AN INVENTORY MODEL FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH QUADRATIC DEMAND UNDER INFLATION AND CUSTOMER RETURNS

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Abstract

In this paper an inventory model is developed for non instantaneous deteriorating products when the demand rate is considered as a quadratic function of time. Further a two parameter weibull deterioration rate with inflation and customer returns are also incorporated. The model is solved analytically by maximizing the total profit. The result is illustrated with a numerical example. The model can be applied to optimize the total profit of business enterprises.

Keywords: *Inventory, Quadratic Demand, Customer Returns, Non instantaneous Weibull Deterioration, Inflation.*

1. Introduction

In today's competitive environment, inventory control plays a crucial role for the success of business enterprises. Traditional inventory models were developed by numerous researchers with one of the assumptions that the items preserved their physical characteristics while they were stored in the inventory. This assumption is evidently true for most items, but not for all. In real life situations, certain items such as foodstuff, pharmaceuticals, chemicals etc. deteriorate quite rapidly with the passing of time and become useless. Wu et al. [3] defined this phenomenon as non instantaneous deterioration and framed an inventory model for non instantaneous deteriorating items with stock dependent demand and permissible delay in payments. In recent years, many researchers have paid their attention towards developing models for non instantaneous deteriorating items including various factors that influence an inventory system and which are more applicable in a realistic environment. Many researchers like [1], [4], [5], [6], [7], [8] made valuable contributions on developing models with varying demand rates and time – varying deterioration rates. While determining optimal inventory polices, the effect of inflation and time value of money should be given due importance. Numerous inventory models were developed by incorporating the above said factors. The empirical findings of Anderson et al. [2] reveal that customer returns increase with both the quantity sold and the price of the product and it plays a vital role in taking optimal decisions. Recently, Ghoreishi et al.[9] have developed an inventory model for non-instantaneous deteriorating items under inflation and customer returns.

In this paper, effort has been taken to develop a finite horizon model incorporating the effect of customer returns, time dependent quadratic demand, weibull rate of deterioration, inflation and time value of money in order to address the realistic circumstances. An optimization algorithm is presented to obtain the optimal values of price and replenishment cycle length by maximizing the total profit. A numerical example is provided to illustrate the model.

2. Notations and assumptions

The following notations and assumptions are used throughout this paper.

2.1. Notations

R	Production rate for the item (units/unit time)
K	Set-up cost per unit
c_1	Holding cost per unit per unit time
c_2	Purchasing price (or the production cost) per unit
p	Selling price per unit (decision variable), where $p > c_2$
T	Duration of the inventory cycle (decision variable)
t_p	Length of the production period in an inventory cycle (decision variable)
t_d	Length of time in which the product exhibits no deterioration, where $t_d \leq t_p$
Q	Production quantity

H Length of the planning horizon

N Number of production cycles during the time horizon H

s Salvage value per unit

r Constant representing the difference between the discount and the inflation rate.

I_0 Maximum inventory level

$I_1(t)$ Inventory level at time $t \in [0, t_d]$

$I_2(t)$ Inventory level at time $t \in [t_d, t_p]$

$I_3(t)$ Inventory level at time $t \in [t_p, T]$

$\theta(t)$ Deterioration rate, $\theta(t) = \alpha\beta t^{\beta-1}$, where $0 < \alpha < 1$ and $\beta > 0$

2.2 Assumptions

i. The planning horizon is finite.

ii. The initial and final inventory levels are both zero.

iii. The deterioration rate $\theta(t) = a\beta t^{\beta-1}$, where $0 < \alpha < 1$ and $\beta > 0$

iv. The production rate, which is finite, is higher than the demand rate.

v. Delivery lead time is zero.

vi. The demand rate, $D(t) = a + bt + ct^2$ (where $a > 0$, $b \neq 0$, $c \neq 0$)

vii. Shortages are not allowed.

viii. We assume that customer returns increase with both the quantity sold and the price. Customers return $RC(p,t) = \gamma D(t) + \delta p(\delta \geq 0, 0 \leq \gamma < 1)$ [2] products during the period for full credit and these units are available for resale in the following period.

ix. The length of the production period is larger than or equal to the length of time in which the product exhibits no deterioration, i.e. $t_p \geq t_d$.

