ROMANIAN JOURNAL OF INFORMATION SCIENCE AND TECHNOLOGY
Volume 21, Number 3, 2018, 219–231

Multihanded and Near Perfect Self Assembling of Tile Assembling Systems

P. Helen CHANDRA[1] and S. THABITHAL[2]

[1]Associate Professor in Mathematics, Jayaraj Annapackiam College for Women, Periyakulam, Theni, Tamilnadu, India
[2]Research Scholar. Jayaraj Annapackiam College for Women, Periyakulam, Theni, Tamilnadu, India
E-mail: chandrajac@yahoo.com

Abstract. In this paper we present multiple handed diamond tile assembly models for the assembly of discrete Sierpinsky triangle. It generalizes the two handed assembly model to allow up to h assemblies to combine in a single step. The problem of strict self assembly of infinite fractals within the self assembly is considered. *Six handed Diamond Assembly Model* is constructed in a near perfect way with scale factor 1. We have further assembled the Sierpinsky triangle within the *Three handed Diamond Assembly Model* with scale factor 3.

Key-words: self assembly; strict self assembly; near perfect assembly; near triangle assembly; base shape; combinable shape.

1. Introduction

Self-assembly is the process by which a collection of relatively simple components, beginning in a disorganized state, spontaneously and without external guidance coalesce to form more complex structures. The process is guided by only local interactions between the components, which typically follow a basic set of rules. In fact, self-assembling systems abound in nature, resulting in everything from the delicate crystalline structure of snowflakes to many of the structurally and functionally varied components of biological systems.

Understanding how to design molecular self-assembly systems that build complex, algorithmically specified shapes and patterns promises to be of fundamental importance for the future of nanotechnology. With the intention of precisely manipulating and organizing matter on the scale of individual atoms and molecules, several artificial self-assembling systems have been designed. In order to model such systems, theoretical models have been developed. One of the most popular among these is the Tile Assembly Model introduced by Erik Winfree. [1]. In [2] the difference between the standard seeded model $(aTAM)$ of tile self-assembly and the seedless two-handed model of tile self-assembly $(2HAM)$ is studied. Most of the results suggest that the two-handed model is more powerful. In particular, it is shown how to simulate any seeded system with a two-handed system that is essentially just a constant factor larger.

Winfree [1] showed that the Sierpinski triangle weakly self-assembles and a striking molecular realization of this self-assembly was achieved by Rothemund et al. [3]. Lathrop et al. [4] proved that the Sierpinski triangle cannot strictly self-assemble. Patitz and Summers [5] exhibited a large class of fractals that cannot strictly self-assemble. The challenging problem is the strict assembly of fractals in a purely growth (non-detaching) model in which the shape of the assembly is exactly the shape of the fractal. The problem of the strict self-assembly of infinite fractals within tile self-assembly is considered in [6]. The tile assembly algorithms for the assembly of the discrete Sierpinski triangle by squared tiles within a class of models called, the h-handed assembly model $(h-HAM)$ is provided. It generalizes the $2-HAM$ to allow up to h assemblies to combine in a single assembly step.

On the other hand, a theoretical model of pasting system called a Triangular Tile Pasting System $(TTPS)$ is considered in [7] to generate two dimensional patterns that are formed by gluing triangular tiles and by iso-array grammars. In [8], diamond tile self assembly model is discussed by simulating a binary counter.

In this paper, we present multiple handed tile assembly models that self-assemble by diamond tiles for the strict self assembly of discrete fractal patterns. The six handed assembly model strictly self assembles the non tree discrete diamond Sierpinsky triangle. It works with 34 tile types and achieves the near perfect assembly with scale factor 1. The three handed assembly model that strictly self assembles the discrete diamond Sierpinsky triangle with scale factor 3 does not near perfectly assemble.

2. Preliminaries

2.1. Tiles

Consider some alphabet of glue types Σ. A tile is a finite edge polygon with some finite subset of border points each assigned some glue type from Σ. Further, each glue type $g \in \Sigma$ has some non-negative integer strength $str(g)$. Finally, each tile may be assigned a finite length string label, e.g., *black, white*, 0 *or* 1. We consider a special class of tiles that are unit diamonds of the same orientation with at most one glue type per edge, with each glue being placed exactly in the center of the tile's edge. Further, for simplicity, we assume each tile center is located at an integer coordinate in $\mathbb{Z}^2$.

