Fuzzy System Reliability Analysis using Trapezoidal Fuzzy Numbers based on Statistical Data

#1B.Ambika, #2G. Michael Rosario

#Research Centre and PG Department of Mathematics
Jayaraj Annapackiam College for Women(Autonomous)
Periyakulam, Tamilnadu, India

ambikakannan@ gmail.com, tony87rio@gmail.com

Abstract--- **In the paper the fuzzy concept is used to find the reliability of serial system and the parallel system. Since the reliability R_j of the subsystem is unknown, using statistical data point estimate R_j is used instead of $\bar{R}_j$. The reliability of the system may fluctuate around the point estimate R_j during a time interval. It follows that using the point estimate of the population reliability R_j is not suitable for the real cases. Therefore, it is more desirable to use the statistical confidence interval. We transfer the statistical confidence interval as the trapezoidal fuzzy number. We fuzzify the reliability of the serial and parallel systems and defuzzifying the fuzzy reliability using the signed distance method. Finally we get a fuzzy estimate reliability of serial and parallel systems.**

Keyword---**Fuzzy Reliability, Statistical Data, Signed Distance,Trapezoidal Fuzzy Number**

I. Introduction

Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes . Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences and the natural sciences. The reliable engineering is one of the important engineering tasks in design and development of technical system. The conventional reliability of a system is defined as the probability that the system performs its assigned function properly during a predefined period under the condition that the system behavior can be fully characterized in the context of probability measures. The reliability of a system can be determined on the basis of tests or the acquisition of operational data. However, due to the uncertainty and inaccuracy of this data, the estimation of precise values of probabilities is very difficult in many systems (e.g. power system, electrical machine, hardware etc., Hammer (2001)[6], El-Hawary (2000)[7]). For this reason the fuzzy reliability concept has been introduced and formulated in the context of fuzzy measures. The basis for this approach is constituted by the fundamental works on fuzzy set theory of Zadeh (1978)[14], Dubois and Prade (1980)[1], Zimmerman (1986)[15] and other. It is an essential tool for formulating the mathematical information of imprecise nature. The theory of fuzzy reliability was proposed and developed by several authors, Cai, Wen and Zhang (1991, 1993)[2-3]; Cai (1996)[4]]; Chen, Mon (1993)[5]; Hammer (2001)[6]; El-Hawary (2000)[7], Onisawa, Kacprzyk (1995)[10]; Utkin, Gurov (1995)[12]. The recent collection of papers by Onisawa and Kacprzyk (1995)[10], gave many different approach for fuzzy reliability. This study presents a new method to analyze fuzzy system reliability using fuzzy number arithmetic operations based on statistical data.

In conventional reliability theory, there are two fundamental hypotheses. (A) The probability assumption: The system behavior is fully characterized in the context of probability measure. (B) The binary-state assumption: At any given time, the system has only two states. One is the functioning state and another is the failed state. In the referenced papers by Cai, Wen and Zhang [2-3] and Cai [4], the authors modified (B) to (B'). (B') The fuzzy-state assumption: At any given time, the system has only two states. One is the fuzzy success state and another is the fuzzy failure state. Cheng and Mon [5] used the α-cut of a triangular fuzzy number to get the interval and find the fuzzy reliability of the serial system and the parallel system. They used fuzzy numbers to find the fuzzy reliability of the serial system and the parallel system. In Mon and Cheng [9], the authors used the α-cut of fuzzy number to derive a non-linear program of the fuzzy system in the serial and parallel cases.

Singer [11], considered the fuzzy reliability of both serial and parallel systems using an approximation of a fuzzy binary operation $\otimes$ with two L-R type fuzzy numbers. In this study statistical methodology is applied. Using statistical approach the fuzzy system is defuzzified and the reliability of the system is estimated in the fuzzy sense. Section 2 gives some basic concepts and definitions of fuzzy sets. Theses definitions will be used in sections 3 and 4.

In section 3 a statistical point estimate is used to examine the reliability of the serial system and that of the parallel system in the crisp case and the concept of the statistical confidence interval converted to a trapezoidal fuzzy number. Section 4 consists of some numerical examples and section 5 is conclusion.

II. BASIC DEFINITIONS

1) Definition: A fuzzy set is called a level α **fuzzy interval**, where $0 \leq \alpha \leq 1$ and we denote it by [p,q;α], if its membership function is

$$\mu_{[p,q;\alpha]}(x) = \begin{cases} \alpha & if \quad p \leq x \leq q \\ 0 & otherwise \end{cases}$$

2) Definition: $\bar{a}$ is called a **fuzzy point**, if its membership function on $R = (-\infty, \infty)$ is

$$\mu_{\bar{a}}(x) = \begin{cases} 1 & if \quad x = a \\ 0 & if \quad x \neq a \end{cases}$$

The family of all fuzzy sets on R which we denote as F_s satisfies the following two conditions.

Let $\widetilde{D} \in F_s$. Then $\widetilde{D}$ satisfies (1) and (2) below.

i) The left and right hand side of the α - level set of $\widetilde{D}, D_l(\alpha)$ and $D_u(\alpha)$ exist. We denote it as $D(\alpha) = [\, D_l(\alpha),\ D_u(\alpha)]$.
ii) $D_l(\alpha)$ and $D_u(\alpha)$are integrable for $\alpha \in [0,1]$.

3) Definition: The left and right hand side of the α - level set of trapezoidal **fuzzy number** $\widetilde{D} = (a_1, b_1, c_1, d_1)$ are

$$D_l(\alpha) = a_1 + (\, b_1 - a_1)\alpha$$

$$D_u(\alpha) = d_1 - (\, d_1 - c_1)\alpha$$

4) Definition: Let $a, 0 \in R$. We define the signed distance of a from 0 as $d^*(a, 0) = a$.