3. The Mathematical Formulation

We consider a production inventory system for non instantaneous deteriorating items in which the planning horizon H is divided into N equal parts of length T = H/N. In each cycle, the inventory system goes like this: During the interval $[0, t_d]$, the inventory level increases due to production. At time t_d, deterioration starts and the inventory level increases due to production until the maximum inventory level is reached at $t = t_p$. During the interval $[t_p, T]$ there is no production and the inventory level decreases due to demand and deterioration until the inventory level becomes zero at $t = T$. The whole process is repeated. The behaviour of the system is depicted in Figure 1.

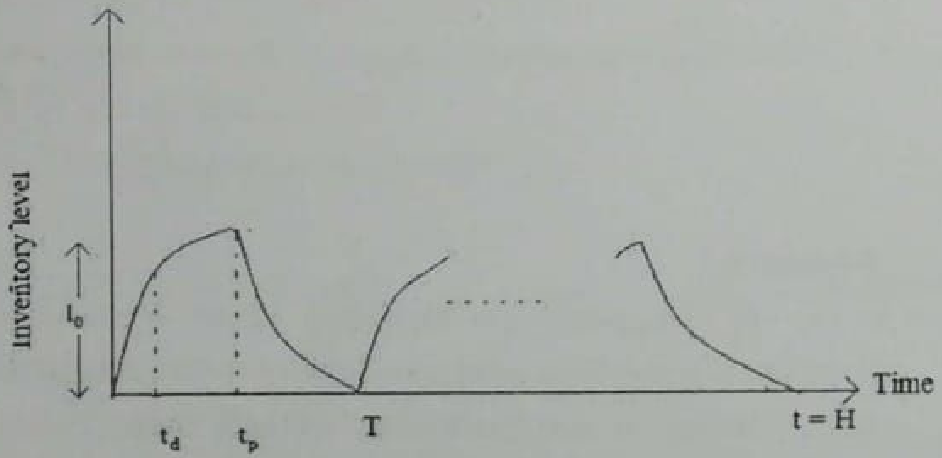

Figure 1: Graphical representation of an inventory system

Therefore the inventory system at any time t can be represented by the following differential equations.

$$\frac{dI_1}{dt} = R - D(t) = (R-a) - bt - ct^2 \quad (1)$$

$$\frac{dI_2}{dt} + \theta I_2(t) = R - D(t) = R - (a + bt + ct^2) \quad (2)$$

$$\frac{dI_3}{dt} + \theta I_3(t) = -D(t) = -(a + bt + ct^2) \quad (3)$$

Using the boundary conditions $I_2(0)=0;\ I_2(t_p)=I_0;\ I_3(T)=0$, the solutions of the differential equations are

$$I_1(t) = (R-a)t - \frac{bt^2}{2} - \frac{ct^3}{3} \quad 0 \le t \le t_p \quad (4)$$

$$I_2(t)=e^{-\alpha t\beta}\left[\begin{array}{l}(R-a)(t-t_p)-\frac{b}{2}(t^2-t_p^2)-\frac{c}{3}(t^3-t_p^3)+\\ \frac{\alpha(R-a)(t^{\beta+1}-t_p^{\beta+1})}{\beta+1}-\frac{ab(t^{\beta+2}-t_p^{\beta+2})}{\beta+2}-\frac{ac(t^{\beta+3}-t_p^{\beta+3})}{\beta+3}\end{array}\right]+I_0e^{\alpha}t_d\le t\le t_p \quad (5)$$

$$I_3(t)=e^{-\alpha t\beta}\left[\begin{array}{l}a(T-t)+\frac{b}{2}(T^2-t^2)+\frac{C}{3}(T^3-t^3)+\\ \frac{aa(T^{\beta+1}-t^{\beta+1})}{\beta+1}+\frac{ab(T^{\beta+2}-t^{\beta+2})}{\beta+2}+\frac{ac(T^{\beta+3}-t^{\beta+3})}{\beta+3}\end{array}\right]t_d\le t\le T \quad (6)$$

$$I_0=e^{-\alpha t_p^{\beta}}\left[\begin{array}{l}a(T-t_p)+\frac{b}{2}(T^2-t_p^2)+\frac{C}{3}(T^3-t_p^3)+\\ \frac{aa(T^{\beta+1}-t_p^{\beta+1})}{\beta+1}+\frac{ab(T^{\beta+2}-t_p^{\beta+2})}{\beta+2}+\frac{ac(T^{\beta+3}-t_p^{\beta+3})}{\beta+3}\end{array}\right]t_d\le t\le T \quad (7)$$