2.2. Diamond Grid

Let $\mathbb{N}$ be the set of natural numbers $\{0, 1, 2, \dots\}, \mathbb{Z} = \mathbb{N} \cup -\mathbb{N}$ is the set of integers and R is the set of real numbers. We are working on the two-dimensional grid of integer positions $Z \times Z$ which is rotated $45°$ clockwise. The directions
$\mathcal{D} = \{ne, nw, se, sw\}$ will be used as functions from $\mathbb{Z} \times \mathbb{Z}$ to $\mathbb{Z} \times \mathbb{Z}$:
$ne(x, y) = (x, y + 1), nw(x, y) = (x - 1, y)$,
$se(x, y) = (x + 1, y)$ and $sw(x, y) = (x, y - 1)$.
We say that (x, y) and $(x^{'}, y^{'})$ are neighbors if $(x^{'}, y^{'}) \in \{ne(x, y), nw(x, y), se(x, y), sw(x, y)\}$.
$ne = sw^{-1}$ and $nw = se^{-1}$. The directions and coordinates of the diamond grid are shown in (Fig.1)

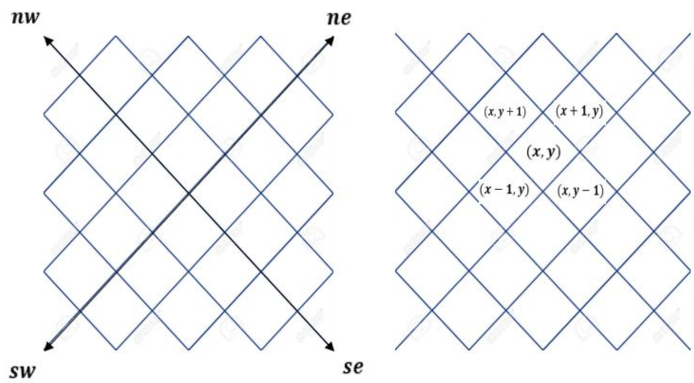


Fig. 1. Coordinate Axes and Coordinates of Diamond Grid.

2.3. Diamond Tiles

Formally, a diamond tile t is a $4-$ tuple $(\sigma_{ne}, \sigma_{nw}, \sigma_{se}, \sigma_{sw}) \in \Sigma^4$ indicating the binding domains on the northeast, northwest, southeast and southwest sides. For $D \in \mathcal{D}$ we write $bd_D(t)$ to refer to the binding domain of the respective side of diamond tile t. According to this definition, diamond tiles may not be rotated;
$(\sigma_{ne}, \sigma_{nw}, \sigma_{se}, \sigma_{sw)} \neq (\sigma_{sw}, \sigma_{ne}, \sigma_{nw}, \sigma_{se})$.

A special binding domain $null$ represents a non-interaction and the special tile *empty = (null, null, null, null)* is used to represent the absence of any other tile. The binding domains determine the interaction between tiles; that is, when two diamond tiles may be placed next to each other. A function $g : \Sigma \times \Sigma \to \mathbb{R}$ where $null \in \Sigma$ is a strength function if $\forall \sigma, \sigma^{'} \in \Sigma, g(\sigma, \sigma^{'}) = g(\sigma^{'}, \sigma)$ and $g(null, \sigma) = 0$. Two diamond tiles that abut on sides labeled σ and $\sigma^{'}$ bind with strength $g(\sigma, \sigma^{'})$, as discussed below. Here, we will only consider g such that mismatched sides have no interaction strength and matching sides have positive strengths given in integral units, in which case the strength of a side labeled by σ is $\hat{g}(\sigma) \in \mathbb{N}$ and $g(\sigma, \sigma^{'}) = \hat{g}(\sigma)$ if $\sigma = \sigma^{'}$, 0 otherwise.

Let $\mathcal{T}$ be a set of tiles containing the special tile *empty*. A configuration of $\mathcal{T}$ is a function $A : \mathbb{Z} \times \mathbb{Z} \to \mathcal{T}$. We write $(x, y) \in A$ iff $A(x, y) \neq empty$. For $D \in \mathcal{D}$, we say the tiles at (x, y) and $D(x, y)$ bind to each other with strength $g_D^A(x, y) = g(bd_D(A(x, y)), bd_{D(-1)}(A(D(x, y)))$. Thick sides have strength 2, thin sides have strength 1. If $g_D^A(x, y) > 0$, then the diamond tiles make a bond. If t is a diamond tile, $A_t^{(x,y)}$ is the configuration such that $A_t^{(x,y)}(x, y) = t$ and all other sites are *empty*. $A_{empty}^{(0,0)}$ is called the *empty* configuration.

The free energy of a configuration C is the sum of all interaction strengths between tiles $G(C) = \frac{1}{2} \sum_{x,y \in \mathbb{Z}} \sum_{D \in \mathcal{D}} g_D^C(x, y)$.

The temperature τ gives the minimal interaction strength required to overcome thermal disruption. A configuration C is a $\tau-$stable assembly if for all non-empty configurations A and B such that $C = A + B, G(C) \geq G(A) + G(B) + \tau$. That is a $\tau-$stable assembly cannot fall apart into two pieces without decreasing the total G by τ or more. Note that for $\tau > 0$, a $\tau-$stable assembly must contain a single connected component. When τ is understood, we simply say that C is an assembly.

2.4. Diamond Tile Assembly System D_{TAS}

A diamond tile system D_{TAS} is defined by the quadruple $\Gamma = (\mathcal{T}, S, g, \tau)$ where $\mathcal{T}$ is a finite set of diamond tiles containing *empty*, S is a set of τ-stable seed assemblies, g is a strength function and $\tau \geq 0$ is the temperature. $|S| = 1$ where $S = A_s^{(0,0)}$ for some seed Diamond tile s.