Now we find the signed distance of $\widetilde{D} \in F_s$. By decomposition theorem, we get $\widetilde{D} = \cup_{0\leq\alpha\leq1} \alpha I_{D(\alpha)}, I_{D(\alpha)}$ is the characteristic function of the α - level set $D(\alpha)$. From Definition 1, we get $I_{D(\alpha)}(x) = \mu_{[\, D_l(\alpha),\ D_u(\alpha);\, \alpha]}, \forall\ x \in R$. Therefore, we have

$$\widetilde{D} = \cup_{0\leq\alpha\leq1} [\, D_l(\alpha),\ D_u(\alpha);\ \alpha].$$

Using Definition 4, for each $\alpha \in [0,1]$, we get the signed distance of $D_l(\alpha)$ and $D_u(\alpha)$ as measured from 0 are $d^*(D_l(\alpha), 0) = D_l(\alpha)$ and $d^*(D_u(\alpha), 0) = D_u(\alpha)$ respectively. Therefore, we may define the signed distance of the interval $[\, D_l(\alpha),\ D_u(\alpha)]$ from 0 by

$$d^*([\, D_l(\alpha), D_u(\alpha)], 0) = \left(\frac{1}{2}\right) [\, D_l(\alpha) +\ D_u(\alpha)].$$

It is clear that for each $\alpha \in [0,1]$, $[D_l(\alpha), D_u(\alpha)]$ and $[D_l(\alpha), D_u(\alpha); \alpha]$ are one-to-one mappings. Similarly, 0 and $\tilde{0}$ are also one-to-one mappings. Therefore, we can define the signed distance from $[D_l(\alpha), D_u(\alpha); \alpha]$ to $\tilde{0}$ as

$$d^*([D_l(\alpha), D_u(\alpha); \alpha], \tilde{0}) = \left(\frac{1}{2}\right)[D_l(\alpha) + D_u(\alpha)].$$

Since $\widetilde{D} \in F_s$, $\int_0^1 D_l(\alpha)\, d\alpha$ and $\int_0^1 D_u(\alpha)\, d\alpha$ both exist.

5) Definition: Let $\widetilde{D} \in F_s$. We define the signed distance of $\widetilde{D}$ measured from $\tilde{0}$ as

$$d(\widetilde{D}, \tilde{0}) = \left(\frac{1}{2}\right)\int_0^1 (D_l(\alpha) + D_u(\alpha))\, d\alpha.$$

Let $\widetilde{D}, \tilde{E} \in F_s$. Their α - level sets are denoted as follows

$$D(\alpha) = [D_l(\alpha), D_u(\alpha)]$$

$$\text{and} \quad E(\alpha) = [E_l(\alpha), E_u(\alpha)] \qquad (1)$$

For each $\alpha \in [0,1]$,

$$[D_l(\alpha), D_u(\alpha); \alpha] \leftrightarrow [D_l(\alpha), D_u(\alpha)]$$

$$\text{and} \quad [E_l(\alpha), E_u(\alpha); \alpha] \leftrightarrow [E_l(\alpha), E_u(\alpha)] \qquad (2)$$

are in one - one correspondence. By decomposition theorem, we have

$$\widetilde{D} = \bigcup_{0 \le \alpha \le 1} [D_l(\alpha), D_u(\alpha); \alpha]$$

$$\text{and} \quad \tilde{E} = \cup_{0 \le \alpha \le 1}[E_l(\alpha), E_u(\alpha); \alpha] \ \tilde{A} \qquad (3)$$

Let $\tilde{A} = (a_1, b_1, c_1, d_1), \tilde{B} = (a_2, b_2, c_2, d_2)$, where $0 < a_1 < b_1 < c_1 < d_1$ and $0 < a_2 < b_2 < c_2 < d_2$, be two trapezoidal fuzzy numbers.

Using definition 3, their left endpoints and right endpoints of the α - cut are

$$A_l(\alpha) = a_1 + (b_1 - a_1)\alpha, A_u(\alpha) = d_1 - (d_1 - c_1)\alpha$$

$$B_l(\alpha) = a_2 + (b_2 - a_2)\alpha, B_u(\alpha) = d_2 - (d_2 - c_2)\alpha$$

respectively, where $0 \le \alpha \le 1$ and $0 \le A_l(\alpha) \le A_u(\alpha), 0 \le B_l(\alpha) \le B_u(\alpha)$

For $\alpha \in [0,1]$, we obtain

$$\tilde{A} \otimes \tilde{B} = \cup_{0 \le \alpha \le 1}[(a_1 + (b_1 - a_1)\alpha)(a_2 + (b_2 - a_2)\alpha)], (d_1 - (d_1 - c_1)\alpha)(d_2 - (d_2 - c_2)\alpha); \alpha] \qquad (4)$$

III. FUZZY SYSTEM RELIABILITY USING TRAPEZOIDAL FUZZY NUMBERS

In this section, we discuss the general serial system and the parallel system using the statistical data.

A. Reliability of Serial System and Reliability of the Parallel System using Point Estimates

Fig.1 represents the subsystems of a serial system, which are $P_1, P_2, \ldots, P_n$. Let us suppose that $R_1, R_2, \ldots, R_n$ $(0 \le R_j \le 1, j = 1,2, \ldots, n)$are the reliability of $P_1, P_2, \ldots, P_n$ respectively, then the reliability of the serial system is

$$\prod_{j=1}^{n} R_j \tag{5}$$

Similarly, when Fig.2 represents the subsystems of a parallel system, which are $P_1, P_2, \ldots, P_n$. Then reliability of parallel system is

$$1 - \prod_{j=1}^{n}(1 - R_j) \tag{6}$$

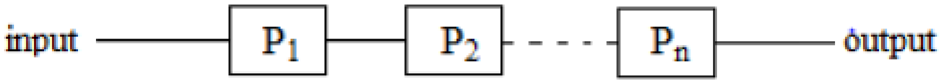

Figure 1: Configuration of serial system.

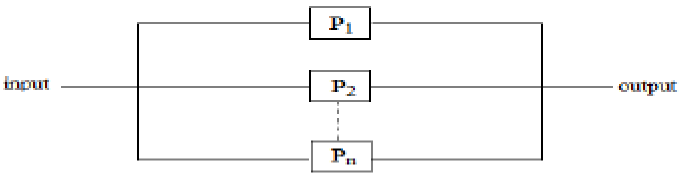

Figure 2: Configuration of parallel system.

Suppose that R_j , $j = 1,2,\ldots,n$ are unknown and for each subsystems $P_j, j = 1,2,,\ldots,n$, we can get n records of reliability of P_j. We use statistical point estimates to examine the reliability of the serial system and that of the parallel system. For this, let us consider that each subsystem P_j is of size n_j $j = 1,2,\ldots,n$ and denote its statistical data by $R_{jq} \in [0,1], q = 1,2,\ldots,n_j$. Let R_j (unknown) be the population reliability of the subsystem P_j. For subsystem P_j using statistical data $R_{jq}, q = 1,2,\ldots,n_j$, we can find their average value.