As the production occurs continuously during the interval $[0, t_p]$, the lot size is given by

$$Q = Rt_p \quad (8)$$

Now, we can obtain the present value of the inventory costs and sales revenue for the first cycle, which include of the following elements.

(a) The present value of the sales revenue for the first cycle is

$$\mathrm{SR} = p\left(\int_0^T D(t)e^{-rt}dt\right)$$

$$= p\left[aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{arT^2}{2}-\frac{brT^3}{3}-\frac{crT^4}{4}\right] \quad (9)$$

(b) The present value of production cost for the first cycle

$$PC = c_2\left(Rt_{(p)}\right) \quad (10)$$

(c) Since production set-up in each cycle is done at the beginning of each cycle, the present value of set-up cost for the first cycle is K, which is a constant value.

(d) The present-value of inventory carrying cost for the first cycle

$$HC = c_1\left(\int_0^{t_d} I_1(t)e^{-rt}dt + e^{-rt}\int_{t_d}^{t_p} I_2(t)e^{-rt}dt + e^{-rt_p}\int_p^T I_3(t)e^{-rt}dt\right)$$

$$HC = c_1\left\{\begin{aligned}&\left(\left((R-a)t_{d^2}\right)2\right) - \frac{\left(bt_{d^3}\right)}{6} - \frac{\left(ct_{d^4}\right)}{12} - \frac{\left(r(R-a)t_{d^3}\right)}{3} + \frac{brt_{d^4}}{8} + \frac{crt_{d^5}}{15} + \\ &\qquad e^{(-rt_d)}\frac{\left((R-a)\xi_1 - b\right)}{2}\frac{\xi_2 - c}{3}\xi_3 + \frac{(R-a)\alpha}{(\beta+1)}\frac{\xi_4 - b\alpha}{(\beta+2)\xi_5}\end{aligned}\right\}$$

$$\xi_1 = \frac{1}{2}\left(t_p^2 - t_d^2\right) - t_p\left(t_p - t_d\right) \quad ; \quad \xi_2 = \frac{1}{3}\left(t_p^3 - t_d^3\right) - t_p^2\left(t_p - t_d\right)$$

$$\xi_3 = \frac{1}{4}\left(t_p^4 - t_d^4\right) - t_p^3\left(t_p - t_d\right) \quad ; \quad \xi_4 = \frac{1}{\beta+2}\left(t_p^{\beta+2} - t_d^{\beta+2}\right) - t_p^{\beta+1}\left(t_p - t_d\right)$$

$$\xi_5 = \frac{1}{\beta+3}\left(t_p^{\beta+3} - t_d^{\beta+3}\right) - t_p^{\beta+2}\left(t_p - t_d\right)$$

$$\xi_6 = \frac{1}{\beta+4}\left(t_p^{\beta+4} - t_d^{\beta+4}\right) - t_p^{\beta+3}\left(t_p - t_d\right)$$

$$\xi_7 = t_p^{\beta}\left(t_p - t_d\right) - \frac{1}{(\beta+1)}\left(t_p^{(\beta+1)} - t_d^{(\beta+1)}\right)$$

$$\xi_8 = \frac{1}{\beta+2}\left(t_p^{\beta+2} - t_d^{\beta+2}\right) - \frac{t_p}{\beta+1}\left(t_p^{\beta+1} - t_d^{\beta+1}\right)$$

$$\xi_{10} = \frac{1}{\beta+3}\left(t_p^{\beta+3} - t_d^{\beta+3}\right) - \frac{t_p^2}{\beta+1}\left(t_p^{\beta+1} - t_d^{\beta+1}\right)$$

