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SIMPLE H ISO-ARRAY SPLICING SYSTEMS

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Abstract

Tom Head [2] introduced the operation of splicing on strings motivated by certain recombinant behavior of DNA molecules. An effective extension of this operation to images was introduced by Helen Chandra et al [3] and H array splicing systems were considered. H iso-array splicing system was introduced by V. Masilamani [6]. Here we introduce a special class of these H iso-array splicing systems allowing only simple splicing rules motivated by the study of Helen Chandra [5] and Mateescu et al [4] and obtain some properties of this case.

Keywords: *DNA computing, Iso-Array, Splicing Operation, Iso-Picture Languages.*

1. Introduction

In syntactic approaches to generation and recognition of images, considered as digitized arrays, several two-dimensional grammars have been proposed and studied. Giammarresi and Restivo [1] have introduced the concept of recognizable rectangular picture languages using rectangular tiles. Motivated by these studies, a new notion of recognizability has been introduced for a class of picture languages called iso-picture languages through iso-triangular tiling systems (ITS) and studied the properties of these languages [7]. One application of the study of iso-picture languages is useful in the generation of interesting kolam patterns. Another application of this study lies in the area of tiling rectangular plane. On the other hand there has been a lot of interest in the study of formal language theory applied to DNA computing. A specific model of DNA recombination is the splicing operation which consists 'cutting' DNA sequences and then 'pasting' the fragments again, under the action of restriction enzymes and ligases. Tom Head defined splicing systems motivated by this behavior of DNA sequences. A new method of splicing on images of rectangular arrays is introduced in [3] as a simple and effective extension of the operation of splicing on strings. Using this notion, iso-picture splicing system has been introduced in [6] to generate iso-pictures.

In this paper, we introduce a new method of applying the simple splicing operation on images of iso-arrays. Simple splicing rule that involve tiles of isosceles right angled triangles are considered. Two-arrays are spliced in four directions. The resulting model called simple iso-array splicing system (SIASS) is simple to handle and generates a new class of iso-array languages called family of simple H iso-array languages (FSHIA). Some closure properties of FSHIA under language theoretic operations union and concatenation, geometric operation reflections about the base lines and rotation by 90°, 180° and 270° are considered.

2. Preliminaries

We recall some of the basic definitions of iso-picture languages and iso-array splicing systems to generate the language L discussed in [6] and [7].

Definition 2.1

Let Σ be an alphabet, # and $ are two special symbols not in Σ. An iso-array over Σ is an isosceles triangular arrangement of tiles , , , and .

A horizontal splicing rule over Σ is of the form $\alpha_1 \# \alpha_2 S \alpha_3 \# \alpha_4$ where $\alpha_1 = U_m\ \lambda$, $\alpha_2 = D_m$ or λ and $\alpha_3 = U_m$ or λ, $\alpha_4 = D_m$ or λ. The set of all horizontal splicing rules is denoted by R.

Similarly, The set of all vertical, right and left splicing rules are denoted by R, R and R.

Definition 2.2

A simple column splicing rule over V is of the form,

$$r = \begin{array}{|c|}\hline a \\ \hline b \\ \hline \end{array} \quad \# \quad \begin{array}{|c|}\hline 1 \\ \hline 1 \\ \hline \end{array} \quad \$ \quad \begin{array}{|c|}\hline a \\ \hline b \\ \hline \end{array} \quad \# \quad \begin{array}{|c|}\hline 1 \\ \hline 1 \\ \hline \end{array}$$

for some $a, b \in V \cup \{\#\}$

A simple row splicing rule over V is of the form,

r =	a	b	#	1	1	$	a	b	#	1	1

for some $a,b \in V \cup \{\#\}$

Definition 2.3

A simple H array scheme is a triplet $\Gamma = (V, R_C, R_r)$ where V is an alphabet

Rc = a finite set of simple column splicing rules and R_r = a finite set of simple row splicing rules. For a given simple H array scheme $\Gamma = (V, R_C, R_r)$ and a language $L \subseteq V^{8}$.

We define, $\Gamma(L) = \left\{ Z \in V^{(*)} \middle| (X^{\#}, Y^{\#}) \middle| - Z^{\#} \text{ for some } X, Y \in L \right\}$

In other words, $\Gamma(L)$ consists of arrays obtained by column or row splicing any two arrays of L using the simple column or row splicing rules. A simple H array splicing system (SHAS) is defined by $S = (\Gamma, I)$ where $\Gamma = (V, R_C, R_r)$ and I is a finite subset of V^{8*}. The language of S is defined by $L(S) = \Gamma^{*}(I)$ and we call it a simple H array splicing language (SHASL) and denote the class of such languages by L (SHASL).