Let $\bar{R}_j = (1/n_j) \sum_{q=1}^{n_j} R_{jq} \in [0,1]$. From the statistical point of view, we know that $\bar{R}_j$ is a point estimate of R_j . Therefore, we use $\bar{R}_j$ instead of the reliability of P_j.

Results:

Let R_j (unknown), $j = 1,2,\ldots,n$ be the population reliability of the subsystem P_j . Then from the point estimate $\bar{R}_j$ of R_j , we have (1) The reliability of the serial system is

$$\prod_{j=1}^{n} \bar{R}_j \tag{7}$$

(2) Then reliability of parallel system is

$$1 - \prod_{j=1}^{n}(1 - \bar{R}_j) \tag{8}$$

B. Fuzzy Reliability using Trapezoidal Fuzzy Numbers

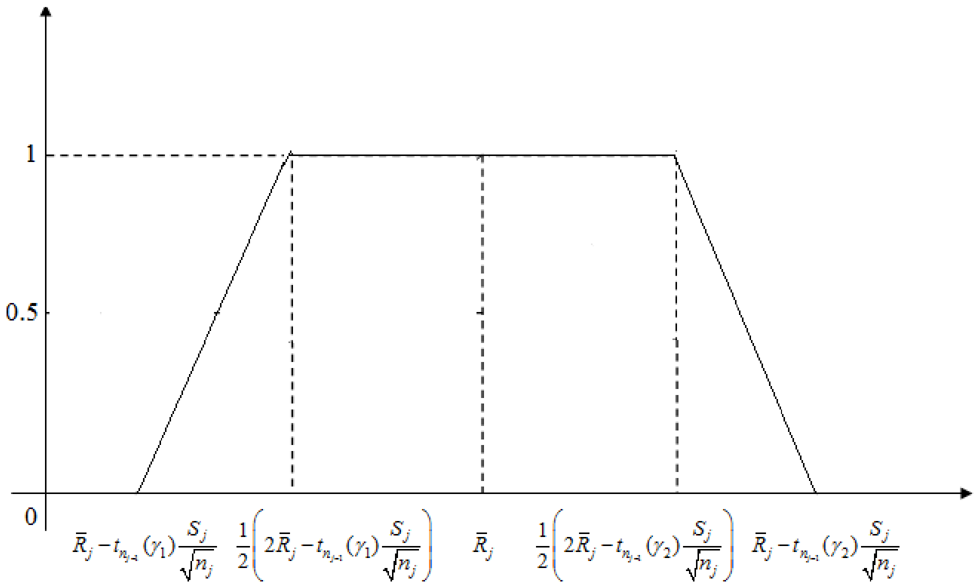

Figure 3: Trapezoidal Fuzzy Number $\bar{R}_j$.

Since the probability distribution of the error between the point estimate $\bar{R}_j$ and R_j is unknown, we use the statistical confidence interval of R_j instead. Let.

$$0 < \gamma_k < 1, k = 1,2, 0 < \gamma < 1, \text{ and } \gamma_1 + \gamma_2 = \gamma. \quad (9)$$

The (1- γ) % confidence interval of R_j for $j = 1,2,\ldots,n$ is

$$[\bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}), \bar{R}_j + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j})] \quad (10)$$

where $s_j{}^2 = 1/(n_j - 1)\sum_{q-1}^{n_j}(R_{jq} - \bar{R}_j)^2$. Let T be a t-distributed random variable with $n_j - 1$ degree of freedom. Then $t_{n_j-1}(\gamma_k)$ satisfies

$$p\left(T \geq t_{n_j-1}(\gamma_k)\right) = \gamma_k, k = 1,2.$$

The decision maker not only chooses γ_1 and γ_2 to satisfy [9], but also satisfies the following conditions, for $j = 1,2,\ldots,n$

$$0 < \bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})(s_j/\sqrt{n_j}) < 1, \text{ and}$$

$$0 < \bar{R}_j - t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j})(s_j/\sqrt{n_j}) < 1 \quad (11)$$

The decision maker must take a suitable value from [10] as an appropriate point estimate of R_j. [10] is an interval and is not a value. Therefore, we can not consider this problem by using the statistical point of view. Instead, we use the fuzzy point of view in the following:

From the Fig.3, we consider

$$a_1 = \bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})$$

$$b_1 = (1/2)(2\bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}))$$

$$c_1 = (1/2)(2\bar{R}_j + t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}))$$

$$d_1 = \bar{R}_j + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j})$$

If the decision maker takes a value that coincides with in the interval $[b_1, c_1]$, then the error is 0. If the value deviates from $[b_1, c_1]$, farther from both sides of this interval the error is bigger.

If the value lies at one of the two endpoints a_1 and d_1 the error will attain a maximum. From the fuzzy point of view, we can transform the error into a confidence level. If the error is 0, then the confidence level is 1.

The farther value is from both side of $[b_1, c_1]$ the confidence level is lesser. At the two endpoints a_1 and d_1 the confidence level will be minimized to 0.

Corresponding to the interval in [10], we characterize the trapezoidal fuzzy number as follows

$$\begin{aligned}\widetilde{R_J} &= (a_1, b_1, c_1, d_1) \\ &= \bar{R_J} - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}), \\ &\quad (1/2)(2\bar{R_J} - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})), \\ &\quad (1/2)(2\bar{R_J} + t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})), \\ &\quad \bar{R_J} + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j}) \qquad (12)\end{aligned}$$

From Fig.3, the membership grade of $[b_1, c_1]$, in R_j is 1. The farther point in intervals (a_1, d_1) from both sides of $[b_1, c_1]$, the membership grade is lower.

The membership grade and the confidence level have the same properties. Therefore, if we make a correspondence between membership grade and confidence level, it is reasonable to set up a trapezoidal fuzzy number in [12] corresponding to [10].

Using Definition 5, we get

$$R_j^* = d(\bar{R_J}, \bar{0}) = \bar{R_J} + \left(\frac{3}{8}\right)[t_{n_j-1}(\gamma_2) - t_{n_j-1}(\gamma_1)](s_j/\sqrt{n_j})$$

belongs to the interval in [10].

