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Generation of Hexagonal Patterns in Finite Interactive System and Scenarios

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Abstract:A basic new type of regular expression for two dimensional languages hasbeen introduced using arbitrary shapes and tiling operations and is represented by a complete2×2 Finite Interactive Systems (FIS) in [1]. It is known that $FIS$are much more complex than finite automata. Motivated by the study in [1, 2, 3], a new type of hexagonal regular expression is presented in [6]. In this paper, hexagonal tiles and scenarios have been introduced based on tiling hexagonal unit cells with colors representing two dimensional hexagonal pictures. A complete 3×3 Hexagonal Finite Interactive System ($HFIS$) is defined and the two dimensional hexagonal languages are recognized by the corresponding regular expressions.

Keywords:Regular expression, Tiling, Scenario, Hexagonal tile, Hexagonal scenario, Finite Interactive System, Hexagonal Finite Interactive System.

1 Introduction

Tiling system as a device for recognizing picture languages and as a possible generalization of finite automata for languages of words is investigated in [4]. Tiling has also been used for the study of two dimensional languages. Finite interactive systems are one of many equivalent presentations of regular two dimensional languages ($2Reg$). On the other hand, extending the notion of regular expression which provides a rich formalism for specifying and analyzing sequential models of computation, regular expressions and context free grammars for picture languages are considered in [5]. Recently, Stefanescu et al. have introduced a new type of regular expressions for two dimensional languages using arbitrary shapes and tiling operations parameterized by restrictions on the connection interfaces [1, 2, 3]. The question of finding an $n2RE$-like representation of FISlanguages has been addressed in [1]. It was shown that a slight extension $x2RE\ of\ n2RE$, considering compositions involving the extreme cells of the words, is powerful enough to represent FIS languages.

A coordinate system is described in [7] providing a natural means for representing hexagonally organized pixels. The operations of the coordinate system and its computations are shown. Motivated by the study in [1, 2, 3] a new type of hexagonal regular expression has been introduced in [6] using operations like linking side borders, land corners and gluing corners for two dimensional hexagonal languages.

In this paper, hexagonal tiles and scenarios are introduced by tiling hexagonal unit cells with colors representing two dimensional hexagonal pictures. A complete 3 x 3 Hexagonal Finite Interactive System ($HFIS$) is defined and the two dimensional hexagonal languages are recognized by the corresponding regular expressions. Different two dimensional hexagonal pictures likekolam patterns have been generated.

2 Preliminaries

In this section we briefly present a new type of regular expressions $n2HRE$ for two-dimensionalhexagonal patterns based on their compositionwhich make use of arbitrary shapes and tiling operations parameterized by restrictions of the connection interfaces [6]. A complete and indecomposable 2×2Finite Interactive System is defined in [1].

2.1 Hexagonal grid [7]

Hexagonal grid is an alternative representation of pixel tessellation scheme for the conventional square grid for sampling and representing discredited images. Each pixel is represented by a horizontal deflection followed by a deflection upward and to the right. These directions are represented by a pair of unit vectors $u$and $v$and this coordinate system is referred as the "h_2" system. Given a pixel with coordinates (u, v)(assumed integer), the coordinates of the neighbors are illustrated in Fig 1.

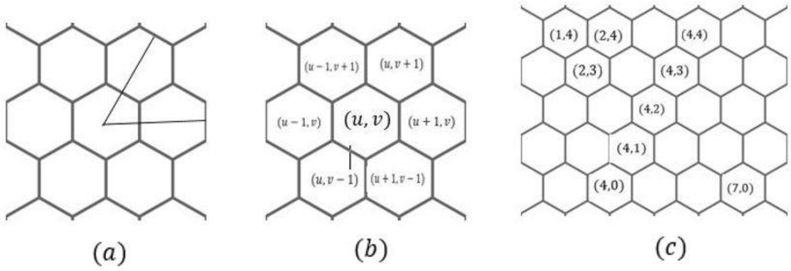


Fig. 1. (a). A coordinate system based on unit vectors u and v, (b) the neighborhood of a hexagonal pixel and (c) labeling of a hexagonal pixel.