Diamond Tile Self-assembly is defined by a relation between configurations: $A \rightarrow B$ if there exists a diamond tile $t \in \mathcal{T}$ and a site (x, y) such that $B = A + A_t^{(x,y)}$ and B is $\tau-$stable. Since $G(A_t^{(x,y)}) \neq 0, G(B) \geq G(A) + \tau$; i.e., a diamond tile may be added to an assembly if the summed strength of its interactions with its neighbors exceeds a threshold set by the temperature. $\rightarrow_\Gamma^\star$ is the reflexive, transitive closure of $\rightarrow_\Gamma$.The diamond tile assembly system defines a partially ordered set, the produced assemblies $D_{Prod(\Gamma)}$ where

$$D_{Prod(\Gamma)} = \{A, \exists\, s \in S\, s.t.\ s \rightarrow_\Gamma^\star A\} \text{ and } A \leq B \text{ iff } A \rightarrow_\Gamma^\star B.$$

Another set, the terminal assemblies $D_{term(\Gamma)}$ is defined as the maximal elements of $D_{Prod(\Gamma)}$:

$$D_{term(\Gamma)} = \{A \in D_{Prod(\Gamma)},\, \nexists\, B\, s.t.\ A < B.\}$$

The produced assemblies include intermediate products of the self-assembly process whereas the terminal assemblies are just the end products and may be considered as the output. If $A \in D_{Prod(\Gamma)} \Rightarrow \exists\, B \in D_{term(\Gamma)}\ s.t.\ A \rightarrow_\Gamma^\star B$ then Γ is said to be haltable, in the sense that every path of self-assembly can eventually terminate. If Γ is haltable and $D_{term(\Gamma)}$ is finite, Γ is said to be halting in the sense that every path of self- assembly does eventually terminate. A halting tile system uniquely produces C if $D_{term(\Gamma)} = \{C\}$. If a tile system uniquely produces C then $D_{Prod(\Gamma)}$ is a lattice: the join of A and B is $A \cup B$ and the meet of A and B is $max\{C' \in D_{Prod(\Gamma)}\, s.t\, C' \leq (A \cap B)\}$. In general, if $D_{Prod(\Gamma)}$ is a lattice, we say that Γ produces a unique pattern – Γ need not be halting nor even haltable.

3. Producible assemblies

We are working on the two-dimensional diamond grid described in 2.2. An assembly is a set of diamond tiles at unique coordinates in $\mathbb{Z}^2$. Stable assemblies will necessarily consist of diamond tiles stacked face to face, forming a subset of the $2D$ diamond grid denoted by the tiled locations in the assembly. We refer to any subset of an assembly A as a subassembly of A. For an assembly A and integer vector $\vec{v} = (v_1, v_2)$, let $A_{\vec{v}}$ denote the assembly obtained by translating each tile in A by vector $\vec{v}$. A shape is any subset of $\mathbb{Z}^2$ and the shape of an assembly A is defined to be the shape consisting of the set of coordinate locations for the centers of each tile in A.

If a set of r assemblies are translated together to form a $\tau-$stale assembly then it is said to be $\tau-$ combinable. Formally, a set of stable assemblies $\{A_1, A_2, \ldots, A_r\}$ is said to be $\tau-$ combinable into an assembly C if there exists a set of assemblies $\{A_{1'}, A_{2'}, \ldots, A_{r'}\}$ such that each $A_{i'}$ is a translation of A_i and $C = \bigcup_{i=1}^{r} A_{i'}$ is a $\tau-$ stable assembly.

3.1. Abstract Diamond Tile Assembly Model

A diamond tile system in the Abstract Tile Assembly Model $(aDAM)$ is an ordered triple $\Gamma = (\mathcal{T}, s, \tau)$ where $\mathcal{T}$ is a finite set of diamond tiles called the tile set containing *empty*, s is an assembly over $\mathcal{T}$ called the τ-stable seed assembly and $\tau \geq 0$ is the temperature. The set of all tiles that are a translation of some tile in $\mathcal{T}$ is denoted as the set $\mathcal{T}^*$. Assembly in the $aDAM$ proceeds by growing from seed assembly s by any sequence of single tile attachments from $\mathcal{T}$ so long as each tile attachment connects with strength at least τ.

For a given $aDAM$ system $\Gamma = (\mathcal{T}, s, \tau)$, the set of producible assemblies for system Γ, $D_{Prod(\Gamma)}$ is defined recursively:
Base: $s \in D_{Prod(\Gamma)}$;
Recursion: For any $A \in D_{Prod(\Gamma)}$ and $b \in \mathcal{T}^*$ such that $C = A \cup \{b\}$ is $\tau-$stable, then $C \in D_{Prod(\Gamma)}$.
we also say $A \rightarrow_1^\Gamma B$ if A may grow into B through a single tile attachment and we say $A \rightarrow^\Gamma B$ if$A$ can grow into B through 0 or more tile attachments. For a shape S, we say a system Γ uniquely assembles S if for all $A \in D_{Prod(\Gamma)}$ there exists a $B \in D_{Prod(\Gamma)}$ of shape S such that $A \rightarrow^\Gamma B$. An assembly sequence denoting the possible sequence of growth for a given system is formally defined to be any sequence (finite or infinite) of assemblies $(A_0, A_1, A_2, \cdots \cdots)$ such that $A_0 = s$ and $A_i \rightarrow_1^\Gamma A_{i+1}$ for each i.

