

Simple Test Tube Splicing on Images

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Abstract—H array splicing system is an effective extension of splicing operation to images. A special class of these H array splicing system allowing only simple splicing rules is also considered. Test tube array system for picture generation has also been studied based on the splicing operations. Motivated by these models here we introduce and investigate a new generative model called Simple Test Tube Array System based on simple parallel splicing operations.

Splicing on strings, Splicing on images, H Array Splicing System, Test Tube Array Splicing System.

I. INTRODUCTION

Formal language theory which is a fundamental branch of theoretical Computer Science has found applications in many areas. Among these DNA computing is an important application area with theoretical results based on language theory have been proposed and intensively investigated by researchers. Head [4,5] introduced a new operation on strings of symbols and defined splicing systems motivated by the behaviour of DNA sequences under the action of enzymes and ligases. Paun et al [10] extended the splicing systems which are computationally universal. Krithivasan [9] extended the concepts of splicing to arrays and defined array splicing systems. Parallel splicing on images of rectangular arrays is introduced in [6,7] as a simple and effective extension of the operation of splicing on strings. As an extension to arrays, a special kind of domino splicing rule called simple array splicing is considered [8].

On the other hand Adleman's experiment [1,2], where he computed an instance of the Hamiltonian path problem with DNA in test tubes paved the way for the number of theoretical models being proposed and examined for their computational power and one among these is Test tube distributed systems based on splicing considered in [3].

A new generative model called test tube array system for picture generation is introduced and investigated [11]. Here we introduce a variant of test tube array system called as Simple Test Tube Array Splicing System for picture generation and some mathematical properties are studied. The generative power is also examined.

II. PRELIMINARIES

A. Simple H Array Splicing System

We now recall simple domino splicing rules and the notion of Simple H Array Splicing Systems [8].

1) *Definition* : A simple domino column splicing rule over V is of the form

$$r = \begin{array}{c} a \\ b \end{array} \quad \not\leftarrow \quad \begin{array}{c} 1 \\ 1 \end{array} \quad \$_c \quad \begin{array}{c} a \\ b \end{array} \quad \not\leftarrow \quad \begin{array}{c} 1 \\ 1 \end{array}$$

for some $a, b \in V \cup \{\#\}$.

A simple domino row splicing rule over V is of the form

$$r = a \quad b \quad \not\leftarrow \quad 1 \quad 1 \quad \$_r \quad a \quad b \quad \not\leftarrow \quad 1 \quad 1$$

for some $a, b \in V \cup \{\#\}$.

2) *Definition* : A Simple H array scheme is a triplet $\Gamma = (V, R_c, R_r)$ where V is an alphabet, R_c = a finite set of simple domino column splicing rules, and R_r = a finite set of simple domino row splicing rules. For a given Simple H array scheme $\Gamma = (V, R_c, R_r)$ and a language $L \subseteq V^{**}$, we define

$$\Gamma(L) = \{Z \in V^{**} / (X\#, Y\#) |^{\Phi} Z\# \text{ or } (X\#, Y\#) |^{\Theta} Z\# \text{ for some } X, Y \in L\}.$$

In other words, $\Gamma(L)$ consists of arrays obtained by column or row splicing any two arrays of L using the simple domino column or row splicing rules.

A Simple H array splicing system (SHAS) is defined by $S = (\Gamma, I)$ where $\Gamma = (V, R_c, R_r)$ and I is a finite subset of V^{**} . The language of S is defined by $L(S) = \Gamma^*(I)$ and we call it a Simple H array splicing language (SHASL) and denote the class of such languages by $\mathcal{L}(SHASL)$.

B. Test Tube Array System

Now we recall Test Tube Array System and languages. [11]

1) *Definition* : A Test Tube Array (TTA) system (of degree $n, n \geq 1$) is a construct

$$\Gamma = ((V, (A_1, R_1, V_1), \dots, (A_n, R_n, V_n))$$

, where V is an alphabet, $A_i \subset V^{**}$,

R_i is a domino column splicing rule over V of the form

$$R_i = \alpha_1 \# \alpha_2 \$ \alpha_3 \# \alpha_4$$

where each $\alpha_i = \begin{pmatrix} a \\ b \end{pmatrix}$ for some $a, b \in V \cup \#$ or a domino row splicing rule over V of the form

$$R_i = \beta_1 \# \beta_2 \$ \beta_3 \# \beta_4$$

where each $\beta_i = (a \ b)$ for some $a, b \in V \cup \#$ and $V_i \subset V$, for each $1 \leq i \leq n$. Each triplet (A_i, R_i, V_i) is called a component of the system, or a tube i : A_i is the set of axioms of the tube i , $R_i \subseteq V^{**} \# V^{**} \$ V^{**} \# V^{**}$, R_i is the set of splicing rules of the tube i and V_i is the selector of the tube i .