This is the estimate value of R_j from a fuzzy point of view. When $\gamma_1 = \gamma_2 = \gamma/2$, then $R_j^* = \bar{R_J}$ for each $j = 1,2,\ldots,n$. The $\alpha -$ level set of $\bar{R_J}, 0 \le \alpha \le 1$ is

$$[\, R_{jl}(\alpha), R_{ju}(\alpha)] \qquad (13)$$

Where

$$\begin{aligned}R_{jl}(\alpha) = \bar{R_J} &- t_{n_j-1}(\gamma_1)](s_j/\sqrt{n_j}) \\ &+(1/2)(2\bar{R_J} - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})) \\ &-\bar{R_J} + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j})\end{aligned}$$

and

$$\begin{aligned}R_{ju}(\alpha) = \bar{R_J} &+ t_{n_j-1}(\gamma_2)](s_j/\sqrt{n_j}) \\ &+\bar{R_J} + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j}) \\ &+(1/2)(2\bar{R_J} - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})) \qquad (14)\end{aligned}$$

$$\bar{R_J} = \cup_{0 \le \alpha \le 1}[R_{jl}(\alpha), R_{jl}(\alpha); \alpha], \qquad (15)$$

$j = 1,2,\ldots,n.$

1) Theorem: Using the Trapezoidal fuzzy numbers $\bar{R_j}, j = 1,2,\ldots,n$, in [12] we have the fuzzy system reliability as follows, (1). Fuzzy reliability of the serial system is

$$\widetilde{R_1} \otimes \widetilde{R_2} \otimes \ldots \otimes \widetilde{R_n}$$

$$\bigcup_{0 \le \alpha \le 1} \left[\prod_{j=1}^{n} R_{jl}(\alpha), \prod_{j=1}^{n} R_{ju}(\alpha); \alpha\right], \qquad (16)$$

where $R_{jl}(\alpha), R_{ju}(\alpha)$ are given in [14]. (2). Fuzzy reliability of the parallel system is

$$\tilde{1} \ominus (\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \ldots \otimes (\tilde{1} \ominus \widetilde{R_n})$$

$$= \bigcup_{0 \le \alpha \le 1} \left[1 - \prod_{j=1}^{n} Q_{ju}(\alpha), 1 - \prod_{j=1}^{n} Q_{jl}(\alpha); \alpha\right], \qquad (17)$$

where $Q_{jl}(\alpha) = 1 - R_{ju}(\alpha)$ and $Q_{ju}(\alpha) = 1 - R_{jl}(\alpha), 0 \le \alpha \le 1, j = 1,2,\ldots,n$.

Proof :

(1). From Eqs. (11) and (14), for each $j = 1,2,\ldots,n$, we have $0 < R_{jl}(\alpha) \le R_{ju}(\alpha)$, For all $\alpha \in [0,1]$.We get (1).

(2). We have $\tilde{1} = (1,1,1,1)$. By [14], we obtain

$$\tilde{1} \ominus \bar{R_j} = \bigcup_{0 \le \alpha \le 1} [Q_{jl}(\alpha), Q_{ju}(\alpha); \alpha], j = 1,2,\ldots,n,$$

where

$$Q_{jl}(\alpha) = 1 - \{\bar{R_j} + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j}) \\ -\{[2\bar{R_j} - t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j})]/2 - \bar{R_j}\}\alpha\}$$

and

$$Q_{ju}(\alpha) = 1 - \{\bar{R_j} + t_{n_j-1}(\gamma_1)](s_j/\sqrt{n_j}) \\ +\bar{R_j} - [2\bar{R_j} - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})]/2\}\alpha\}$$

$0 \le \alpha \le 1$.

From [11], we get $0 < Q_{jl}(\alpha) < Q_{ju}(\alpha)$,
For all $\alpha \in [0,1]$ and $j = 1,2,\ldots,n$.
We have

$$[(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \ldots \otimes (\tilde{1} \ominus \widetilde{R_n})]$$

$$\bigcup_{0 \le \alpha \le 1} \left[\prod_{j=1}^{n} Q_{jl}(\alpha), \prod_{j=1}^{n} Q_{ju}(\alpha); \alpha\right],$$

We obtain

$$\tilde{1} \ominus [(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \ldots \otimes (\tilde{1} \ominus \widetilde{R_n})]$$

$$= \bigcup_{0 \le \alpha \le 1} \left[1 - \prod_{j=1}^{n} Q_{ju}(\alpha), 1 - \prod_{j=1}^{n} Q_{jl}(\alpha); \alpha\right],$$

This completes the proof of (2).

2) Theorem: Under the condition of Theorem 1, we have the following results. (1). By Definition 5, we can defuzzify [16] to get the estimate reliability of serial system in the fuzzy sense as follows

$$d(\widetilde{R_1} \otimes \widetilde{R_2} \otimes \ldots \otimes \widetilde{R_n}, \tilde{0}) \\ = (1/2) \int_0^1 \left[\prod_{j=1}^{n} R_{jl}(\alpha) + \prod_{j=1}^{n} R_{ju}(\alpha)\right] d\alpha.$$

(2). By Definition 5, we can defuzzify [17] to get the estimate reliability of parallel system in the fuzzy sense as follows

$$d(\tilde{1} \ominus [(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \ldots \otimes (\tilde{1} \ominus \widetilde{R_2})], \tilde{0})$$
$$= (1/2)\int_0^1 (1 - \prod_{j=1}^{n} Q_{ju}(\alpha) + 1 - \prod_{j=1}^{n} Q_{jl}(\alpha))d\alpha.$$

Proof:
By Theorem 1 and Definition 5, we have desired result.

C) Numerical Examples

1) Example : Consider the following serial system and parallel system and to find the estimate reliability using Trapezoidal fuzzy number. In Jing et al [8], let $\gamma = 0.02,\ \gamma_1 = 0.011, \gamma_1 = 0.009$.