2.2 Regular Expressions for Hexagonal Patterns [6]

Let Σ be a finite alphabet. A general 2-dimensional hexagonal pictureis a set of hexagonal unit cells in the 2-dimensional space filled with symbols from the given alphabet. Let $h_a$be a hexagonal unit cell filled with letter 'a' over Σ . The set of all hexagonal pictures over $\Sigma$is denoted by$\Sigma_{h_2}^{**}$.A ***general composition operator*** on a two dimensional hexagonal pictures is defined as follows. Given two hexagonal pictures, get new pictures by putting them together such that no interior cell of the first picture may overlap an interior cell of the other.

Hexagonal regular expressions are defined by placing the connection points on the borders of the composed pictures. These constraints act on the following three types of elements named as Side borders, Land corners and Golf corners. We use the symbol || for Side border, ∧ for Land corners and ∨ for Golf corners.

Side Borders: Elements in $SB = \{NW, NE, EE, SE, SW, WW\}$ where NW stands for North West border, NE for North East border, EE for East border, SE for South East border, SW for South West border and WW for West border. The Side borders are denoted by symbols as$||_{NW}, ||_{NE}, ||_{EE}, ||_{SE}, ||_{SW}, ||_{WW}$.

Land Corners: Elements in $LC = \{nw, nn, ne, se, ss, sw\}$, where nw stands for northwest land corner, nn for north land corner,ne for north-east land corner, se for south-east land corner, ss for south land corner and sw for south-west land corner. The Land Corners are denoted by symbols as $\wedge_{nw}, \wedge_{nn}, \wedge_{ne}, \wedge_{se}, \wedge_{ss}, \wedge_{sw}$.

Golf Corners: Elements in $GC = \{nw', nn', ne', se', ss', sw'\}$, where nw' stands for north-west golf corner, $nn'$for north golf corner, ne' for north-east golf corner, se' for south-east golf corner, ss' for south golf corner and sw' for south-west golf corner. The Golf Corners are denoted by symbols as $\vee_{nw'}$, $\vee_{nn'}$, $\vee_{ne'}$, $\vee_{se'}$, $\vee_{ss'}$, $\vee_{sw'}$.

Extreme Cell: A cell is an *extreme c*ell if it touches at most one other cell in the interior area. A side $(NW\ /\ NE\ /\ EE\ /\ SE\ /\ SW\ /\ WW)$ or a corner on the border of a picture is *extreme* if it belongs to an extreme cell and does not touch another cell (side elements with an end point touching another cell are also excluded – they are not extreme).

Gluing combinations: The constraints on gluing the borders are independently allowed on one or more of the following *Gluing combinations* $(\boldsymbol{x},\boldsymbol{y})$**:** x and y are different and either they are both in $\{||_{EE}, ||_{WW}\}$ or in $\{||_{NE}, ||_{SW}\}$ or in $\{||_{SE}, ||_{NW}\}$ or both are land corners in$\{\wedge_{nw}, \wedge_{nn}, \wedge_{ne}, \wedge_{se}, \wedge_{ss}\ and\ \wedge_{sw}\}$ or both are combinations of golf - land corners for the same directions { e.g. $(\wedge_{ss'}\ ,\ \wedge_{ss})$}.The following are the possible combinations:

Linking Side Borders:

$L1 = \{\ (||_{EE}, ||_{WW}), (||_{WW}, ||_{EE}), (||_{SE}, ||_{NW}), (||_{NW}, ||_{SE}),\ (||_{SW}, ||_{NE})\ ,\ (||_{NE}, ||_{SW})\ \}.$

Linking Land Corners:

$$L2 = \{(\wedge_{nn}, \wedge_{sw}), (\wedge_{nn}, \wedge_{se}), (\wedge_{ne}, \wedge_{nw}), (\wedge_{ne},\wedge_{ss}), (\wedge_{se}, \wedge_{sw}), (\wedge_{se}, \wedge_{nn})$$