$$\text{Let } B = V^{**} - \cup_{i=1}^n V_i^{**}.$$

The pair $\sigma_i = (V_i, R_i)$ is the underlying H array scheme associated with the component of the system. An n -tuple $(L_1, \dots, L_n), L_i \subset V^{**}, 1 \leq i \leq n$ is called a configuration of the system: L_i is also called the contents of the i^{th} tube. For two configurations $(L_1, \dots, L_n), (L'_1, \dots, L'_n)$, we define $(L_1, \dots, L_n) \Rightarrow (L'_1, \dots, L'_n)$ iff, for each $1 \leq i \leq n$, $L'_i = \cup_{j=1}^n (\sigma_j^*(L_j) \cap V_i^{**}) \cup (\sigma_i^*(L_i) \cap B)$. The language generated by Γ is

$$L(\Gamma) = \{w \in V^{**} \mid w \in L_1^{(t)} \text{ for some } (A_1, \dots, A_n) \Rightarrow^* (L_1^{(t)}, \dots, L_n^{(t)}, t \geq 0)\},$$

where \Rightarrow^* is the reflexive and transitive closure of the relation \Rightarrow .

III. SIMPLE TEST TUBE ARRAY SYSTEM

Now we introduce Simple Test Tube Array System and languages.

A. Definition and Example

1) *Definition* : A Simple Test Tube Array System (*STTAS*) (of degree $n, n \geq 1$) is a construct

$$\Gamma = ((V, (A_1, R_1, V_1), \dots, (A_n, R_n, V_n)),$$

where V is an alphabet, $A_i \subset V^{**}$, R_i is a simple column or row splicing rule over V of the form

$$R_i = \begin{matrix} a & & & & a & & & & \\ b & \# & 1 & \$ & b & \# & 1 & \text{or} & \\ & & & & & & & & \end{matrix}$$

$$R_i = \begin{matrix} a & b & \# & 1 & 1 & \$ & a & b & \# & 1 & 1 \end{matrix}$$

for some $a, b \in V \cup \{\#\}$ and $V_i \subset V$, for each $1 \leq i \leq n$. Each triplet (A_i, R_i, V_i) is called a component of the system, or a tube i : A_i is the set of axioms of the tube i , R_i is the set of simple splicing rules of the tube i and V_i is the selector of the tube i . Let $B = V^{**} - \cup_{i=1}^n V_i^{**}$.

The pair $\sigma_i = (V_i, R_i)$ is the underlying H array scheme associated with the component of the system. An n -tuple $(L_1, \dots, L_n), L_i \subset V^{**}, 1 \leq i \leq n$ is called a configuration of the system: L_i is also called the contents of the i^{th} tube.

For two configurations $(L_1, \dots, L_n), (L'_1, \dots, L'_n)$, we define $(L_1, \dots, L_n) \Rightarrow (L'_1, \dots, L'_n)$ iff, for each $1 \leq i \leq n$, $L'_i = \cup_{j=1}^n (\sigma_j^*(L_j) \cap V_i^{**}) \cup (\sigma_i^*(L_i) \cap B)$.

This means that the contents of each tube is spliced according to the associated set of rules (we pass from L_i to $\sigma_i^*(L_i), 1 \leq i \leq n$), and the result is redistributed among n tubes according to the selectors V_1, \dots, V_n . The part which cannot be redistributed (does not belong to some $V_i^{**}, 1 \leq i \leq n$) remains in the tube. Because we have imposed no restrictions on the alphabets V_i (eg., we did not require that they are pairwise disjoint), when an array in $\sigma_j^*(L_j)$ belong to several languages V_i^{**} , then copies of this array will be distributed to all tubes with this property.

One of the components of the system Γ is designed as the master one and the union of its contents in all possible computations is the result of the system computations. We consider the first tube is the master. Hence the language generated by Γ is

$$L(\Gamma) = \{w \in V^{**} \mid w \in L_1^{(t)} \text{ for some } (A_1, \dots, A_n) \Rightarrow^* (L_1^{(t)}, \dots, L_n^{(t)}, t \geq 0)\},$$

where \Rightarrow^* is the reflexive and transitive closure of the relation \Rightarrow .