3. Simple H iso–array splicing system

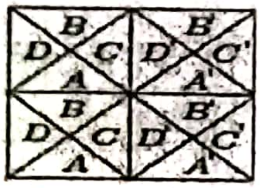
We define simple splicing rules and the main notion of simple H iso–array splicing system.

Definition 3.1

A simple horizontal splicing rule over Σ is of the form $\alpha \# \alpha_1 S \alpha \# \alpha_2$ where $\alpha = U_m$ or λ, $\alpha_1 = D_O$ and $\alpha_2 = D'_O$. The set of all simple horizontal splicing rule is denoted by R_{O_e}.

Example

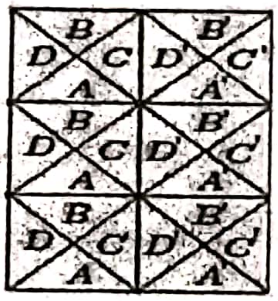
Let $\Sigma =$  and the axiom

A = 

Let $x_1 = A$ and $x_2 = A$ and the simple splicing rules are

$R_{O_s} = \{A\#1S\,A\#1,\ A'\#S\,A'\#1\}$

Then $(x_1, x_2)\, x_3$ where a member generated by the system is given by

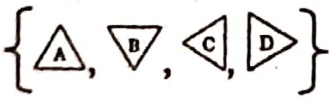
$x_3 =$ 

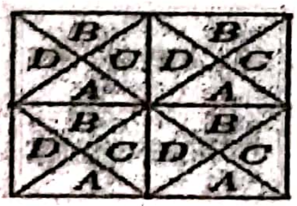
Definition 3.2

A simple vertical splicing rule over Σ is of the form $\alpha\ \#\ \alpha_1 S\alpha\#\alpha_2$ where $\alpha = L_m$ or λ,

$\alpha_1 = R_O$ and $\alpha_2 = R_O'$. The set of all simple vertical splicing rule is denoted by R_{O_s}.

Example

Let $\Sigma =$ 

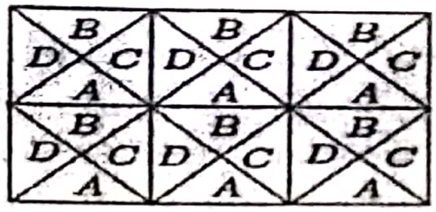
$A_0 =$ 

Let $p_1 = A_O$ and $p_2 = A_O$ and the simple splicing rules are

$R_{O_s} = \{C\#1SC\#1\}$

Then $(p_1, p_2) \perp p_3$ where a member generated by the system is given by

$p_3 =$



Defintion 3.3

A simple right splicing rule over $\sum$ is of the form $\alpha\#\alpha_1 S\alpha\#\alpha_2$ where

(i) $\alpha = D_m$ or λ, $\alpha_1 = U_O$ and $\alpha_2 = U'_O$ (or)

(ii) $\alpha = R_m$ or λ, $\alpha_1 = U_O$ and $\alpha_2 = U'_O$ (or)

(iii) $\alpha = D_m$ or λ, $\alpha_1 = L_O$ and $\alpha_2 = L'_O$ (or)

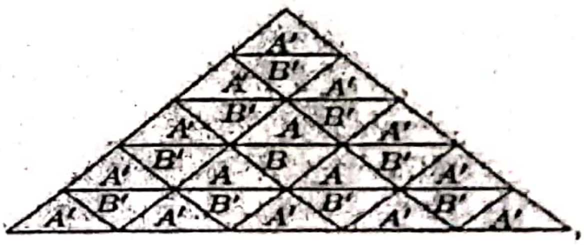
(iv) $\alpha = R_m$ or λ, $\alpha_1 = L_O$ and $\alpha_2 = L'_O$

The set of all right splicing rules is denoted by R_{O_4}.

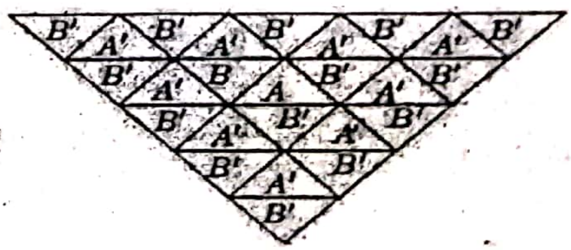
Example

Let $\sum =$  and the axiom

$d_1 =$

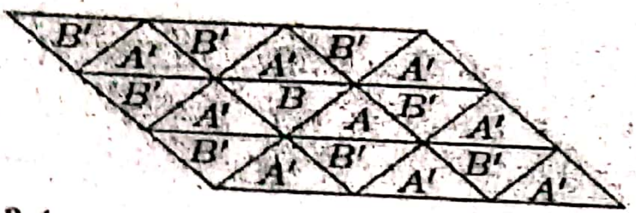


$d_2 =$



$R_{O_4} = \{B'\#1"S"B'\#1,\ B\#1S\ B\#1\}$

$(d_2, d_1) \perp d_3$,

$d_3 =$ 

Definition 3.4

The simple left splicing rule over Σ is of the form $\alpha\#\alpha_1 S\alpha\#\alpha_2$ where