$(\wedge_{ss}, \wedge_{nw}), (\wedge_{ss}, \wedge_{ne}), (\wedge_{sw}, \wedge_{se}), (\wedge_{sw}, \wedge_{nn}), (\wedge_{nw}, \wedge_{ne}), (\wedge_{nw}, \wedge_{ss})\}$

Linking Golf Corners:

$L3 = \{(\vee_{ss'}, \wedge_{ss}), ((\vee_{nn'}, \wedge_{nn}), (\vee_{sw'}, \wedge_{sw}), (\vee_{nw'}, \wedge_{nw}), (\vee_{se'}, \wedge_{se}), (\vee_{ne'}, \wedge_{ne})\}$

Constricting formulae: On each of the above eligible gluing combinations (x, y) we give a constraint consisting of a propositional logic formula (**Constricting formulae**)$F \in PL(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ that is , a Boolean formula built up starting with the following atomic formulae:

$$\varphi_1(x,y) = 'x < y'\ ,\ \varphi_2(x,y) = 'x = y',\ \varphi_3(x,y) = 'x > y'\ ,\ \varphi_4(x,y) = 'x \# y'$$

The meaning of the *connectors i*s the following: " $<$ " - left is included into the right; " $=$ " - left is equal to the right; " $>$ "- left includes the right; “#” - left and right overlaps, but no one is included in the other.The notation, $\varphi_0(x,y) = "x!\,y"$ is also used where “!” means empty intersection. Actually, this is a derived formula$\varphi_1(x,y) \vee \varphi_2(x,y) \vee \varphi_3(x,y) \vee \varphi_4(x,y)$.

Restricted compositions

A *restriction formula φ* is a Boolean combination in $PL(F_{1,\dots\dots,}F_n)$, where F_i are constricting formulas involving certain eligible glueing combinations $(x_i, y_i) \in$ *Connect*. A *restricted composition operation* $-(F)-$ is the restriction of the general composition to composite pictures satisfying F. A hexagonal picture $h \in f.g$ belongs to $f\ (F)\ g$ if for all glueing combinations (x_i, y_i)occurring in F the contact of the $x_i$border of $f$andy_i border of g satisfy F_i. The non-restricted general composition $f.g$ is the same as $f(.)g$.

The set of hexagonal expressions obtained using all the operators defined so far are denoted by $x2HRE$; they represent *Two dimensional Hexagonal Regular Expressions extended with* *composition operators on extreme cells*. Dropping the operators involving the extreme cells, the basic new type of regular expressions $n2HRE$ for two dimensional hexagonal pictures are obtained.

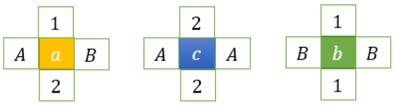
Hexagonal regular expression: Let Σ be a finite alphabet. The set $REG\ (\Sigma_{h_2})$ of simple hexagonal regular expressions over Σ with typical element r is defined by

$$r ::= h_a \mid (r_1 \,||\, r_2) \mid (r_1 \wedge r_2) \mid (r_1 \vee r_2) \mid (r_1(F)r_2) \mid (r_1(.)r_2) \mid r^*(F)$$

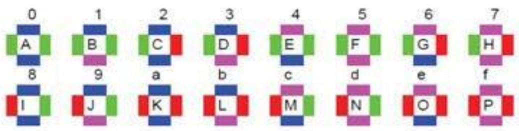
Here h_a stands for a hexagonal picture filled with letter $a \in \Sigma$. r_1 and r_2 are regular expressions. The language generated by such an expression is defined as follows. For all $a \in \Sigma$ $L(h_a) = \{h_a\}$. For two expressions r_1 and r_2 we define $L(r_1||r_2) = L(r_1)||L(r_2)$ and so on. The classes of languages definable by such expressions will be denoted by $REG(\Sigma_{h_2})$.