The Language generated by Simple Test Tube Array System is denoted by *STTAL* and the class of all such Languages by $\mathcal{L}(STTAL)$

2) *Example* : Consider the system

$$\Gamma = (V, (A_1, R_1, V_1), (A_2, R_2, V_2), (A_3, R_3, V_3)) \text{ with } V = \{a, b, c, d, e, f\}, A_1 = \{(cabd)_n, (gb^2e)_n\}, V_1 = \{a, b, c, d\},$$

$$R_1 = \left\{ \begin{matrix} \# & \# & \# & \# & \# & b & \# & 1 & b & \# & 1 \\ b & \# & 1 & \$ & b & \# & 1 & & b & \# & 1 \\ & & & & & & & & & & \end{matrix} \right\}$$

$$V_2 = \{a, b, c, e\}, A_2 = \{(ca^2g)_n\}$$

$$R_2 = \left\{ \begin{matrix} \# & \# & \# & \# & \# & a & \# & 1 & a & \# & 1 \\ a & \# & 1 & \$ & a & \# & 1 & & a & \# & 1 \\ & & & & & & & & & & \end{matrix} \right\}$$

$$A_3 = \{(bd)_n\}, R_3 = R_1, V_3 = \{a, b, c, e\}$$

We start from the configuration

$$(A_1, A_2, A_3) = (\{(cabd)_n, (gb^2e)_n\}, \{(ca^2g)_n\}, \{(bd)_n\})$$

During the Iteration I, Splicing is possible in tubes 1,2 and 3 simultaneously.

In tube 1,

$$\begin{aligned} & ((cab)_n|(d)_n, (gb)_n|(be)_n) \vdash ((cab^2e)_n, (gbd)_n) \\ & ((cab)_n|(d)_n, (gbb)_n|(e)_n) \vdash ((cabe)_n, (gb^2d)_n) \\ & ((cab)_n|(d)_n, (cab)_n|(d)_n) \vdash ((cabd)_n, (cabd)_n) \\ & ((gb)_n|(be)_n, (gb)_n|(be)_n) \vdash ((gb^2e)_n, (gb^2e)_n) \\ & ((gbb)_n|(e)_n, (gb)_n|(be)_n) \vdash ((gb^3e)_n, (gbe)_n) \\ & ((gb)_n|(be)_n, (gbb)_n|(e)_n) \vdash ((gbe)_n, (gb^3e)_n) \\ & ((gbb)_n|(e)_n, (gbb)_n|(e)_n) \vdash ((gb^2e)_n, (gb^2e)_n) \end{aligned}$$

In tube 2,

$$\begin{aligned} & ((ca^2)_n|(g)_n, (ca^2)_n|(g)_n) \vdash ((ca^2g)_n, (ca^2g)_n) \\ & ((ca)_n|(ag)_n, (ca)_n|(ag)_n) \vdash ((ca^2g)_n, (ca^2g)_n) \\ & ((ca)_n|(ag)_n, (ca^2)_n|(g)_n) \vdash ((cag)_n, (ca^3g)_n) \\ & ((ca^2)_n|(g)_n, (ca)_n|(ag)_n) \vdash ((ca^3g)_n, (cag)_n) \end{aligned}$$

In tube 3,

$$((b)_n|(d)_n, (b)_n|(d)_n) \vdash ((bd)_n, (bd)_n)$$

The generated arrays $(cabe)_n$ and $(cab^2e)_n$ in tube 1 are distributed to tubes 2 and 3. The remaining generated arrays that cannot be distributed will remain in tube 1. The generated arrays in tube 2 cannot be distributed to tube 1 or tube 3 and they remain in tube 2 itself. The generated array $(bd)_n$ in tube 3 is transmitted to tube 1.

$$\text{Now } A_1 = \left\{ \begin{array}{l} (cabd)_n, (gbd)_n, (gbe)_n, (gb^2d)_n, \\ (gb^2e)_n, (gb^3e)_n, (bd)_n \end{array} \right\}$$

$$A_2 = \{(cabe)_n, (cab^2e)_n, (cag)_n, (ca^2g)_n, (ca^3g)_n\}$$

$$A_3 = \{(cabe)_n, (cab^2e)_n, (bd)_n, \}$$

Similarly the process can be iterated. After the iteration 2,

$$A_1 = \left\{ \begin{array}{l} (cabd)_n, (gbd)_n, (gbe)_n, (gb^2d)_n, (gb^2e)_n, \\ (gb^3e)_n, (gb^4e)_n, (gb^5e)_n, (gb^6e)_n, (bd)_n \end{array} \right\}$$

$$A_2 = \left\{ \begin{array}{l} (cabe)_n, (cab^2e)_n, (cab^3e)_n, \\ (cag)_n, (ca^2g)_n, (ca^3g)_n \end{array} \right\}$$

$$A_3 = \{(ca^s b^t e)_n, (bd)_n, s, t \leq 3, n \geq 2\}$$

After the third iteration,

$$A_1 = \{(ca^s b^t d)_n, \dots, s, t \geq 1, n \geq 2\}$$

Consequently, by the repeated process of splicing , tube

1 contains the required array. We consider that the first tube is the master. Hence, the language generated by Γ is

$$\mathcal{L}(\Gamma) = \{(ca^s b^t d)_n, \quad s, t \geq 1, n \geq 2\}$$

B. Closure Properties

1) *Proposition* : The class $\mathcal{L}(STTAS)$ is closed under reflection on the base and right leg and rotations by 90° , 180° and 270° .