$$\xi_{10} = \frac{1}{\beta+4}\left(t_p^{\beta+4} - t_d^{\beta+4}\right) - \frac{t_p^3}{\beta+1}\left(t_p^{\beta+1} - t_d^{\beta+1}\right)$$

$$\xi_{11} = \frac{1}{\beta+1}\left(t_p^{\beta+1} - t_d^{\beta+1}\right) \quad ; \quad \xi_{12} = \frac{t_p^{\beta}}{\beta+1}\left(t_p^{\beta+1} - t_d^{\beta+1}\right) - \frac{1}{2\beta+1}\left(t_p^{2\beta+1} - t_d^{2\beta+1}\right)$$

$$\xi_{13} = \frac{1}{3}\left(t_p^3 - t_d^3\right) - \frac{t_p}{2}\left(t_p^2 - t_d^2\right) \quad ; \quad \xi_{14} = \frac{1}{4}\left(t_p^4 - t_d^4\right) - \frac{t_p^2}{2}\left(t_p^2 - t_d^2\right)$$

$$\xi_{15} = \frac{1}{5}\left(t_p^5 - t_d^5\right) - \frac{t_p^3}{2}\left(t_p^2 - t_d^2\right) \quad ; \quad \xi_{16} = \frac{1}{\beta+3}\left(t_p^{\beta+3} - t_d^{Gb+3}\right) - \frac{t_p^{\beta+1}}{2}\left(t_p^2 - t_d^2\right)$$

$$\xi_{17} = \frac{1}{\beta+4}\left(t_p^{\beta+4} - t_d^{\beta+4}\right) - \frac{t_p^{\beta+2}}{2}\left(t_p^2 - t_d^2\right) \quad ; \quad \xi_{18} = \frac{1}{\beta+5}\left(t_p^{\beta+5} - t_d^{\beta+5}\right) - \frac{t_p^{\beta+3}}{2}\left(t_p^2 - t_d^2\right)$$

$$\xi_{19} = \frac{t_p^{\beta}}{2}\left(t_p^2 - t_d^2\right) - \frac{1}{\beta+2}\left(t_p^{\beta+2} - t_d^{\beta+2}\right)$$

(e) Assume that returns from period $i-1$ are available for resale at the beginning of period i (except the first period in which there is no cycle previous to it). Also, it is assumed that the salvage value of the product at the end of the last period ($i = N$) is s. Therefore, the present value of return cost and resale revenue for each cycle is obtained as follows

$$PRC_i = \left\{\left(p\left[\gamma\left(aT + \frac{(bT^2)}{2} + \frac{cT^3}{3} - \frac{(arT^2)}{2} - \frac{(brT^3)}{3} - \frac{(crT^4)}{4}\right)@ + \delta p\left(T - \frac{(rT^2)}{2}\right)\right]\right)\right\} \quad (12)$$

Consequently, the present value of total profit, denoted by $f(p,t_p,T;N)$, is given by

$$f(p,t_p,T;N) = \begin{array}{l} \sum_{i=0}^{N-1}(SR - PC - K - HC - PRC)e^{-r \cdot i \cdot T} + Se^{-r \cdot H}\int_0^T(\gamma D(t) + \delta p)dt \\ -c_2\int_0^T(\gamma D(t) + \delta p)dt \end{array} \quad (13)$$

which we want to maximize subject to the following constraints $p > 0$, $0 < t_p < T$, $N \in N$. Thus, the objective to determine the values of t_p, p and N that maximizes $f(p,t_p,T;N)$ subject to $p > 0$ and $0 < t_p < T$, where N is a discrete variable, p and t_p are continuous variables, can be reduced to maximizing $f\left(p,t_p,\frac{H}{N};N\right)$. Owing to the complexity of the profit function, it is difficult to show analytically the validity of the sufficient conditions, $\left(\frac{\partial^2 f}{\partial p \partial t_p}\right)^2 - \left(\frac{\partial^2 f}{\partial t_p^2}\right)\left(\frac{\partial^2 f}{\partial p^2}\right) < 0$ and one of $\left(\frac{\partial^2 f}{\partial t_p^2}\right) < 0$ or $\left(\frac{\partial^2 f}{\partial p^2}\right) < 0$. Since N is a discrete variable, the following algorithm can be used to determine the optimal values of p, t_p and N for the proposed model.