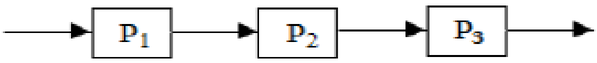

Figure 4: Serial System

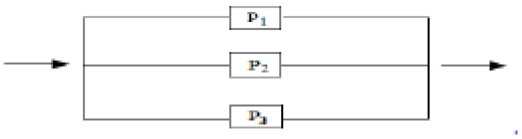

Figure 5: Parallel System

Here $\overline{R}_j = (1/n_j)\sum_{q=1}^{n_j} R_{jq}$ and $s_j^2 = (1/(n_j-1))\sum_{q=1}^{n_j}(R_{jq} - \overline{R}_j)^2$. From the table of the t-distribution with $n_j - 1$ degrees of freedom, $j = 1,2,3$, we get the following data,

$t_9(\gamma_1) = 2.7017, t_{19}(\gamma_1) = 2.5212;\ t_{14}(\gamma_1) = 2.5921, t_9(\gamma_2) = 2.9068, t_{19}(\gamma_2) = 2.6034; t_9(\gamma_2) = 2.6946.$

From Tables 1 and 2 we have the following Trapezoidal fuzzy numbers,

Subsystem	Sample Size	Sample Mean	Sample Standard Deviation
P_1	$n_1 = 10$	$\overline{R}_1 = 0.8$	$s_1 = 0.02$
P_2	$n_2 = 20$	$\overline{R}_2 = 0.75$	$s_2 = 0.03$
P_3	$n_3 = 15$	$\overline{R}_3 = 0.9$	$s_3 = 0.01$

Table 1

Table 1. Gives the statistical data for , $\widetilde{R}_j = 1,2,3$.

j	Degrees Of Freedom (k)	a_1	b_1	c_1	d_1
1	9	0.7829	0.7914	0.8092	0.8184
2	19	0.7331	0.7415	0.7587	0.7675
3	14	0.8933	0.8967	0.9035	0.9070

Table 2. Two Endpoints and Midpoints for Trapezoidal Fuzzy number

$$\widetilde{R_1} = (0.7829, 0.7914, 0.8092, 0.8184),$$
$$\widetilde{R_2} = (0.7331, 0.7415, 0.7587, 0.7675),$$
$$\widetilde{R_3} = (0.8933, 0.8967, 0.9035, 0.9070).$$

From [14], the α −level fuzzy sets of $\widetilde{R_j}, j = 1,2,3$
$0 \leq \alpha \leq 1$ are

$$R_{1l}(\alpha) = 0.7829 + 0.0085\alpha, \qquad R_{1u}(\alpha) = 0.8184 - 0.0092\alpha,$$
$$R_{2l}(\alpha) = 0.7331 + 0.0084\alpha, \qquad R_{2u}(\alpha) = 0.7675 - 0.0088\alpha,$$
$$R_{3l}(\alpha) = 0.8933 + 0.0033\alpha, \qquad R_{3u}(\alpha) = 0.9070 - 0.0035\alpha.$$

(1) Since $0 < R_{jl}(\alpha) < R_{ju}(\alpha)$ for all $0 \leq \alpha \leq 1, j = 1,2,3$, from Theorem 1(1), the fuzzy reliability of the serial system is

$$\widetilde{R_1} \otimes \widetilde{R_2} \otimes \widetilde{R_3} = \cup_{0 \leq \alpha \leq 1}\left[\prod_{j=1}^{3} R_{jl}(\alpha), \prod_{j=1}^{3} R_{ju}(\alpha); \alpha\right],$$

From Theorem 2(1), we get the estimate reliability of the serial system in the fuzzy sense as

$$d\left(\widetilde{R_1} \otimes \widetilde{R_2} \otimes \widetilde{R_3}, \bar{0}\right) = (1/2)\int_0^1\left[\prod_{j=1}^{3} R_{jl}(\alpha) + \prod_{j=1}^{3} R_{ju}(\alpha)\right]d\alpha = 0.54078.$$

From [7], we get the crisp case reliability of the serial system to be

$$\prod_{j=1}^{3} \bar{R_j} = o.54.$$

(2) From Theorem 1(2), we have

$$Q_{1l}(\alpha) = 0.1816 + 0.0092\alpha, \qquad Q_{1u}(\alpha) = 0.2171 - 0.0085\alpha,$$
$$Q_{2l}(\alpha) = 0.2325 + 0.0088\alpha, \qquad Q_{2u}(\alpha) = 0.2669 - 0.0084\alpha,$$
$$Q_{3l}(\alpha) = 0.0930 + 0.0035\alpha, \qquad Q_{3u}(\alpha) = 0.1067 - 0.0033\alpha.$$

Since $0 < Q_{jl}(\alpha) < Q_{ju}(\alpha)$ for all $0 \leq \alpha \leq 1, j = 1,2,3$. Fuzzy reliability of the parallel system is

$$\tilde{1} \ominus \left[\left(\tilde{1} \ominus \widetilde{R_1}\right) \otimes \left(\tilde{1} \ominus \widetilde{R_2}\right) \otimes \left(\tilde{1} \ominus \widetilde{R_3}\right)\right] = \cup_{0 \leq \alpha \leq 1}\left[1 - \prod_{j=1}^{3} Q_{ju}(\alpha), 1 - \prod_{j=1}^{3} Q_{jl}(\alpha); \alpha\right].$$

From Theorem 2(2), we get the estimate reliability of the parallel system in the fuzzy sense as

$$d(\tilde{1} \ominus \left[\left(\tilde{1} \ominus \widetilde{R_1}\right) \otimes \left(\tilde{1} \ominus \widetilde{R_2}\right) \otimes \left(\tilde{1} \ominus \widetilde{R_3}\right)\right], \tilde{0})$$

$$= (1/2)\int_0^1 [1 - \left[\prod_{j=1}^{3} Q_{ju}(\alpha) + 1 - \prod_{j=1}^{3} Q_{jl}(\alpha)\right]]d\alpha = 0.994977.$$

From [8], we find the crisp case reliability of the parallel system to be $1 - \prod_{j=1}^{3}\left(1 - \bar{R_j}\right) = 0.995$.

2) Example: Let us illustrate an example given by Mon and Cheng[9] Two grinding machines, are working next to each other. Let us find the probability that people coming into the vicinity of the machines are injured mainly by getting a chip into their eye, by using Trapezoidal fuzzy number.
The most endangered persons are the operators, who are obliged to wear safety glasses but often fail to do this. Further endangered are persons coming into the vicinity of the machines, the persons bringing and carrying away items, further those entering the area for other reasons.