2.3 Finite interactive systems (FIS) [1]

Finite interactive systems are a two-dimensional extension of finite automata, well suited for recognizing two-dimensional words. A finite interactive system [1] is defined by: a set S of states (denoted by numbers 1, 2, ...) and a set C of classes (denoted by capital letters $A, B, \ldots$); a set T of transitions of the following form: $(A, 1) \to a \to (B, 2)$, where a is a letter of a given alphabet $\sum$ and $A, B, 1, 2$ are as above; specification of the initial/final states and classes. A FIS is complete if it specifies a transition on $(c_1, s_1) \to t \to (c_2, s_2)$ for any pair $((c_1, s_1), (c_2, s_2))$ in $((C \times S) \times (C \times S))$. A useful cross/tile representation may be used; it is based on showing the transitions and stating which states and classes are initial/final. $S1$ is an example with $1, A$ initial and $2, B$ final.

$S1$:


A complete 2×2 FIS is specified by the following transitions:



The states / classes of this FIS will be denoted using the initials of the colors: the classes are g ($green$) and r (red), while the states are b ($blue$) and m ($magenta$).

3 3×3 Finite Interactive System

In this section, a new FIS based model of two dimensional regular languages in terms of tiles and scenarios are presented and several hexagonal patterns are generated.

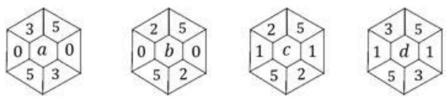
3.1 Hexagonal Tiles and Scenarios

Let Σ be a finite alphabet. A hexagonal tile is a hexagonal cell labelled with symbol from the given alphabet and enriched with additional information on each border. This information is represented abstractly as an element from a finite set and is called a border label. The role of border labels is to impose local gluing constraints on self-assembling tiles: two neighbouring cells, sharing a side border (east-west or north east -south west ornorth west - south east) should agree on the label on that border.A hexagonal scenario is similar to a two-dimensional hexagonal picture, but: (1) each hexagonal cell is replaced by a tile; and (2) east-west or north east -south west or north west - south east neighbouring cells have the same label on the common border.

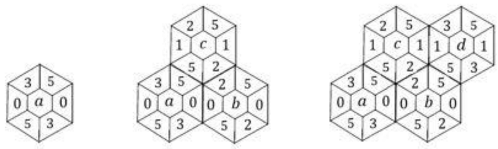
Graphically, a hexagonal scenario is obtained using the hexagonal tiles representing the transitions and identifying the matching classes or states of the neighbouring cells. The labels on the north east and north west borders represent north memory states, while the south east and south west borders represent south memory states and the ones on the west and east borders represent interaction classes. The selected labels on the external borders are called initial for south west, west and north west borders and final for north east, east and south east borders.

3.2 Example

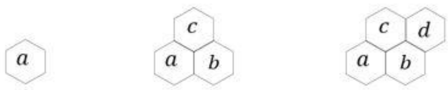
- Tiles



- Scenarios



- Accepted pictures



3.3 Hexagonal Finite Interactive System ($HFIS$)

Let Σbe a finite alphabet. A Hexagonal Finite Interactive System ($HFIS$) over $\Sigma$is defined by:

- a set S =$\{s_1, s_2, s_3, s_4\}$ of states and a set C = $\{c_1, c_2\}$ of classes;
- a set$T$ of transitions of the form: $(s_1,\ c_1,\ s_2)\ \rightarrow a\ \rightarrow (s_3,\ c_2,\ s_4)$ where a is a symbol of a given alphabet Σ ;
- specification of the initial/final states and classes.

Let $s_a$be the scenario of direct transition where $a$is the labelled symbol over the alphabet. The set of all scenarios representing hexagonal picture is denoted by $\Sigma^{**}_{s_2.}$.

A $HFIS$ is complete if it specifies a transition $(s_1,\ c_1,\ s_2)\ \rightarrow t\ \rightarrow (s_3,\ c_2,\ s_4)$ for any pair $((s_1,\ c_1,\ s_2)\ (s_3,\ c_2,\ s_4))$ in $\big((S\ \times C\ \times S) \times (S \times C\ \times S)\big)$.

