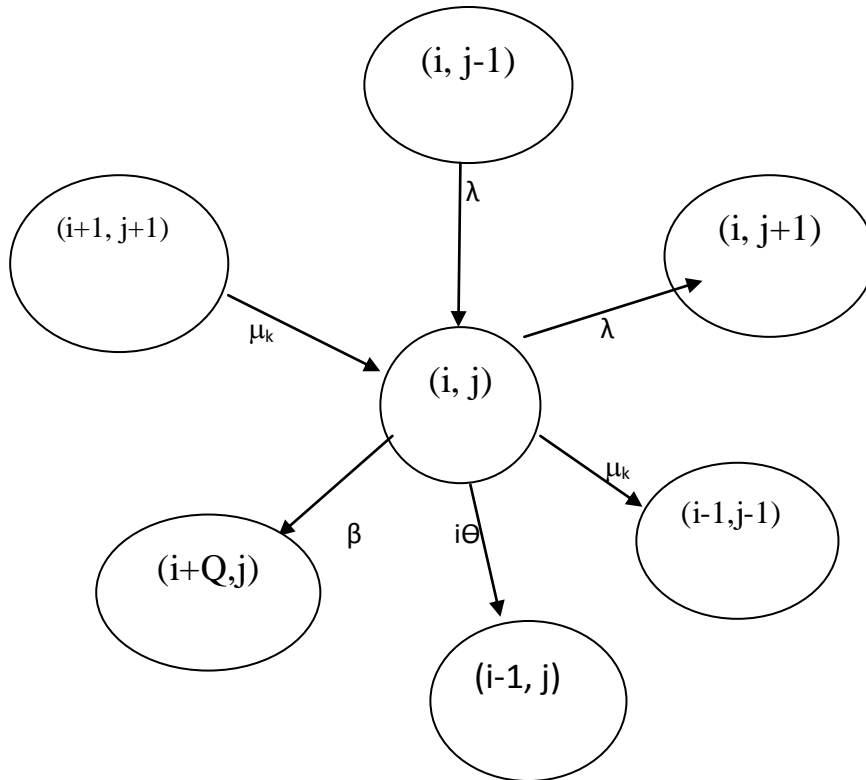
fig(2)

$$(\lambda + \eta)\pi^f(0,0) = \alpha\pi^f(0,1) + \theta\pi^f(1,0) + \beta\pi^f(1,1) \tag{2}$$

$$(\lambda + \eta)\pi^f(0,j) = \lambda\pi^f(0,j-1) + \theta\pi^f(1,j) + \beta^f\pi^f(1,j+1) \quad 1 \leq j \leq N-1 \tag{3}$$

$$\eta\pi^f(0,N) = \lambda\pi^f(0,N-1) + \theta\pi^f(1,N) \tag{4}$$

For $1 \leq i \leq s$

$$(\lambda + \eta + i\theta)\pi^f (i,0) = (i+1)\theta\pi^f (i+1,0) + \beta^f \pi^f (i+1,1) \tag{5}$$

$$(\lambda + \eta + i\theta + \beta^f)\pi^f (i,j) = \lambda\pi^f (i,j-1) + (i+1)\theta\pi^f (i+1,j) + \beta^f \pi^f (i+1,j+1), 1 \le j \le N-1 \tag{6}$$

$$(\eta + i\theta + \beta^f)\pi^f (i,N) = \lambda\pi^f (i,N-1) + (i+1)\theta\pi^f (i+1,N) \tag{7}$$

For $s+1 \le i \le Q-1$

$$(\lambda + i\theta)\pi^f (i,0) = (i+1)\theta\pi^f (i+1,0) + \beta^f \pi^f (i+1,1) \tag{8}$$

$$(\lambda + i\theta + \beta^f)\pi^f (i,j) = \lambda\pi^f (i,j-1) + (i+1)\theta\pi^f (i+1,j) + \beta^f \pi^f (i+1,j+1), 1 \le j \le N-1 \tag{9}$$

$$0(i\theta + \beta^f)\pi^f (i,N) = \lambda\pi^f (i,N-1) + (i+1)\theta\pi^f (i+1,N) \tag{10}$$