Proof: We first prove that $\mathcal{L}(STTAS)$ is closed under reflection.

Let $\Gamma = (V; (A_1, R_1, V_1), (A_2, R_2, V_2), \dots, (A_n, R_n, V_n))$ and $B = (B_1 B_2, \dots, B_n)$ where $A_i \subseteq V^{**}$, $V_i \subseteq V$ be a Simple Test Tube Array System with rules in R_i of the form

$$p = \begin{array}{c} a_{i1} \\ b_{i1} \end{array} \quad \& \quad \begin{array}{c} \lambda_1 \\ \lambda_2 \end{array} \quad \$ \quad \begin{array}{c} a_{i1} \\ b_{i1} \end{array} \quad \& \quad \begin{array}{c} \lambda_3 \\ \lambda_4 \end{array} \quad (i = 1, 2, \dots, n)$$

to describe a picture language L .

The picture language consisting of images which are reflections of arrays of L on the base can be obtained by Simple Test Tube Array System consisting of rules of the form

$$p' = \begin{array}{c} b_{i1} \\ a_{i1} \end{array} \quad \& \quad \begin{array}{c} \lambda_2 \\ \lambda_1 \end{array} \quad \$ \quad \begin{array}{c} b_{i1} \\ a_{i1} \end{array} \quad \& \quad \begin{array}{c} \lambda_4 \\ \lambda_3 \end{array} \quad (i = 1, 2, \dots, n)$$

corresponding to p .

Similarly, the reflections of arrays of L on the right leg can be obtained by an Simple Test Tube Array System with modified rules

$$p' = \begin{array}{c} \lambda_3 \\ \lambda_4 \end{array} \quad \& \quad \begin{array}{c} a_{i1} \\ b_{i1} \end{array} \quad \$ \quad \begin{array}{c} \lambda_1 \\ \lambda_2 \end{array} \quad \& \quad \begin{array}{c} a_{i1} \\ b_{i1} \end{array} \quad (i = 1, 2, \dots, n)$$

corresponding to p .

We next prove that $\mathcal{L}(STTAS)$ is closed under rotations by 90° , 180° and 270° . We mention only the modified rules of $R_i (i = 1, 2, \dots, n)$

$$b_{i1} \ a_{i1} \ \& \ \lambda_2 \ \lambda_1 \ \$ \ b_{i1} \ a_{i1} \ \& \ \lambda_4 \ \lambda_3, \ (i = 1, 2, \dots, n)$$

for rotation by 90° ,

$$\begin{array}{c} \lambda_4 \\ \lambda_3 \end{array} \ \& \ \begin{array}{c} b_{i1} \\ a_{i1} \end{array} \ \$ \ \begin{array}{c} \lambda_2 \\ \lambda_1 \end{array} \ \& \ \begin{array}{c} b_{i1} \\ a_{i1} \end{array} \ (i = 1, 2, \dots, n)$$

for rotation by 180° and

$$\lambda_3 \ \lambda_4 \ \& \ a_{i1} \ b_{i1} \ \$ \ \lambda_1 \ \lambda_2 \ \& \ a_{i1} \ b_{i1} \ (i = 1, 2, \dots, n)$$

for rotation by 270° .

2) *Proposition* : The class $\mathcal{L}(STTAS)$ is not closed under union and row and column concatenation.

C. Generative Power

1) *Theorem* : Rectangles of ' a ' surrounded by ' x 's can be generated by a Simple Test Tube Array System.