A tile representation is used which is based on showing the transitions and stating which states and classes are initial/final. The states/classes of this $HFIS$ is denoted by the initials of the colors: The classes c_1 and $c_2$are g $(green)$ and b $(blue)$ while the $swne$ memory states $s_1$and s_3 $are\ p$ $(purple)$ and m $(magenta)$ and the $nwse$ memory states$s_2$and s_4 are r (red) and o $(orange)$.

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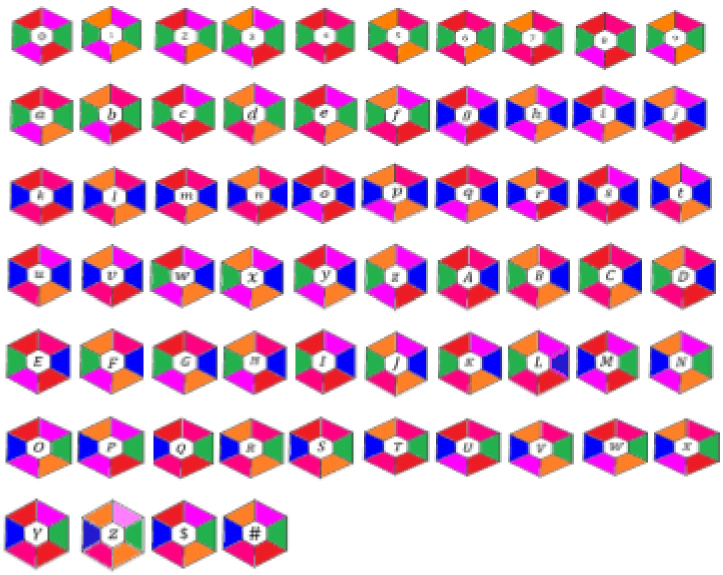
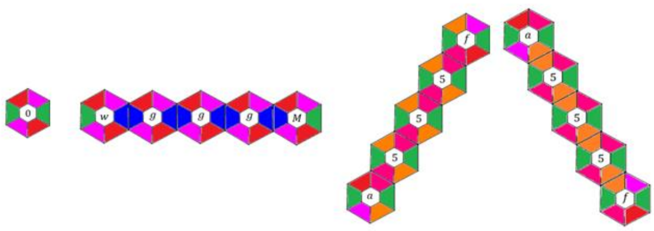
A scenario is called indecomposable if all its south west and north east borders are labelled with s_1 and s_3 west and east borders with c_1 and c_2 and the north west and south east borders with s_2 and s_4 respectively and it doesn't contain any sub-scenarios with this property.

A complete $3 \times 3HFIS$ is specified by the following 64 transitions:



Suppose we want to construct the recognized two dimensional hexagonal pictures corresponding to scenarios going from $m(g)r$ (initial) used for south west, west and north west borders to $m(g)r$ (final) used for north east, east, south east borders. The decomposable scenarios include the following combinations.



(1) (2) (3) (4)

The first scenario s_0 specifies the direct passing from $p(g)r$ to $p(g)r$. The second scenario, corresponding to the expression

$$s_w(||_{EE}=||_{ww})(s_g^*(||_{EE}=||_{WW}))(||_{EE}=||_{WW})s_M$$

is a generic indecomposable scenario describing the case when one keeps fixed the swne memory state p $(purple)$, the $nwse$ memory state r (red) and goes from the class g $(green)$ to the other class b $(blue)$, loops there and finally comes back to g $(green)$. The third scenario corresponding to the expression

$$s_a(||_{NE}=||_{SW})(s_5^*\ ||_{NE}=||_{SW})\ ||_{NE}=||_{SW})s_f$$

describe the case when the class g $(green)$ is fixed, the swne memory state goes from p $(purple)$ to m $(magenta)$, $nwse$ memory state from r (red) to o $(orange)$, loops there and finally comes back to p $(purple)$and r (red). The case of the fourth scenario is similar. Let us denote the regular expressions for the pictures by S_1, S_2, S_3 and S_4 corresponding to the 4 types of scenarios.