3.2. Multiple Handed Diamond Tile Assembly Model

The multiple handed Diamond tile assembly model $(hH - DAM)$ generalizes the two-handed assembly model to allow for potentially more than two hands. It allows up to a given number h pre-built assemblies to come together to form a new producible assembly. Formally, a multiple handed diamond tile assembly system is an ordered triple $\Gamma = (\mathcal{T}, \tau, h)$ where $\mathcal{T}$ is a finite set of diamond tiles called the tile set containing *empty*, $\tau \geq 0$ is the temperature and h is a positive integer called the number of hands. Assembly in an $hH - DAM$ system $(\mathcal{T}, \tau, h)$ proceeds by repeatedly combining up to h assemblies at a time to form new $\tau-$ stable assemblies.

For a given $(hH - DAM)$ system $\Gamma = (\mathcal{T}, \tau, h)$, the set of producible assemblies for system Γ, $D_{Prod(\Gamma)}$ is defined recursively:
Base: $\mathcal{T}^* \in D_{Prod(\Gamma)}$
Recursion: For any set of r assemblies, $0 \leq r \leq h$, if a producible set of assemblies $(A = A_1, A_2, \cdots A_r) \subseteq D_{Prod(\Gamma)}$ is $\tau-$combinable into B, then $B \in D_{Prod(\Gamma)}$.

Initially, the assembly set $\mathcal{T}$ consist only of stable assemblies. We further restrict $\mathcal{T}$ to consist of singleton tile assemblies and thus refer to $\mathcal{T}$ as the tile set of the system and we refer to $|T|$ as the tile complexity of Γ. In the case of $h = 2$, we have the standard two handed Assembly model in which assembly proceeds by repeatedly combining any pair of combinable assemblies. A system Γ is said to finitely assemble an infinite shape $X \subset \mathbb{Z}^2$ if every finite producible assembly of Γ has a possible way of growing into an assembly that places tiles exactly on those points in X.

4. Strict Self-Assembly of Diamonds

In this section, we present two constructions for the strict self assembly of Diamond tiles to produce the non-tree discrete Diamond Sierpinski Triangle. We first construct a $6H - DAM$ (Six Handed Diamond Assembly Model) that strictly assembles the Diamond tiles and produce the non-tree Sierpinski triangle at optimal scale 1 and 34 distinct diamond tiles. Further, this construction achieves near perfect assembly. Next, we construct a $3H - DAM$ using only three hands and works at scale 3.

4.1. Discrete Diamond Sierpinski Triangle

The discrete Diamond Sierpinski Triangle can be defined as the set $D_\infty = \bigcup_{i=0}^{\infty} D_i$ (Fig.2) where $D_0 = \{(0,0), (-1,0), (0,1), (-1,1)\}$, $D_{i+1} = D_i \cup (D_i + 2^i V)$ and $V = \{(-2,0), (0,2)\}$. D_∞^c and D_i^c are factor c scalings of D_∞ and D_i.

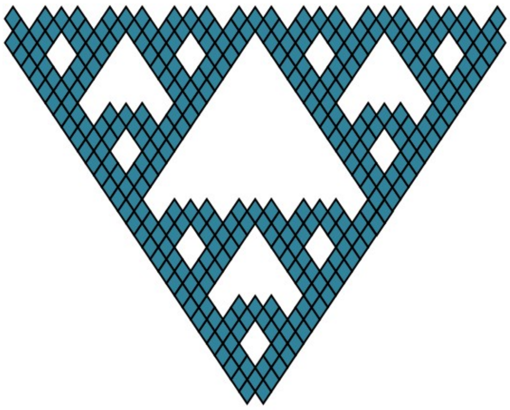


Fig. 2. Scale 1 Sierpinski triangle.(D_i^1).

4.2. Near Perfect Assembly of Diamonds

An infinite fractal pattern $P = \bigcup_{i=0}^{\infty} P_i$, where sets P_{i+1} are obtained by a function $f(P_i)$, is said to be near perfectly assembled by an $hH - DAM$ $\Gamma = (T, \tau, h)$ of Diamond tiles if the system assembles P and if :

1. There exists a c such that for all i, there exists an assembly of Diamond tiles $A \in D_{Prod(\Gamma)}$ of shape D_A such that $\mid P_i \setminus D_A \mid \leq c$. In other words, for every P_i, there exists at least one producible assembly of Diamond tiles $A \in D_{Prod(\Gamma)}$ whose shape is a subset of P_i smaller by at most a constant number of points independent from i.
2. For any producible assembly of Diamond tiles $A \in D_{Prod(\Gamma)}$, the shape of some translation of A is a subset of some P_i, smaller by at most a constant number of points independent from i.