(i) $\alpha = U_m$ or λ, $\alpha_1 = D_O$, $\alpha_2 = D'_O$ (or)

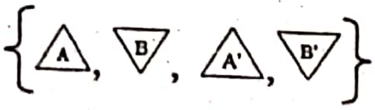
(ii) $\alpha = U_m$ or λ, $\alpha_1 = L_O$, $\alpha_2 = L'_O$ (or)

(iii) $\alpha = L_m$ or λ, $\alpha_1 = R_O$, $\alpha_2 = R'_O$ (or)

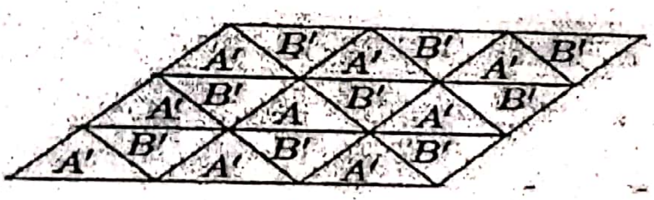
(iv) $\alpha = R_m$ or λ, $\alpha_1 = D_O$, $\alpha_2 = D'_O$.

The set of all simple left splicing rules is denoted by $R_{O_{\cdot}}$.

Example

Let $\Sigma =$  and the above axiom $\{d_1, d_2\}$. The splicing rule is $R_{O_{\cdot}} = \{A'\#1"S"\ A'\#1,\ A\#1S\ A\#1\}$

$(d_1, d_2) \perp d_4$,

$d_4 =$ 

Definition 3.5

A simple H iso-array scheme is a tuple $\sigma = (\Sigma, R_{O_{\cdot}}, R_{O_{\cdot}}, R_{O_{\cdot}}, R_{O_{\cdot}})$ where Σ is an alphabet, $R_{O_{\cdot}}$ is a finite set of simple horizontal splicing rules.

Similarly $R_{O_{\cdot}}, R_{O_{\cdot}}, R_{O_{\cdot}}$ are finite sets of simple vertical, left and right splicing rules.

A simple H iso-array splicing system is defined by $S = (\sigma, N)$ where N is a finite subset of $\sum_{I}^{**}$.

We define $\sigma(N)=\{Z \in \Sigma_I^{(**)} / (X^*, Y^*) \perp Z^* \text{ for some } X, Y \in N\}$

In other words, $\sigma(N)$ consists of iso-arrays obtained by column or row splicing any two iso-arrays of L using the simple column or row splicing rules. A simple H iso-array splicing system (SHIAS) is defined by $S=(\sigma, N)$ where $\sigma=(\Sigma, R_{O_\cdot}, R_{O_\cdot}, R_{O_\cdot}, R_{O_\cdot})$ and N is a finite subset of $\sum_I^{**}$. The language of S is defined by $L(S)=\sigma^*(N)$ and we call it a simple H iso-array splicing language (SHIASL) and denote the class of such languages by L (SHIASL).

Definition 3.6

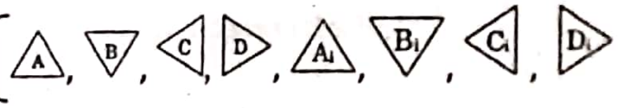
The family of iso-picture languages generated by these simple splicing systems is denoted by FSHIA.

Theorem: 3.7

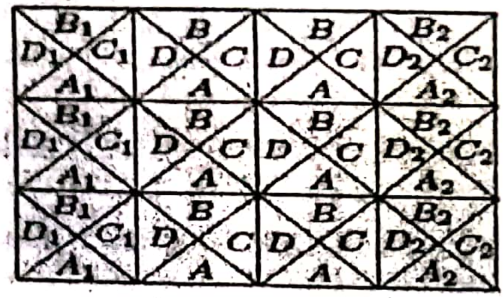
(i) The class FSHIA is not closed under union.

(ii) FSHIA is not closed under horizontal and vertical concatenation.