For $Q \le i \le S-1$,

$$(\lambda + i\theta)\pi^f (i,0) = (i+1)\theta\pi^f (i+1,0) + \beta^f \pi^f (i+1,1) + \eta\pi^f (i-Q,0) \tag{11}$$

$$(\lambda + i\theta + \beta^f)\pi^f (i,j) = \lambda\pi^f (i,j-1) + (i+1)\theta\pi^f (i+1,j) + \beta^f \pi^f (i+1,j+1) + \eta\pi^f (i-Q,j), 1 \le j \le N-1 \tag{12}$$

$$(i\theta + \beta^f)\pi^f (i,N) = \lambda\pi^f (i,N-1) + (i+1)\theta\pi^f (i+1,N) + \eta\pi^f (i-Q,N) \tag{13}$$

$$(\lambda + S\theta)\pi^f (S,0) = \eta\pi^f (s,0) \tag{14}$$

$$(\lambda + S\theta + \beta^f)\pi^f (S,j) = \lambda\pi^f (S,j-1) + \eta\pi^f (s,j), \quad 1 \le j \le N-1 \tag{15}$$

$$(S\theta + \beta^f)\pi^f (S,N) = \lambda\pi^f (S,N-1) + \eta\pi^f (s,N) \tag{16}$$

The above set of equations together with the condition

$$\sum_{(i,j) \in E} \pi^f (i,j) = 1 \tag{17}$$

determine the steady-state probabilities uniquely

3.2 System Performance Measures

1. $\pi^f(j, r)$ also gives the long-run fraction of time the system is in the state (j, r), the average inventory level is given by

$$\bar{I}^f = \sum_{j=1}^{s} j \sum_{r=0}^{N} \pi^f(j,r) \tag{18}$$

2. The expected cost due to different service rates utilized is

$$\bar{\Gamma}_{I^f} = \sum_{(j,r)\in E} \Gamma_{f(r)} \pi^f(j,r), \text{ where } \Gamma_{f(r)} = \eta_n \text{ if } f(r) = n \tag{19}$$

3. The reorder rate is given by $\quad \bar{\alpha}_1^{\;f} = \mu^f \sum_{r=0}^{N} \pi^f(s+1,r) \tag{20}$

4. The mean waiting time is given by

$$\bar{\alpha}_2^{\;f} = \sum_{r=1}^{N} \frac{r}{\mu^f} \sum_{j=0}^{S} \pi^f(j,r) + \frac{1}{\beta} \sum_{m=1}^{[N/S]} \sum_{r=1}^{mS} m \sum_{j=0}^{S} \pi^f(j,r) \tag{21}$$

5. The balking rate is given by

$$\bar{\alpha}_3^{\;f} = \lambda \sum_{j=0}^{S} \pi^f(j,N) \tag{22}$$

6. The Expected Perishable rate

$$\alpha_4^{\;f} = \sum_{j=1}^{S} \sum_{r=0}^{N} j\theta \Pi^f \tag{23}$$

The long-run expected cost rate when policy f is adopted is given by

$$C^f = h\bar{I}^f + c_1 \alpha_1^f + c_2 \alpha_2^f + g\alpha_3^f + \beta_k \bar{\Gamma}_{I^f} + p\alpha_4^f \tag{24}$$

Where in the steady state, for a given policy f,

$\bar{I}^f$ is the average inventory level, α_1^f is the expected reorder rate, α_2^f is the average waiting time for a customer; α_3^f is the expected balking rate, $\alpha_4{}^f$ the expected perishable rate and $\bar{\Gamma}_{I^f}$ is the expected cost per unit time associated with using the different rates.

Our objective is to find an optimal policy $f *$ for which $C^{f*} \leq C^f$ for every f .

Hence the average cost rate of the system is given by

$$C^f = h\sum_{j=1}^{S} j \sum_{r=0}^{N} \pi^f(j,r) + c_1\mu^f \sum_{r=0}^{N} \pi^f(s+1,r) + c_2 \sum_{r=1}^{N} \frac{r}{\mu_r{}^f} \sum_{j=0}^{S} \pi^f(j,r)$$
$$+ \frac{c_2}{\beta} \sum_{m=1}^{[N/S]} \sum_{r=1}^{mS} m \sum_{j=0}^{S} \pi^f(j,r) + g\lambda \sum_{j=0}^{S} \pi^f(j,N) + \sum_{(j,r)\in E} \Gamma_{f(r)}\pi^f(j,r)$$

(24)

4. Linear programming problem

4.1 LPP Formulation

Let us define the variables D(j, r, k) as

D(j, r ,k)= Pr [decision is k │ state is (j,r)].