3.4 Pretzel shape scenario

Formally, a pretzel picture is a set of cells such that each cell has precisely two touching cells around and no proper subset has this property. Examples of pretzel shape scenarios representing the hexagonal pictures are shown in Fig. 3.

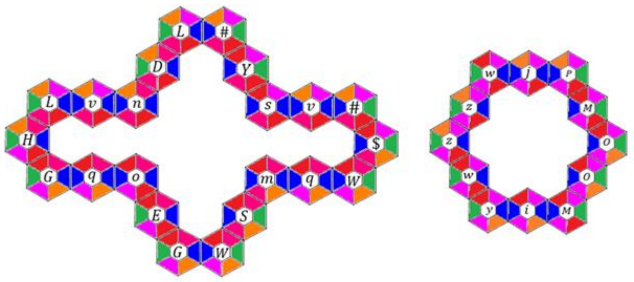
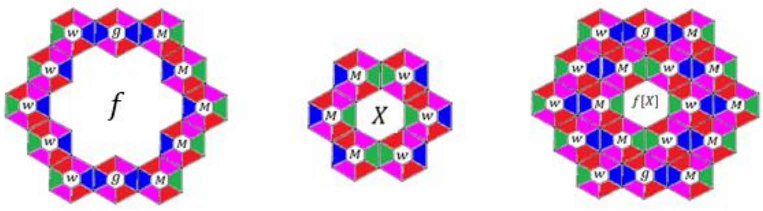


Fig. 3. Pretzel shape scenarios

3.5 $HFIS$ Scenario

Recursively, filling the inner/outer area of the pretzel shape scenario by the scenario representing the hexagonal picture, we obtain the $HFIS$ scenario representing the two dimensional hexagonal pictures passing from one specification of states/class to another specification. Let f be an expression for the pretzel shape scenario representing the hexagonal picture passing from $p(g)r$ to $p(g)r$. We denote by $f[X]$, the result of filling the pretzel hole by X where X also will be the pretzel shape or any other scenario representing the hexagonal picture.



An expression for defining $f[X]$ will be

$$f[X] = [(||_{WW}>||_{EE})\&(||_{NW}>||_{SE})\&(||_{SW}>||_{NE})\&(||_{EE}>||_{WW})\&(||_{SE}>||_{NW})\&(||_{NE}>||_{SW})]X$$

Let us denote the expression $f[X]$by $s_5.L(X)$ is the two dimensional hexagonal language which satisfies the regular expression

$$X = (s_1 + s_2 + s_3 + s_4 + s_5)^*(.) \quad (1)$$

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corresponding to the scenarios passing from $p(g)r$ to $p(g)r$. Hence we have achieved and formally state the result obtained so far as follows.

3.6 Theorem

The two dimensional hexagonal language consists of hexagonal pictures recognized by the complete 3×3 $HFIS$ can be represented by $x2HRE$ formalism and a mechanism for solving the equation 1

3.7 Example

Kolam patterns engraved on tilings are generated by the 3×3 $\boldsymbol{HFIS}$.
By assembling the pretzel shape scenario representing a hexagonal picture passing from $p(g)r$to $p(g)r$ and filling the inside and outside area in the six direction by a scenario (tile) passing directly from $p(g)r$to $p(g)r$, the kolam like star pattern is generated (Fig 4.(a))