4.3. $6H - DAM$ of Diamond Sierpinski Triangle

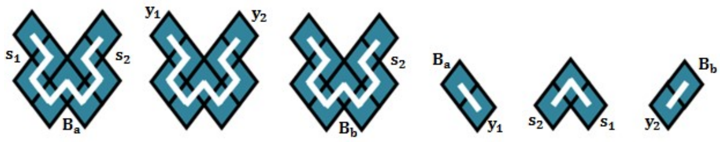
We provide a $6H{-}DAM$ of Diamond tiles that assembles the discrete $Diamond\ Sierpinski\ Triangle$ at scale 1 using 34 distinct diamond tiles. The system also assembles the $Diamond\ Sierpinski\ Triangle$ in an idealized manner that we call *near perfect assembly*. We say that a system near perfectly assembles an infinite fractal if it will produce assemblies that differ from some finite stage of the fractal only by a constant number of points. In other words, if we take any producible assembly in the system, we will find one stage of the fractal that is almost equal to the shape of the chosen assembly with only a constant number of points missing. The inverse also will be true; if we pick any stage of the fractal, we will find one or more assemblies that are almost equal to that stage.

4.3.1. Theorem

There exists a $6H - DAM$ of Diamond tiles that near perfectly assembles the discrete Diamond Sierpinski Triangle at scale factor 1 and has a tile complexity of 34.

Proof: We first prove that condition 1 is satisfied by presenting the diamond tiles in the tile set and describing how they will combine to produce assemblies with a shape that is 3 points away from an D_i, for all D_i. We then present that condition 2 is also satisfied by doing induction on the size (number of tiles) of the producible assemblies and proving that the shape of all producible assemblies of size greater than 34 must be a subset of a stage of the Diamond Sierpinski Triangle, smaller by only 3 points.

Consider a $6H - DAM$ $\Gamma = (T, \tau, 6)$ with tile types T as shown in Fig. 3 and $\tau = 2$. The tile set T (made up of 34 Diamond tiles) is designed to meet both conditions required for near perfect assembly of the discrete diamond Sierpinski triangle. The near perfect assembly feature of the system is achieved by taking advantage of the six hands and the temperature to prevent any other wrong assemblies from combining. Each of the large producible assemblies is made up of six pieces, which must be assembled before they combine into the next stage D_i of the Sierpinski triangle. White lines show a necessary sequence in which the pieces must be constructed in order to achieve near perfect assembly captured by a single stroke path with single protrusions in the path of helper tile β.



(a) The "base" assemblies b1, b2 and b3. (b) The "helper" assemblies α, β and γ.

Fig. 3. Diamond Tile set T in $6H - DAM$.

Condition 1

To satisfy condition 1 of near perfect assembly of diamond tiles, all stages D_i of the discrete diamond Sierpinski triangle need to have at least one assembly with a shape that is almost, by a constant number of points, equal to that stage. Consider the first stage D_0 of the triangle. An assembly of either b_1 or b_2 or b_3 with the left and bottommost four diamond tiles will form the (2×2) diamond tile shape exactly, with no points of difference (Fig. 4). The next stage D_1 consists of three copies of the D_0 shape, one staying in the original's position, one translated towards north west direction by 2 points and another one translated towards north east direction by 2 points. The shape of b_1 or b_2 or b_3 is the closest to D_1 and it is smaller than this stage by 3 tiles (It is missing the one leftmost, bottommost and rightmost tile) as shown in Fig. 4.

D_2 follows the same pattern, consisting of three copies of D_1 translated by the specified amount. The assembly with the biggest subset of D_2 will be composed of the three base assemblies, glued together by the set of α, β and γ helper assemblies shown in Fig.3(b). These six assemblies will need to form a loop (Fig. 4) joining them all together such that the strength of its interactions with its neighbors exceeds a threshold set by the temperature 2 or the assembly will not be $\tau-$stable. The resulting assembly's shape is a subset of D_2 smaller by 3 points captured by a single stroke path with single protrusions in the path of helper tile β.

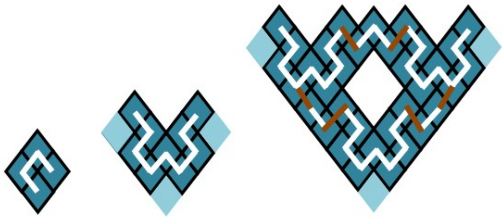


Fig. 4. Assemblies D_0, D_1 and D_2.

This process is continued for all of the following stages to meet Condition 1 in the same way: three copies of the previous assembly are used to satisfy the previous stage, all joined together by a new set of α, β and γ assemblies. It is shown in Fig.5 for a small part of the system, but it will be repeated for any D_i . The floating pieces are separated to easily distinguish between the six different assemblies that combined to form a new iteration from the previous one. These specific assemblies are smaller than an D_i for all D_i by 3 diamond tiles, and therefore Γ satisfies condition 1 of near perfect assembly of diamond tiles of the Sierpinski triangle.