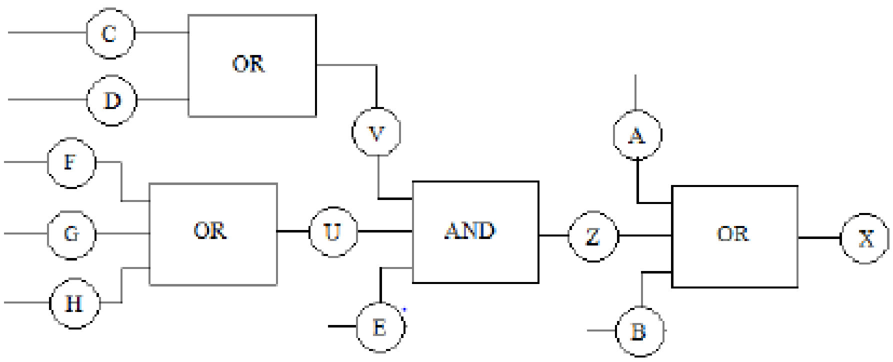

Figure 6: System X

j	Symbols	Basic Event	Populations Reliability (unknown)	Sample Size	Sample Mean	Sample Standard Deviation
1	A	Operator 1 fails to wear safety glasses	R_A	$n_A = 10$	$\bar{R}_A$=0.1	s_A=0.004
2	B	Operator 2 fails to wear safety glasses	R_B	$n_B = 10$	$\bar{R}_B$=0.2	s_B=0.004
3	C	Machine 1 is operating	R_C	$n_C = 10$	$\bar{R}_C$=0.8	s_C=0.004
4	D	Machine 2 is operating	R_D	$n_D = 10$	$\bar{R}_D$=0.6	s_D=0.01
5	E	Persons entering the area without safety glasses	R_E	$n_E = 10$	$\bar{R}_E$=0.9	s_E=0.02
6	F	Persons entering the endangered area bringing material	R_F	$n_F = 10$	$\bar{R}_F$=0.5	s_F=0.004
7	G	Persons entering the area carrying away made product	R_G	$n_G = 10$	$\bar{R}_G$=0.6	s_G=0.004
8	H	Persons entering the area for other reasons	R_H	$n_H = 10$	$\bar{R}_H$=0.5	s_H=0.001

Table 3 Statistical data of the Basic Events Contributing to The Accident.

j	Symbols	a_1	b_1	c_1	d_1
1	A	0.0965	0.0983	0.1018	0.1036
2	B	0.1965	0.1983	0.2018	0.2036
3	C	0.7965	0.7983	0.8018	0.8036
4	D	0.5914	0.5957	0.6046	0.6091
5	E	0.8829	0.8915	0.9092	0.9184
6	F	0.4965	0.4983	0.5018	0.5037
7	G	0.5965	0.5983	0.6018	0.6034
8	H	0.4992	0.4996	0.5005	0.5009

Table 4. Two Endpoints and Midpoints for Trapezoidal Fuzzy Number

$$U = F + G + H,$$
$$V = C + D,$$
$$Z = E \times U \times V,$$
$$X = A + B + Z \qquad (18)$$

$$\widetilde{R_1} = (0.0965, 0.0983, 0.1018, 0.1036)(= \widetilde{R_A}),$$
$$\widetilde{R_2} = (0.1965,0.1983,0.2018,0.2036)(= \widetilde{R_B}),$$
$$\widetilde{R_3} = (0.7965, 0.7983, 0.8018, 0.8036)(= \widetilde{R_C}),$$
$$\widetilde{R_4} = (0.5914,0.5957,0.6046,0.6091)(= \widetilde{R_D}),$$
$$\widetilde{R_5} = (0.8829,0.8915,0.9092,0.9184)(= \widetilde{R_E}),$$
$$\widetilde{R_6} = (0.4965,0.4983,0.5018,0.5037)(= \widetilde{R_F}),$$
$$\widetilde{R_7} = (0.5965,0.5983,0.6018,0.6034)(= \widetilde{R_G}),$$
$$\widetilde{R_8} = (0.4991,0.4996,0.5005,0.5009)(= \widetilde{R_H}).$$

From [14], the α −level sets of $\widetilde{R_j}$, $j = 1,2,\ldots,8, 0 \le \alpha \le 1$ are

$$R_{1l}(\alpha) = 0.0965 + 0.0018\alpha, R_{1u}(\alpha) = 0.1036 - 0.0018\alpha,$$
$$R_{2l}(\alpha) = 0.1965 + 0.0018\alpha, R_{2u}(\alpha) = 0.2036 - 0.0018\alpha,$$
$$R_{3l}(\alpha) = 0.7965 + 0.0018\alpha, R_{3u}(\alpha) = 0.8036 - 0.0018\alpha,$$
$$R_{4l}(\alpha) = 0.5914 + 0.0043\alpha, R_{4u}(\alpha) = 0.6091 - 0.0045\alpha,$$
$$R_{5l}(\alpha) = 0.8824 + 0.0086\alpha, R_{5u}(\alpha) = 0.9184 - 0.0092\alpha,$$
$$R_{6l}(\alpha) = 0.4965 + 0.0018\alpha, R_{6u}(\alpha) = 0.5037 - 0.0019\alpha,$$
$$R_{7l}(\alpha) = 0.5965 + 0.0018\alpha, R_{7u}(\alpha) = 0.6034 - 0.0016\alpha,$$
$$R_{8l}(\alpha) = 0.4991 + 0.0005\alpha, R_{8u}(\alpha) = 0.5009 - 0.0004\alpha.$$

Since $0 < R_{jl}(\alpha) \le R_{ju}(\alpha)$ for all $0 < \alpha \le 1, j = 1,2,\ldots,8$, by [18], Theorem 1(2), we have the following results. From [14], the α −cut of $\bar{R}_j$ at the left and right endpoints are $R_{jl}(\alpha)$, $R_{ju}(\alpha)$ and $Q_{jl}(\alpha)$, $Q_{ju}(\alpha)$. By Theorem 1(2), we obtain

$$Q_{jl}(\alpha) = 1 - R_{ju}(\alpha)$$
$$Q_{ju}(\alpha) = 1 - R_{jl}(\alpha)$$

For [18], $U = F + G + H$, through [17] in Theorem 1(2), we have

$$Q_{6l}(\alpha) = 0.4963 - 0.0018\alpha, Q_{6u}(\alpha) = 0.5035 + 0.0019\alpha;$$
$$Q_{7l}(\alpha) = 0.3966 - 0.0016\alpha, Q_{7u}(\alpha) = 0.4035 + 0.0018\alpha,$$
$$Q_{8l}(\alpha) = 0.4991 - 0.0004\alpha_{;}, Q_{8u}(\alpha) = 0.5009 + 0.0005\alpha. \qquad (19)$$

and we get

$$\widetilde{R_U} = \cup_{0 \le \alpha \le 1}\left[1 - \prod_{j=6}^{8} Q_{ju}(\alpha), 1 - \prod_{j=6}^{8} Q_{jl}(\alpha) ; \alpha\right].$$