Proof: Let $\Gamma = (V, (A_1, R_1, V_1), (A_2, R_2, V_2))$ with $V = \{x, a, y\}, V_1 = \{x, a\},$

$$A_1 = \left\{ \begin{array}{cccc} x & x & x & y & x & x & x \\ x & a & x & , & y & a & a & x \\ x & x & x & y & x & x & x \end{array} \right\},$$

$$R_1 = (R_{r_1}, R_{c_1}), R_{r_1} = \phi$$

$$R_{c_1} = \left\{ \begin{array}{cccc} \# & \# & \# & \# & \# & ; & x & \# & 1 & \$ & x & \# & 1 & ; \\ x & \# & 1 & \$ & x & \# & 1 & ; & a & \# & 1 & \$ & a & \# & 1 & ; \\ a & \# & 1 & \$ & a & \# & 1 & ; & x & \# & 1 & \$ & x & \# & 1 & ; \\ a & \# & 1 & \$ & a & \# & 1 & ; & \# & \# & \# & \# & \# & \# & \# & \# \\ a & \# & 1 & \$ & a & \# & 1 & ; \\ x & \# & 1 & \$ & x & \# & 1 & ; \end{array} \right\}$$

$$A_2 = \left\{ \begin{array}{cccc} y & y & y & y \\ x & a & a & x \\ x & a & a & x \\ x & x & x & x \end{array} \right\}, V_2 = \{x, a\},$$

$$R_2 = (R_{r_2}, R_{c_2}), R_{c_2} = \phi,$$

$$R_{r_2} = \left\{ \begin{array}{l} \# x \# 1 \$ \# x \# 1 ; \\ x a \# 1 1 \$ x a \# 1 1 ; \\ a a \# 1 1 \$ a a \# 1 1 ; \\ a x \# 1 1 \$ a x \# 1 1 ; \\ x \# \# 1 1 \$ x \# \# 1 1 \end{array} \right\}$$

L is the language consisting of pictures of the form in Figure 1, where white square is interpreted as a and black as x .

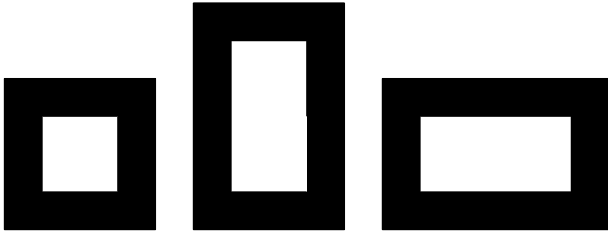


Figure 1. Rectangles of 'a's surrounded by 'x's

2) *Theorem* : L shapes can be generated by a Simple Test Tube Array System.

Proof: Let $\Gamma = (V, (A_1, R_1, V_1), (A_2, R_2, V_2))$ with $V = \{x, y, \cdot\}, V_1 = \{x, \cdot\},$

$$A_1 = \left\{ \begin{array}{cccc} x & \cdot & y & \cdot & \cdot & \cdot \\ x & x & , & y & x & x & x \end{array} \right\},$$

$$R_1 = (R_{r_1}, R_{c_1}), R_{r_1} = \phi$$

$$R_{c_1} = \left\{ \begin{array}{cccc} \# & \# & \# & \# & \# & ; & \cdot & \# & 1 & \$ & \cdot & \# & 1 & ; \\ \cdot & \# & 1 & \$ & \cdot & \# & 1 & ; & x & \# & 1 & \$ & x & \# & 1 & ; \\ \cdot & \# & 1 & \$ & \cdot & \# & 1 & ; & x & \# & 1 & \$ & x & \# & 1 & ; \\ \cdot & \# & 1 & \$ & \cdot & \# & 1 & ; & \# & \# & \# & \# & \# & \# & \# & \# \end{array} \right\}$$

$$A_2 = \left\{ \begin{array}{ccc} y & y & y \\ x & \cdot & \cdot \\ x & \cdot & \cdot \\ x & x & x \end{array} \right\}, V_2 = \{x, \cdot\},$$

$$R_2 = (R_{r_2}, R_{c_2}), R_{c_2} = \phi$$

The picture language M consisting of all $m \times n$ arrays ($m \geq 2, n \geq 2$) describing token L of 'x's as depicted in Figure 2.

$$\begin{array}{cccc} x & \cdot & \cdot & \cdot \\ x & \cdot & \cdot & \cdot \\ x & \cdot & \cdot & \cdot \\ x & \cdot & \cdot & \cdot \\ x & \cdot & \cdot & \cdot \\ x & \cdot & \cdot & \cdot \\ x & x & x & x \end{array}$$

Figure 2. Token L of 'x's.

IV. CONCLUSION

In this paper, we have introduced a variant of Test Tube Array System. We have studied some closure properties such as union, concatenation and rotation on family of languages generated by Simple Test Tube Array System. Some models are generated. It is worth comparing with other models of picture description. Further application and properties of this system could be studied. It will be of interest to examine the possibility to incorporate in P system and compare the resulting model with the model proposed here. It is a question on the minimum number of tubes required to solve the problem.

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