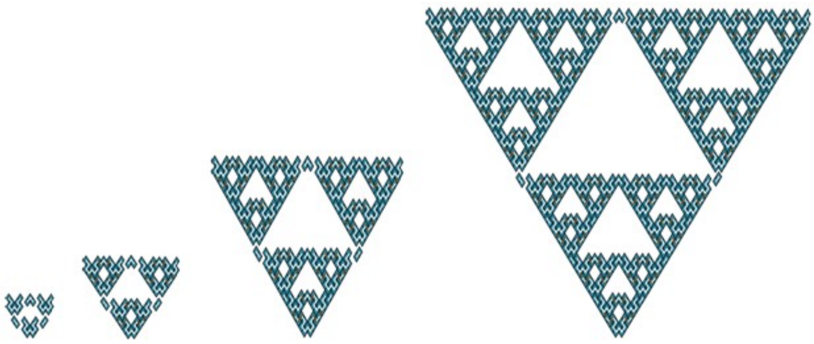


Fig. 5. Stages of growth for $6H - DAM$.

Condition 2

To see how Γ satisfies condition 2, we first define near-triangle assemblies. Near-triangle assemblies are a class of assemblies whose shape is a subset of some stage of the Diamond Sierpinski Triangle D_i with the same 3 corner points missing (the one leftmost, rightmost and bottommost point). Near-triangle assemblies must also have the same set of exposed glues at the same relative points. One example of a near-triangle assembly of diamond tiles is the third assembly in Fig.4 that is close to stage D_2. The shape of this assembly is missing the 3 corner points. Further, tiles at points adjacent to these missing points expose glues that allow only helper tiles to attach. We use notation A_i to denote the near-triangle assembly of diamond tiles with shape close to D_i . Note that all near-triangle assemblies with shape close to D_i are of size $4 \times 3^i - 3 + (1)$.

Condition 2 is proved by induction on the size of the assemblies, that all producible assemblies of size greater than 34 must be near-triangle assemblies. It is clear that assemblies smaller than 33 are base or helper assemblies, which can only combine to form near-triangle assembly A_2 . Consider the near-triangle assembly shown in Fig. 4 (third assembly) of size $33 + (1)$ as the base case. Since the base case (A_2) is a near-triangle assembly we assume all producible assemblies of size greater than $33 + (1)$ up to assembly A_n of size $4 \times 3^n - 3 + (1)$, are near-triangle assemblies of some D_i for $i \leq n$. Assembly A_n is a near-triangle assembly, which means that the 3 corner points in D_n are missing from A_n and it also exposes the same glues as all other near-triangle assemblies.

Only helper tiles can be attached to the exposed glues on near-triangle assemblies, and helper tiles are attached only to near-triangle assemblies or base pieces, so the assembly of size $4 \times 3^n - 3 + (1)$, (or any bigger producible assembly) has to follow the same process as the one shown in Fig. 5 , where three equal-sized near triangle assemblies must be combined with the three helper assemblies otherwise it is not producible. The process must be followed because if the three near triangle assemblies are not the same size, the distance between at least two assemblies is not filled and the loop described before is not closed as shown in Fig. 6 resulting an assembly that is not $\tau-$stable. Any other assembly of size $4 \times 3^n - 3 + (1)$, is not producible since the same type of attachments must happen. In fact, we can be sure that A_n can only grow into A_{n+1} which is a near-triangle assembly of size $4 \times 3^{n+1} - 3 + (1)$. Otherwise A_n cannot grow into an assembly that is not A_{n+1}, so Γ meets condition 2 of near perfect assembly.

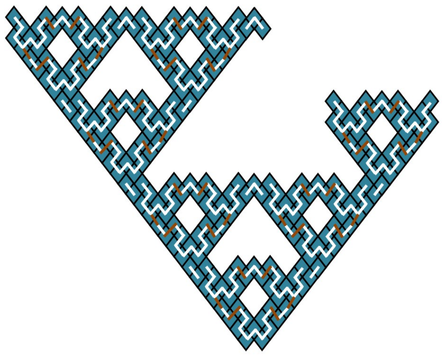


Fig. 6. Unstable Near triangle assemblies .

Result We have attained the result as the shape of any producible assembly smaller than size 34 is smaller than an D_i by at most 3 points and the shape of anything larger than 34 is smaller than an D_i by 3 points. These are the only producible assemblies satisfying both the conditions. We conclude that Γ near perfectly assembles the discrete Diamond Sierpinski Triangle.

4.4. $3H - DAM$ of Diamond Sierpinski Triangle

We present here a $3H - DAM$ of Diamond Tiles that strictly self-assembles the discrete Diamond Sierpinski Triangle at scale factor 3. The system does not near perfectly assemble the Sierpinski triangle, since it relies on many filler assemblies that increase in number as other assemblies grow.

4.4.1. Theorem

There exists a $3H - DAM$ $\Gamma = (T, \tau, 3)$ that self-assembles D^3_∞ with $|T| = 954$.

Proof: First, we describe the components needed and the formation to assemble the model.

Base Shape: A base shape of D_i^3 is an assembly of diamond tiles with shape D_i^3 that is missing tiles along the outer edges and the corners of the triangle as shown in Fig. 7(a).

Combinable shape: A combinable shape of D_i^3 is an assembly of diamond tiles with shape D_i^3 that is missing tiles along two outer edges of the triangle and corner tiles where these edges meet as shown in Fig. 7(b).