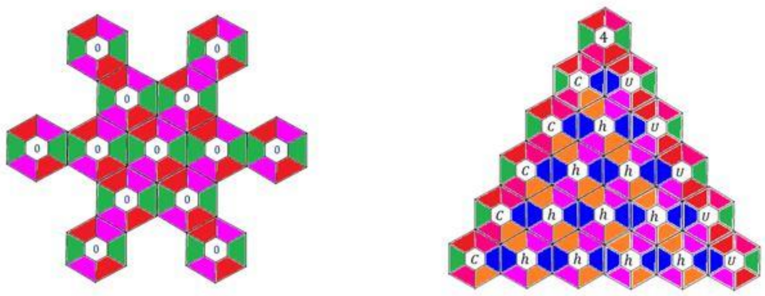

Fig. 4. (a) Star (b) Triangle

Let $\sum = \{0\}$ and f be the pretzel shape scenario representing the hexagonal picture passing from $p(g)r\ to\ p(g)r$. Let us assemble $f$with the direct scenario s_0 by the relation

$$g = f[(\|_{WW} \# \|_{EE})\&(\|_{NW} \# \|_{SE})\&(\|_{SW} \# \|_{NE})$$

$$\&(\|_{EE} \# \|_{WW})\&(\|_{SE} \# \|_{NW})\&(\|_{NE} \# \|_{SW})]s_0$$

Now the scenario g is assembled with the tile s_0 inside and the kolam like star pattern $g[0]$ is generated.

Similarly, assembling scenario passing from $m(g)r\ to\ m(g)r$ in the form of a shape Λ and filling inside by scenario passing from $p(b)o\ to\ p(b)o$ the kolam like triangular pattern is generated. (Fig 4.(b))

3.8 Theorem

Kolam patterns in the form of hexagonal grid are generated by the 3×3 $HFIS$.

Process: Building the six directions of the two dimensional hexagonal grid and recursively filling the triangular pattern inside, kolam like hexagonal pattern is generated. (Fig. 5)

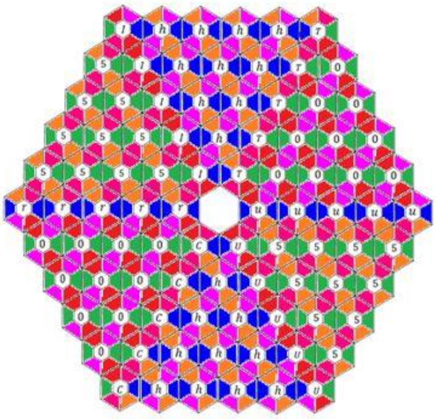

Fig. 5. Hexagonal Grid

Let $\sum = \{ 0, 5,\ h,\ r,\ u,\ C,\ I,\ T, U\}$. The scenario s_r^* passing from $p(b)o$ to $m(b)r$, s_I^* passing from $m(g)r$ to $p(b)r$, s_0^* passing from $m(b)o$ to $m(g)r$, s_u^* passing from $m(b)r$ to $p(b)o$, s_C^* passing from $m(g)r\ to\ m(b)o$ and the scenario s_U^* passing from $p(b)r\ to\ m(g)r$ are assembled by the following relations.

$s_1 = s_r^*(||_{NE}\ \#\ ||_{SW})s_I^*(||_{EE}\ \#\ ||_{WW})s_0^*(||_{SE}\ \#\ ||_{NW})s_u^*$

$s_2 = s_C^*(||_{EE}\ \#\ ||_{WW})s_U^*$

$s_3 = s_1(||_{SE}\ \#\ ||_{NW})\&\ (||_{SW}\ \#\ ||_{NE})s_2$

Let f be the scenario which represents the regular expression s_3.

The scenario s_h in the form of a triangle represents the regular expression $s_h^*(.)$ and the triangle scenarios s_0 and $s_5$are similar.$X = f[s_h, s_0, s_5]$ is the scenario in the form of a hexagonal grid, a kolam like hexagonal pattern with triangles and $L(X)$ is the language of hexagonal kolam patterns over $\sum$.

4 Conclusion

Interactive computation has a long tradition and there are many successful approaches to deal with the intricate aspects of this type of computation. This paper focuses on a new hexagonal model about finite interactive system consisting of tiles and scenarios. A complete 3×3 $HFIS$is described. Some new operations on hexagonal pictures and hexagonal picture languages have been proposed and the families of languages that can be generated using the operations are also deliberated. Comparisons with some known results in the area of two dimensional languages are to be included in future.

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