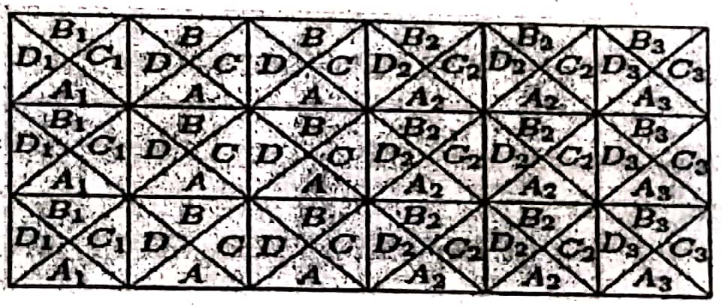
Proof(i)

Let $\Sigma = \{$  $\overline{1} \le i \le 3\}$

L_1 = The set of all iso-picture of rectangles with three rows and at least 4 columns, where each square is any of the form , ,  and all the squares in the first column should have , the last column should have  and the intermediate column should have .

I = 

L_2 = The set of all iso-picture of rectangles over $\sum$ with 3 rows and at least 6 columns where each square is of the form , , ,  with tiles in the first column as , the last column with  and the remaining columns of rectangular array of size (m,n) except the second and third column is denoted by 

J = 

The following simple splicing system will generate the language L_1

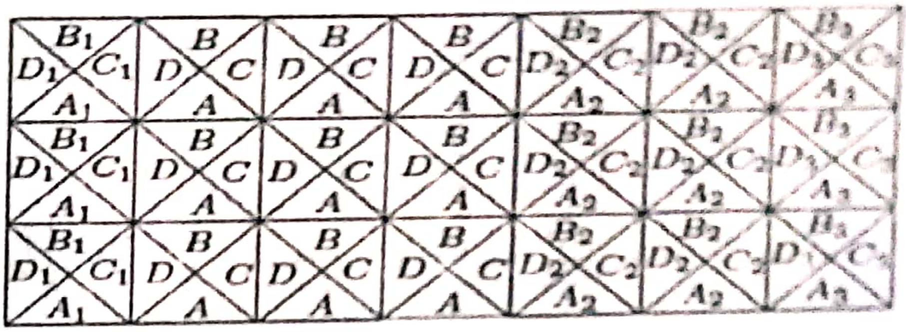
Let $S = (\sigma, N)$ where $\sigma = \left(\Sigma, R_{O_{\cdot}}, R_{O_{\cdot}}, R_{O_{\cdot}}, R_{O_{\cdot}}\right)$ such that

$R_{O_{\cdot}} = R_{O_{\cdot}} = R_{O_{\cdot}} = \phi$ and $R_{O_{\cdot}} = \{C\#1S\ C\#1\}$ and $N = \{I\}$

Similarly for L_2, the simple splicing system will have rules

$R_{O_{\cdot}} = \{C_2\#1S\ C\ 2\#1\}$ and $N = \{J\}$

Since any simple splicing that produces the images in L_1 and L_2 will produce an image u which is not in $L_1 \cup L_2$.

u= 

Hence FSHIA is not closed under union.

Similarly, we can prove that (ii) and hence FSHIA is not closed under horizontal and vertical concatenation.

Theorem 3.8

The class of FSHIA is closed under rotations by 90°, 180° and 270°.

Proof

It is known that the class of FHIA is closed under rotations by 90°, 180° and 270° [6]. Similar proof can be given to FSHIA also.

4. Conclusion

In this paper, we have introduced a new model called simple H iso-array splicing system. Some closure properties such as union, concatenation and rotation of FSHIA are studied. The study initiated in this paper might prove useful to analyze better the structure of images.

5. References

[1] D. Giammarresi and A. Restivo (1997), Two Dimensional languages, In Handbook of Formal Languages, G. Rozenberg and A. Salomaa eds, Vol. 3, pp. 215-267, Springer verlog, 1997.

[2] T. Head (1987), Formal Language Theory and DNA: An analysis of the generative capacity of specific recombinant behaviours, Math. Biology, 49, 737-759.

[3] P. Helen Chandra, K.G. Subramanian and D.G. Thomas, (2004),Parallel Splicing on Images, International Journal of patter Recognition and Artificial Intelligence 18, 6, 1071-1091.

[4] M.Mateescu, Gh. Paun, G. Rozenberg and A. Salomaa, (1998) Simple Splicing Systems, Discrete Applied Math., 84, 145-163.

[5] P. Helen Chandra, K.G. Subramanian and A. Roslin Sagaya Marry, (2005), Splicing on Images, Computational Mathematics, 20, pp. 126-131.

[6] V. Masilamani, D.K. Sheena Christy, D.G. Thomas, T. Kalyani, (2010): Parallel Splicing On Iso-Arrays, Proceedings of Fifth IEEE International Conference on Bio-Inspired Computing Theories and Applications Vol. 2, 1535-1542.

[7] T. Kalyani, V.R. Dare and D.G. Thomas, (2004), Local and Recognizable Iso-Picture Languages, Lecture Notes in Computer Science, 3316, 738-743.