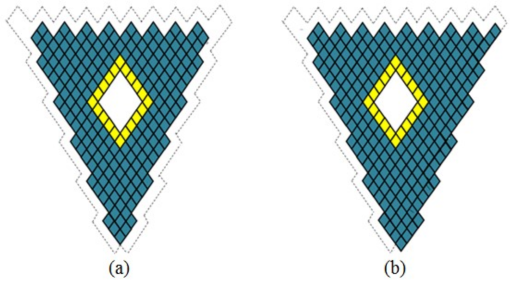


Fig. 7. (a) base shape and (b) combinable shape.

Formation of Base Pieces: The process of the base assembly begins by assembling the three pieces of assemblies shown in Fig. 8. These are the combinable shapes of D_1^3. There are three sets in total of these combinable shapes of D_1^3, two of which are not pictured here. The yellow combinable shape of D_1^3 is the left triangle in the next iteration. The white path traces the process of these assemblies, where tiles connected by the path are connected by unique $strength-2$ glues. This pattern ensures that each combinable shape of D_1^3 is completely assembled before either can assemble into the base shape of D_2^3. Once these combinable shapes of D_1^3 have been assembled completely, they can be attached to their two corresponding combinable shapes of D_1^3.

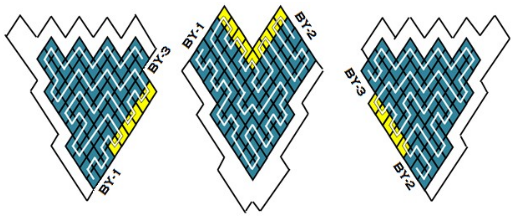
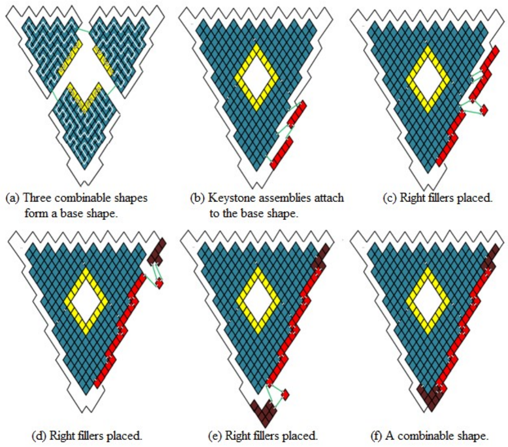


Fig. 8. Yellow combinable shapes of D_1^3.

Each combinable shape of D_1^3 has two strength-1 glues that are used to construct a base shape of D_2^3 using cooperative binding. Each $strength-1$ glue attaches to one triangle each, forming a Γ-stable assembly, as shown in Fig. 9(a). The other two sets of combinable shapes of D_1^3 not showed here exhibit the same behavior. These three different base shapes of D_2^3 will be described as yellow, blue and red. Fig. 8 presents the yellow combinable shapes of D_1^3 that make up the base shape of D_2^3.



(a) Three combinable shapes form a base shape. (b) Keystone assemblies attach to the base shape. (c) Right fillers placed. (d) Right fillers placed. (e) Right fillers placed. (f) A combinable shape.

Fig. 9. The process of keystone attachment and edge growth.

Keystone Attachments: Keystone tiles are assemblies that cooperatively attach to their corresponding exposed glues, in a nondeterministic manner. For each base shape of D_1^3, one of three keystones will attach, followed by the filling of an edge of the assembly. This method of assembly is used to produce three different combinable shapes of D_1^3. The assembled base shapes of D_2^3, described in the previous paragraph, have two exposed glues on a single edge which are sites of keystone tile attachment. Each assembled base shape of D_2^3 has three keystone tiles that can attach at their exposed edges. We will describe these keystone tiles by the colors yellow, blue and red. These three keystone tiles are each split into two sets, so that they must be assembled using three hands (two for the keystone tiles and one for the base shape of D_2^3 to be properly assembled, as shown in Fig. 9(b). Each of the three different assembled base shapes of D_2^3 expose keystone glues on differing edges; a yellow base shape will attach their keystone tiles on the right edge, a blue base shape on the left edge and a red base shape on the top edge.

Connector Glues: When the keystone tiles have been attached to a base shape of D_2^3, they will place tiles along the edge of the assembly, matching the color of the keystone tiles that have attached. This process is shown in Fig. 9(b)-9(f). Whitespace is the area of the assembly that is part of the shape D_i^3 but is not covered by tiles. Each attachment of these tiles that fill the whitespace along the edge takes three hands to assemble, one for the base shape of D_2^3 (along with the already attached keystone and edge filling tiles) and two for the tiles that fill in the whitespace, which consists of a corner and the fitting assembly. The new combinable shape of D_2^3 will have exposed glues that will provide a method of attaching to the corresponding combinable shapes of D_2^3. This process is shown for a yellow base shape of D_2^3 in 9; in the case of the other base shape of D_2^3, blue base shape looks like a mirror of the yellow base shape and having the keystone attach by filling tiles along the left edge and a red base shape consists of having the keystone attach atop the assembly to grow filling tiles along the top edge. The produced combinable shapes of D_2^3 have exposed strength-1 glues that will allow them to attach to two other combinable shapes of D_2^3, forming a base shape of D_2^3. Three combinable shapes of D_2^3 will only attach to one another if they have attached the same type of keystone tile, as shown in Fig. 10.