We may refer to $f\left(p, t_p, \frac{H}{N}; N\right)$ and, for the sake of convenience, just denote it by $f\left(p, t_p, N\right)$.

4. The optimal solution procedure

Using the following algorithm the optimal values of t_p, p and N.

Step 1 Let $N = 1$

Step 2 The necessary conditions for optimality are

$$\frac{\partial}{\partial p} f\left(p, t_p, N\right) = 0 \text{ and } \frac{\partial}{\partial t_p} f\left(p, t_p, N\right) = 0 \qquad (14)$$

Step 3 For different integer values of N, derive t_{p^*} and p^* from equations (14). Substitute $\left(p^*, t_{p^*}, N^*\right)$ to equation (13) to obtain $f\left(p^*, t_{p^*}, N^*\right)$.

Step 4 Add one unit to N and repeat steps 2 and 3 for the new N. If there is no increase in the last value of $f\left(p, t_p, N\right)$, then consider the previous one which has the maximum value. The point $\left(p^*, t_{p^*}, N^*\right)$ and the value $f\left(p^*, t_{p^*}, N^*\right)$ constitute the optimal solution and satisfy the following conditions $\Delta f\left(p^*, t_{p^*}, N^*\right) < 0 \Delta f\left(p^*, t_{p^*}, N^* - 1\right)$

where $\Delta f\left(p^*, t_{p^*}, N^*\right) = \left(p^*, t_{p^*}, N^* + 1\right) - \left(p^*, t_{p^*}, N^*\right)$. We substitute $\left(p^*, t_{p^*}, N^*\right)$ into equation (8) to derive the N^{th} production lot size.

5. Numerical example

To illustrate the solution procedure let us consider the following numerical example. Consider the following parameters: R=800 units/unit time, c_1 =\$8/unit/unit time, c_2 =\$10/unit, t_d =0.04 unit time, K = 250/production run, r = 0.9, a = 500, b = 150, c = 300, H = 40 unit time, α = 0.9, β = 0.9, γ = 0.9, s = \$3/unit, δ = 0.9.

N	t_p	T	P	$f(p,t_p,N)$
1	0.8207	40	1285	1227
2	0.8214	20	3436	4141
3*	0.8220*	13.33*	1646*	5978*
4	0.8226	10	9986	1576

The optimal solution is $p^* = 1646$, $t_{p^*} = 0.8220$, $T^* = 13.33$, $Q^* = 658$, $f(p^*,t_{p^*},N^*) = 5978$. If we plot the total profit function for different values of p which varies from 500 to 4500 and for a fixed $t_{p^*} = 0.8220$, we get a strictly concave graph of the total profit function which is given in Figure 2.

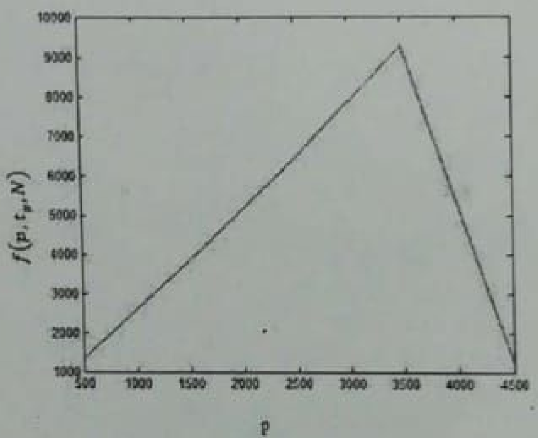

Figure 2: Total profit $f(p,t_p,N)$ vs. p

6. Conclusion

In this paper, we have developed a finite horizon inventory model with time dependent quadratic demand for non-instantaneous deteriorating items to address a realistic environment. The effects of inflation and customer returns are also considered. An analytical formulation of the model and an optimal solution procedure are also presented. Graphical representation of the concavity of the profit function is provided by considering the numerical example.

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