Then for any stationary policy f, we have D(j, r, k) =0 or 1.Suppose D(j, r, k) were continuous variable (instead of integers), then the semi-Markov decision problem can be reformulated as a linear programming problem . For this purpose we consider the class of all randomized , time –invariant Markovian policies for which the probability functions D(j, r, k) satisfy

$$0 \leq D(j,r,k) \leq 1$$

and

$$\sum_{k \in A_i}{}' D(j.r,k) = 1, 0 \le r \le N, 0 \le j \le S; i = 1,2$$

The linear programming problem is best expressed in terms of the variable

$y(j,r,k)$, which are defined as

$$y(j,r,k) = D(j,r,k)\pi^f(j,r) \tag{25}$$

As y(j, r, k)=Pr[state is (j ,r) and decision is k],for any given f, we have

$$\pi^f(j,r) = \sum_{k \in A} y(j,r,k) \quad (j,r) \in E \tag{26}$$

Expressing $P^f(j,r)$ in terms of $y(j,r,k)$ in (22) we obtain the following linear

programming problem:

Minimize

$$C = h\sum_{k=1}^{K} j \sum_{j=1}^{S}\sum_{r=1}^{N} y(j,r,k) + h\sum_{j=1}^{S} jy(j,0,0) + c_1\sum_{k=1}^{K}\sum_{r=1}^{N} \mu_k\, y(s+1,r,k) + g\lambda y(0,N,0) + c_2\sum_{k=1}^{K}\sum_{r=1}^{N} \frac{r}{\mu_k}\sum_{j=0}^{S} y(j,r,k)$$

$$+ c_2\sum_{k=1}^{K}\frac{1}{\beta}\sum_{m=1}^{\frac{N}{S}}\sum_{r=1}^{mS} m\sum_{j=0}^{S} y(j,r,k) + g\lambda\sum_{k=1}^{K}\sum_{j=1}^{S} y(j,N,k) + \sum_{(j.r)\in E, r\neq 0}\sum_{k=1}^{K} \eta_k\, y(j,r,k) \qquad ......(27)$$

The constraints of the linear programming problem are as follows:

a) From (25), we have

$$y(j,r,k) \ge 0 \quad (j,r) \in E_l, k \in A_l, l = 1,2 \tag{28}$$

b) Since $\displaystyle\sum_{(j,r)\in E} P^f(j,r) = 1$ we have from (26)

$$\sum_{l=1}^{2} \sum_{(j,r)\in E_l} \sum_{k\in A_l} y(j,r,k) = 1 \tag{29}$$

c) The remaining constraints are the balance equations. After discarding the last equation as redundant in the set of equations, we have

$$\lambda y(S,0,0) = \beta \sum_{j=0}^{S} y(j,0,0), \tag{30}$$

$$\sum_{k=1}^{K}(\lambda + \mu_k) y(S,r,k) = \beta \sum_{k=1}^{K}\sum_{j=0}^{S} y(j,r,k) + \lambda y(S,r-1,k), 1 \le r \le N-1, \tag{31}$$

$$\sum_{k=1}^{K}\mu_k y(S,N,k) = \beta \sum_{k=1}^{K}\sum_{j=0}^{S} y(j,N,k) + \sum_{k=1}^{K}\lambda y(S,N-1,k), \tag{32}$$

$$\lambda y(j,0,0) = \sum_{k=1}^{K}\mu_k y(j+1,1,k), \quad s+1 \le j \le S\text{-}1, \tag{33}$$

$$\sum_{k=1}^{K}(\lambda + \mu_k) y(j,r,k) = \sum_{k=1}^{K}\mu_k y(j+1,r+1,k) + \sum_{k=1}^{K}\lambda y(j,r-1,k),$$

$$s+1 \le j \le S\text{-}1; \ 1 \le r \le N\text{-}1 \tag{34}$$

$$\sum_{k=1}^{K}\mu_k y(j,N,k) = \sum_{k=1}^{K}\lambda y(j,N-1,k), \qquad \sum_{j=1}^{N} \gamma_k y(j,M,k) = \sum_{j=1}^{N}\lambda y(j,N-1,k), s+1 \le$$

$$j \le S-1 \tag{35}$$

$$(\lambda + \beta) y(j,0,0) = \sum_{k=1}^{K}\mu_k y(j+1,1,k), \ 1 \le j \le s \tag{36}$$

$$\sum_{k=1}^{K}(\lambda + \beta + \mu_k) y(j,r,k) = \sum_{k=1}^{K}\mu_k y(j+1,r+1,k) + \sum_{k=1}^{K}\lambda y(j,r-1,k),$$

$$1 \le j \le s \ ; 1 \le r \le N\text{-}1 \tag{37}$$

$$\sum_{k=1}^{K}(\beta + \mu) y(j,N,k) = \sum_{k=1}^{K}\lambda y(j,N-1,k), \ 1 \le j \le s \tag{38}$$

$$(\lambda + \beta)\, y(0,0,0) = \sum_{k=1}^{K} \mu_k \, y(1,1,k), \tag{39}$$

$$\sum_{k=1}^{K}(\lambda + \beta)\, y(0,r,k) = \sum_{k=1}^{K} \mu_k \, y(1,r+1,k) + \sum_{k=1}^{K} \lambda y(0,r-1,k), \ \ 1 \leq r \leq \text{N-1} \tag{40}$$

As we can see from the lemma below solving the linear programming problem gives the optimal solution when the $y(j,r,k)s$ are constrained to be integers.