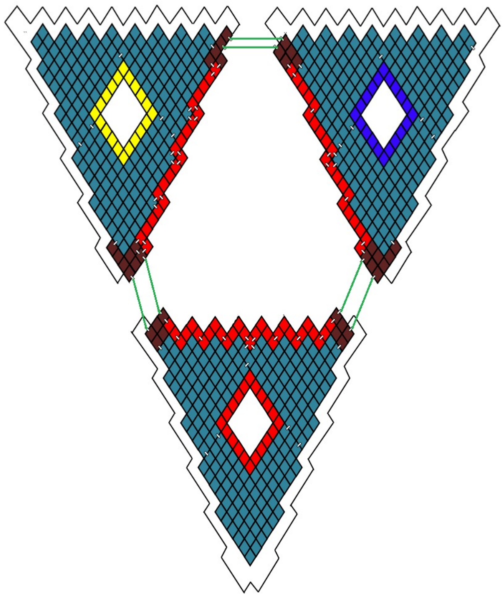


Fig. 10. Different combinable shapes to the next iteration.

Infinite Assembly: A base shape of D_i^3 will form a combinable shape of D_i^3, and three combinable shapes of D_i^3 will form a base shape of D_{i+1}^3 in order to self- assemble the infinite diamond Sierpinski triangle. If a base shape of D_i^3 was designated yellow (using keystone assemblies in stage D_{i-1}^3), the tiles will be placed along its right edge; if blue, it will be placed along its left edge; if red, it will be placed along its top edge. The filling tiles are designed to fill in arbitrarily long edges using a constant tile set. The three-resulting red, blue and yellow combinable shapes of D_i^3 will be attached to form a base shape of D_{i+1}^3 if they have chosen the same keystone. The nondeterministic attachment of keystones allows for three combinable shapes of D_i^3 to produce three different base shapes of D_{i+1}^3, which will become complementary combinable shapes of D_{i+1}^3. These three combinable shapes of D_{i+1}^3 can produce base shapes of D_{i+2}^3. The nondeterministic keystone attachment and arbitrary-length filling tiles allow us to self-assemble the discrete Diamond Sierpinski Triangle using a 3-handed system at scale factor 3 with 954 tile types; 624 tiles for the base pieces and 330 tiles for the keystone assemblies and filler tiles.

5. Conclusion

We have focussed our problem of multihanded self assembly of diamond tiles on two discrete fractals in algorithmic self-assembly theory. Pure growth models must decide how to build a shape through a single mechanism, the placement of tiles, which inherently uses up geometric space. We have presented two models that strictly self assemble the non tree Sierpinsky triangle. The $6-$handed model uses 34 diamond tile types, works at scale factor 1 and achieves near perfect assembly. The 3 handed model works with scale factor 3 and uses 954 tile types; 624 tiles for the base pieces and 330 tiles for the keystone assemblies and filler tiles to construct the

Sierpinsky triangle. We note that D^c_∞ cannot be strictly assembled by any $aDAM$ system which will be considered as a future work.

References

[1] E. WINFREE, *Algorithmic Self-assembly of DNA*, Ph.D. thesis, California Institute of Technology, 1998.

[2] S. CANNON, E. D. DEMAINE, M. L. DEMAINE, S. EISENSTAT, M. J. PATITZ, R. T. SCHWELLER, S. M. SUMMERS and A. WINSLOW, *Two hands are better than one (up to constant factors): Self-assembly in the 2ham vs. atam,* International Symposium on Theoretical Aspects of Computer Science, Kiel, Germany, 2013, pp. 172–184.

[3] P. W. K. ROTHEMUND, N. PAPADAKIS, and E. WINFREE, *Algorithmic self-assembly of DNA Sierpinski triangles*, PLoS Biology , **2**(12), Dec. 2004.

[4] J. I. Lathrop, J.H. Lutz and S.M. Summers, *Strict self-assembly of discrete Sierpinski triangles*, Theoretical Computer Science, **410**, 4-5, pp. 384–405, April 2009.

[5] Matthew J. PATITZ and Scott M. SUMMERS, *Self-assembly of discrete self-similar fractals*, Natural Computing, **9**(1), 135–172, March 2010.

[6] C. T. CHALK, D. A. FERNANDEZ, A. HUERTA, Mario A. MALDONADO, R. T. SCHWELLER and L. SWEET, *Strict Self-Assembly of Fractals using Multiple Hands*, Algorithmica, **76**(1), pp. 195–224, Sept. 2016.

[7] K. BHUVANESWARI and T. KALYANI, *Triangular tile pasting P system for pattern generation*, International Conference on Applied Mathematics and Theoretical Computer Science, Tamilnad, India, 2013, pp. 26–30.

[8] F. Keerthika ROSY, *An Innovative Study on Diamond Tile Pasting system,* M. Phil. Dissertation, JAC, 2017.