For [18], V = C+D, through [17] in Theorem 1(2), we find

$$Q_{3l}(\alpha) = 0.1964 - 0.0018\alpha;\ Q_{3u}(\alpha) = 0.2035 + 0.0018\alpha;$$
$$Q_{4l}(\alpha) = 0{:}3909 - 0.0092\alpha;\ Q_{4u}(\alpha) = 0.4086 + 0.0043\alpha; \quad (20)$$

and we get

$$\widetilde{R_V} = \cup_{0 \le \alpha \le 1}\left[1 - \prod_{j=3}^{4} Q_{ju}(\alpha),\, 1 - \prod_{j=3}^{4} Q_{jl}(\alpha)\,;\alpha\right].$$

For [18], $Z = E \times U \times V$, by [16] in Theorem 1(1) and $0 < R_{5l}(\alpha) \le R_{5u}(\alpha)$,

$0 < 1 - \prod_{j=6}^{8} Q_{ju}(\alpha) < 1 - \prod_{j=6}^{8} Q_{jl}(\alpha)$ and
$0 < 1 - \prod_{j=3}^{4} Q_{ju}(\alpha) < 1 - \prod_{j=3}^{4} Q_{jl}(\alpha)$, we obtain

$$\widetilde{R_Z} = \cup_{0 \le \alpha \le 1}\left[R_{5l}(\alpha)(1 - \prod_{j=6}^{8} Q_{ju}(\alpha))\,(1 - \prod_{j=3}^{4} Q_{ju}(\alpha))\right],$$
$$R_{5u}(\alpha)(1 - \prod_{j=6}^{8} Q_{jl}(\alpha))\,(1 - \prod_{j=3}^{4} Q_{jl}(\alpha)\,;\alpha).$$

For [18], $X = A + B + Z$, by [17] in Theorem 1(2), we have

$$Q_{1l}(\alpha) = 0.8964 - 0.0018\alpha, Q_{1u}(\alpha) = 0.9035 + 0.0018\alpha;$$
$$Q_{2l}(\alpha) = 0.7964 - 0.0018\alpha, Q_{2u}(\alpha) = 0.8035 + 0.0018\alpha; \quad (21)$$

Since $\tilde{1} \ominus \widetilde{R_A} = \cup_{0 \le \alpha \le 1}[Q_{1l}(\alpha), Q_{1u}(\alpha);\ \alpha]$ and

$\tilde{1} \ominus \widetilde{R_B} = \cup_{0 \le \alpha \le 1}[Q_{2l}(\alpha), Q_{2u}(\alpha);\ \alpha]$ we have

$$\tilde{1} \ominus \widetilde{R_A} = \cup_{0 \le \alpha \le 1}\ [1 - R_{5u}(\alpha)(1 - \prod_{j=6}^{8} Q_{jl}(\alpha))\left(1 - \prod_{j=3}^{4} Q_{jl}(\alpha)\right),$$
$$1 - R_{5l}(\alpha)(1 - \prod_{j=6}^{8} Q_{ju}(\alpha))\left(1 - \prod_{j=3}^{4} Q_{ju}(\alpha)\,;\alpha\right)]$$

Because $0 < Q_{jl}(\alpha) < Q_{ju}(\alpha), j = 1{,}2$ and

$$0 = 1 - R_{5u}(\alpha)(1 - \prod_{j=6}^{8} Q_{jl}(\alpha))\left(1 - \prod_{j=3}^{4} Q_{jl}(\alpha)\right)$$
$$= 1 - R_{5l}(\alpha)(1 - \prod_{j=6}^{8} Q_{ju}(\alpha))\left(1 - \prod_{j=3}^{4} Q_{ju}(\alpha)\right),$$

For all, $\alpha \in [0, 1]$, we get

$$\left(\tilde{1} \ominus \widetilde{R_A}\right) \otimes \left(\tilde{1} \ominus \widetilde{R_B}\right) \otimes \left(\tilde{1} \ominus \widetilde{R_Z}\right) = \cup_{0 \le \alpha \le 1}[LH(\alpha), RH(\alpha);\ \alpha],$$

where

$$LH(\alpha) = Q_{1l}(\alpha)Q_{2l}(\alpha)[1 - R_{5u}(\alpha)\left(1 - \prod_{j=6}^{8} Q_{jl}(\alpha)\right)\left(1 - \prod_{j=3}^{4} Q_{jl}(\alpha)\right)],$$
$$RH(\alpha) = Q_{1u}(\alpha)Q_{2u}(\alpha)[1 - R_{5l}(\alpha)\left(1 - \prod_{j=6}^{8} Q_{ju}(\alpha)\right)\left(1 - \prod_{j=3}^{4} Q_{ju}(\alpha)\right)].$$

Therefore, from [17] in Theorem 1(2), we obtain the fuzzy reliability of X

$$\widetilde{R_X} = \tilde{1} \ominus [\left(\tilde{1} \ominus \widetilde{R_A}\right) \otimes \left(\tilde{1} \ominus \widetilde{R_B}\right) \otimes \left(\tilde{1} \ominus \widetilde{R_Z}\right)]$$
$$= \cup_{0 \le \alpha \le 1}[1 - RH(\alpha), 1 - LH(\alpha);\ \alpha].$$