The optimal solution of the above linear programming problem yields a deterministic policy.

From equations (25) and (26), we have

$$D(j, r, k) = \frac{y(j,r,k)}{\sum_{k=0}^{K} y(j,r,k)} \tag{41}$$

Since the decision problem is completely ergodic, every basic feasible solution to the above linear programming problem has the property that for each $(j,r) \in E$, $D(j,r,k)$ is 1 for exactly one value of k and zero for all other values of k. Thus, given the amount of inventory on-hand and the number of customers in the system, we have to choose the service rate μ_k for which $D(j,r,k)$ is 1. Hence any basic feasible solution of the linear programming yields a deterministic policy.

5. Numerical illustration and discussion

In this section, we illustrate the method described in section 4 through numerical examples. We use the simplex package in TORA for solving the linear programming problem. In all cases the computational time was less than 10 s on a PENTIUM 3.0 alpha machine.

It is our conjecture that the service rate to be employed depends only on the number of customers in the system and not on the inventory level. This conjecture can

be proved for the zero-leadtime case assuming that the same μ is used for all states. Then the expected cost rate is,

$$C(\mu) = \frac{c_1}{S[m+\alpha(1)/\lambda]} + h\frac{S+1}{2} + c_2\sum_{k=1}^{K}\sum_{j=1}^{S}kmP(j,k) + g\sum_{j=0}^{S}P(j,N) + \eta(\gamma)\sum_{j=1}^{S}\sum_{r=1}^{N}P(j,r), \quad (42)$$

Where m=1/μ and

$$\alpha(1) = \frac{1-(\lambda/\mu)}{1-(\lambda/\mu)^N} \qquad (43)$$

For $0 \le j \le S; 1 \le r \le N,$

$$P(j,r) = \left[\frac{\lambda}{\mu}\right]^{r-1}P(j,0) \qquad (44)$$

For $0 \le j \le S,$

$$P(j,0) = \frac{1}{S}\frac{1-(\lambda/\mu)}{1-(\lambda/\mu)^{N+1}} \qquad (45)$$

In the above equations, $\eta(\mu)$, an increasing function of μ, is the cost associated with service rate μ. By differentiating C(μ) with respect to μ, it is easy to verify that the stationary value of μ is independent of the inventory level. When different service rates are used for different states ,we were unable to prove the conjecture . However, from the numerous numerical examples we ran, we found that the service rate is insensitive to changes in the inventory level and replenishment rates. Furthermore, service rates also seem to be independent of ordering costs, inventory carrying costs and balking costs.

The only three parameters that have appreciable effect on the service rates are the arrival rate, the waiting time costs and the number of customers in the system .

Typical numerical results , from one of the examples we studied, are summarized in Tables 1-2, where n is the number of customers in the system. The

first two tables show the effect of changes in waiting time costs .In the above two tables, we use $\mu_1=1, \mu_2=2, \mu_3=3, \mu_4=4$.

Two immediate conclusions can be drawn from the computational results in Tables 1-2:

(1) As the cost of waiting ,c_2, increases, larger service rates are utilized for larger values of n. For examples, in Table 1,when $c_2=1$, $\mu_1=1$ is used for all n values, expect for n=9,10, where $\mu_2=1$ is utilized .When $c_2=10$, $\mu_1=1$ is not used at all and μ_2, μ_3, μ_4 are used for respectively $1 \leq n \leq 3, 4 \leq n \leq 6$ and $7 \leq n \leq 10$

(2) As the arrival rate λ is increasing , larger service rates are utilized for larger values of n . We note that for a service facility ,a large number of customers (n) waiting is equivalent to poor service. Therefore the conclusions above are quite important as they show that the model can be used to decrease the average waiting time (providing good service) by selecting the appropriate service rates in a cost effective way.

Tables 1

Optimal rates for $c_1=50, h=0.1, g=2$, p=3
$\eta_1 = 1, \eta_2 = 2, \eta_3 = 3, \eta_4 = 4, S = 25, s = 6, N = 10, \lambda = 7, \beta = 0.3$

C_2	$\mu_1=1$	$\mu_2=2$	$\mu_3=3$	$\mu_4=4$
1	$1 \leq n \leq 8$	$9 \leq n \leq 10$	_	_
2	$1 \leq n \leq 7$	$8 \leq n \leq 10$	_	_
3	$1 \leq n \leq 6$	$7 \leq n \leq 10$	_	_
4	$1 \leq n \leq 5$	$6 \leq n \leq 9$	n=10	_
5	$1 \leq n \leq 4$	$5 \leq n \leq 8$	$9 \leq n \leq 10$	_

6	$1 \le n \le 3$	$4 \le n \le 7$	$8 \le n \le 9$	n=10
7	$1 \le n \le 3$	$4 \le n \le 6$	$7 \le n \le 8$	$9 \le n \le 10$
8	$1 \le n \le 2$	$3 \le n \le 5$	$6 \le n \le 7$	$8 \le n \le 10$
9	$n = 1$	$2 \le n \le 4$	$5 \le n \le 7$	$8 \le n \le 10$
10	_	$1 \le n \le 3$	$4 \le n \le 6$	$7 \le n \le 10$

Tables 2

Optimal rates for c_1=50,h=0.1,g=2, p=3

$\eta_1 = 1, \eta_2 = 2, \eta_3 = 3, \eta_4 = 4, S = 25, s = 6, N = 10, \lambda = 7, \beta = 0.3$

C_2	$\mu_1=1$	$\mu_2=2$	$\mu_3=3$	$\mu_4=4$
1	$1 \le n \le 9$	n=10	_	_
2	$1 \le n \le 8$	$9 \le n \le 10$	_	_
3	$1 \le n \le 7$	$8 \le n \le 9$	n=10	_
4	$1 \le n \le 6$	$7 \le n \le 8$	$9 \le n \le 10$	_
5	$1 \le n \le 4$	$5 \le n \le 6$	$7 \le n \le 10$	_
6	$1 \le n \le 4$	$5 \le n \le 6$	$7 \le n \le 9$	n=10
7	$1 \le n \le 4$	n=5	$6 \le n \le 8$	$9 \le n \le 10$
8	$1 \le n \le 3$	n=4	$5 \le n \le 7$	$8 \le n \le 10$
9	$1 \le n \le 2$	_	$3 \le n \le 7$	$8 \le n \le 10$
10	n=1	n=2	$3 \le n \le 6$	$7 \le n \le 10$

In Table1, the costs associated with the different rates are linear functions of the rates ($\eta_i = \mu_i$ and $\eta_i = 2\mu_i$, respectively). In Table 2 where there is a steeper increase in a cost for changing the rate from μ_1 to μ_2 ($\eta_1 = \mu_1 = 1, \eta_2 = 3$), the number of states for which rate is μ_2 is smaller than that in Table 1. For example when $c_2=9$, there is no state in which rate μ_2 is employed and when $c_2=1, 7,8,10$, there is only one state in which rate μ_2 is employed.

The case where $\mu_n = n\mu$ (,n is an integer) can be interpreted as a system in which n service are used each serving at a rate μ. Hence, the problem of determining the number of servers to be employed as a function of the state of the system can be solved by using our model by simply taking $\mu_n = n\mu$

6.Conclusions and future research

Analysis of inventory control at service facilities is fairly recent. Most of the earlier work in this direction has been on the determination of ordering policies or on finding optimal stocking levels for a given policy . We approach the problem in a different manner . Given an ordering policy , we determine the service rates to be employed as a function of the number of customers in the queue and the amount of inventory on hand so that the long-run expected cost rate is minimized . The rationale is that quite often it is possible to control the service rate by changing the number of servers or by using a faster or slower server, whereas there may be constraints such as limited storage size and type of vendors that make it difficult to change the stocking level or the frequency of ordering. As such, determination of optimal service rates is an important problem in the service industry.

In our problem, we use an (s, S) ordering policy. Policies such as one-to-one ordering, or (R,Q) systems or any other fixed ordering policy can be analyzed with methodology. We used the tools of Markov decision processes to analyze the problem and linear programming to determine the optimal service rates.

The main contribution of the paper is the determination of the control policy that indicates the specific optimal service rate to be used for every possible state of the

system . We made two interesting observations. One is that the service rate to the employed is insensitive to change in inventory level and depended only on the number of customers waiting for service. The second is that the parameters which influence the service rates are the customer arrival rates and waiting time costs . It is not intuitive that the inventory level ,replenishment rate and inventory carrying costs have no effect on the service rate .

We are currently studying service system with perishable items having zero leadtimes. We plan to extend the analysis to general leadtimes.

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