From [22] and Definition 5, we have the estimate reliability of the system X in the fuzzy sense as follows

$$d(\widetilde{R_Z}, \tilde{0}) = (1/2)\int_0^1 (1 - RH(\alpha), 1 - LH(\alpha);\ \alpha)d\alpha$$

$$= (1/2)\int_0^1 [Q_{lu}(\alpha)Q_{2u}(\alpha) - Q_{lu}(\alpha)Q_{2u}(\alpha)Q_{5l}(\alpha)$$

$$+ Q_{lu}(\alpha)Q_{2u}(\alpha)Q_{5l}(\alpha)(\prod_{j=6}^{8} Q_{ju}(\alpha) + \prod_{j=3}^{4} Q_{ju}(\alpha))$$

$$- Q_{lu}(\alpha)Q_{2u}(\alpha)Q_{5l}(\alpha)(\prod_{j=6}^{8} Q_{ju}(\alpha) + \prod_{j=3}^{4} Q_{ju}(\alpha))$$

$$(1/2)\int_0^1 [Q_{1l}(\alpha)Q_{2l}(\alpha) - Q_{ll}(\alpha)Q_{2l}(\alpha)Q_{5u}(\alpha)$$

$$+ Q_{ll}(\alpha)Q_{2l}(\alpha)Q_{5u}(\alpha)(\prod_{j=6}^{8} Q_{jl}(\alpha) + \prod_{j=3}^{4} Q_{jl}(\alpha))$$

$$- Q_{ll}(\alpha)Q_{2l}(\alpha)Q_{5u}(\alpha)(\prod_{j=6}^{8} Q_{jl}(\alpha) + \prod_{j=3}^{4} Q_{jl}(\alpha)]d\alpha \quad (23)$$

By $R_{5l}(\alpha), R_{5u}(\alpha)$, Eq.(4.8) $j = 6,7,8$, [20], $Q_{jl}(\alpha), Q_{ju}(\alpha)$, j = 6,7,8, [20], $Q_{jl}(\alpha), Q_{ju}(\alpha), j = 3,4$, [21] and $Q_{jl}(\alpha), Q_{ju}(\alpha), j = 1,2$, [23] can found as

$$d(\widetilde{R_Z}, \tilde{0}) = 0.75786496.$$

This result is the estimate reliability of the system X.

IV. CONCLUSION

In this study, statistical data is used to estimate the reliability of a system which may be serial or parallel. By fuzzy methods confidence intervals can be transformed to any fuzzy number. Decomposition theorem is used to write fuzzy set as the union of level α intervals and signed distance method is used to defuzzify the fuzzy reliability and to obtain the estimate reliability of the system.

This is illustrated by an example which have 3 subsystem as an extension, we also illustrated by an example which have 7 subsystem, which is a combination of serial and parallel system. When the number of subsystems are increasing, to obtain optimal value of system reliability, trapezoidal fuzzy number can be extended to trapezoidal fuzzy number.

We can also verify whether their results are accepted or rejected. Jing et al [8] used trapezoidal fuzzy number to estimate the reliability of a system and using statistical data. In this study, we use trapezoidal fuzzy number to estimate the reliability of a system and using statistical data. The reliability of the serial system is $R_1 = 0.5407$ (in 4.2). But the crisp value for the reliability of the serial system is $R' = 0.54$. Then

$$\frac{(R' - R_1)}{R'} \times 100\% = -0.129\%$$

Thus $R_* = 0.5407$ is near to the crisp case $R' = 0.54$. That is to say, the defuzzification is significant in this study. Similarly, the reliability of the parallel system is $R_2 = 0.9952$. From the crisp case, the reliability of the parallel system is $R'' = 0.995$.

And

$$\frac{(R'' - R_2)}{R''} \times 100\% = -0.0201\%$$

Thus $R_2 = 0.9952$ is near to the crisp case $R'' = 0.995$. That is to say, the defuzzification is significant. Hence the results are accepted for the reliability of serial system and parallel system using trapezoidal fuzzy number.

REFERENCES

[1] D. Dubois, and H. Prade, “Fuzzy Sets and Systems:Theory and Applications”, Academic Press, 1980.

[2] K.-Y. Cai, C.-Y. Wen, and M.-L Zhang, “Fuzzy variables as a basic for a theory of fuzzy reliability in the possibility context”, Fuzzy Sets and Systems,

42, pp. 45-172, 1991.

[3] K.-Y. Cai, C.-Y. Wen, and M.-L Zhang, "Fuzzy states as for a theory of fuzzy reliability", Microelectron Reliability, 33, pp. 2253-2263, 1993.

[4] K.-Y. Cai, "System failure engineering and fuzzy methodology" , An introductory overview, Fuzzy Sets and Systems, 83, pp. 113-133, 1996.

[5] C.-H. Chen, and L. Mon, "Fuzzy system reliability analysis by interval of con dence", Fuzzy Sets and Systems, 56, pp. 29-35, 1993.

[6] M. Hammer, "Application of fuzzy theory to electrical machine reliability", Zeszyty Naukowe Politechniki Slaskiej, seria: Elektryka 177, pp. 161-166, 2001.

[7] M.E. El-Hawary," Electric power applications of fuzzy systems.IEEE Press Series on Power Engineering", 2000.

[8] Jing-Shing Yao, Jin-Shieh Su and Teng-San Shih, "Fuzzy system reliability analysis using tri-angular fuzzy numbers based on statistical data", Journal of information science and engineering, 24, pp. 1521-1535, 2008.

[9] D.L. Mon, and C.H. Cheng, "A general and easy computational method for function of fuzzy numbers", Int. J. Systems Sci., 26, (4), pp. 983-990, 1995.

[10] T. Onisawa, and J. Kacprzyk, "Reliability and safety analysis under fuzziness.Physica", Verlag , Heidelberg, 1995.

[11] D. Singer, "A fuzzy set approach to fault tree and reliability analysis", Fuzzy Sets and Systems, 34, pp. 145-155, 1990.

[12] L.V. Utkin, and S.V. Gurov, "A general formal approach for fuzzy reliability analysis in the possibility context", Fuzzy Sets and Systems, 83, pp. 203-213, 1995.

[13] J.V. Virant, and N. Zimic, "Attention to time in fuzzy logic", Fuzzy Sets and Systems, 82, pp. 39-49, 1996.

[14] L. A. Zadeh, "Fuzzy set as a basis for theory of possibility", Fuzzy Sets and Systems, 1, pp. 3-28, 1978.

[15] H.J. Zimmermann, "Fuzzy set theory and its applications", KluwerNijho , Boston, 1986.

[16] G. Michael Rosario, and et al ., "Fuzzy Reliability Evaluation of Weaving Machine in Textile Industry", International Journal of Mathematical Archive, 9, pp. 1, 20-25, 2018.

[17]. G. Michael Rosario, and et al., "An Inventory Modal for Deteriorating Item with depend demand in both fuzzy and crisp sense", Mathematical sciences, International Research journal, Spl. Issue ISSN: 2278-8697, 7, 2018.