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MDP IN SUPPLY CHAIN: INVENTORY SYSTEM WITH SERVICE FACILITY AT RETAIL NODE WITH IMPATIENT CUSTOMER

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Abstract

In this article, we study a MDP based inventory control in a two stage Supply Chain. We consider a two stage Supply Chain having Distribution Centre (DC) and Retail Vendor (RV) with their respective inventory systems. We consider the service facility at the retailer node having finite waiting space, inventory is replenished from distribution centre. Customers arrive according to a Poisson arrival process to retail node. The individual customer's unit demand is satisfied after a random time at service which is assumed to have Exponential distribution. The lead time of reorders are assumed to have independent exponential distributions. The impatient customers renege from the queue with exponential rate. The joint probability distribution of the number of customers in the finite waiting room and the inventory level in system is obtained. System performance measures are computed and total expected cost rate is calculated. Optimal reorder quantity is obtained. Numerical examples are provided to illustrate the model.

Introduction

Supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products and the distribution of these finished products to customers. Supply Chain exists in both service and manufacturing organizations, but the complexity of the chain may vary greatly from industry to industry.

Inventory decision is an important component of the supply chain management, because Inventories exist at each and every stage of the supply chain as raw material or semi-finished or finished goods. They can also be as Work-in-process between the stages or stations. Since holding of inventories can cost anywhere between 20% to 40% of their value, their efficient management is critical in Supply Chain operations.

The usual objective for a multi-echelon inventory model is to coordinate the inventories and the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. The reason is that a key concept of supply chain management is that a company should strive to develop an informal partnership relation with its suppliers and retailers that enable them jointly to maximize their total profit. Information technology has a substantial impact on supply chains. Scanners collect sales data at the point-of-sale, and Electronic Data Interchange (EDI) allows these data to be shared immediately with all stages of the supply chain. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature. As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency.

The first quantitative analysis in inventory studies started with the work of Harris [9]. ClarkandScarf[4] had put forward the multi-echelon inventory first. They analyzed N-echelon pipelining system without considering a lot size, recent developments in two-echelon models may be found in Q.M. He and Sven Axsäter[1] proposed an approximate model of inventory structure in SC. One of the oldest papers in the field of continuous review multi-echelon inventory system is a basic and seminal paper written by Sherbrooke in 1968. He assumed (S-1,S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Continuous review models of multi-echelon inventory system in 1980's concentrated more on repairable items in a Depot-Base system than as consumable items(see Graves[7,8], Moinzadeh and Lee[10]). All these papers deal with repairable items with batch ordering. Seifbarghy and Jokar analyzeda two echelon inventory system with one warehouse and multiple retailers controlled by continuous review (R, Q) policy. A Complete review was provided by Benita M. Beamon (1998) [2]. The supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic work of Clark and Scarf(1960)[5]. In the case of continuous review perishable inventory models with random lifetimes for the items, most of the models assume instantaneous supply of order. The assumption of positive lead times further increases the complexity of the analysis of these models and hence there are only a limited number of papers dealing with positive lead-times. A continuous review perishable inventory system at Service Facilities was studied by Elango(2001)[6]. A continuous review(s,S) policy with positive lead times in two-echelon Supply Chain was considered by K.Krishnan and C.Elango[9].

Berman and Kim [11] analyzed a problem in a stochastic environment where customers arrive at service facilities according to a Poisson Process. The service times are exponentially distributed with mean inter-arrival time assumed to be larger than the order quantity is known. Berman and Sapna [3] studied an inventory control problem at a service facility requiring one item of the inventory and assumed Poisson arrivals, arbitrarily distributed service times and zero lead times. They assumed finite waiting room. Under a specified cost structure, the optimal ordering quantity that minimizes the long run expected cost rate has been derived.

In this paper we considered inventory system maintained in a service facilities at Retailer vendor in tandem supply chain having Retailer vendor (RV) and Distribution Centre (DC). One item from inventory at RV is used to serve the customer. (S, S) policy is adopted at Retail Vender node. We assume that customers arrive according to a Poisson arrival process to retail node. The individual customer's unit demand is satisfied after a random time at service which is assumed to have Exponential distribution. The lead time of reorders are assumed to have independent exponential distributions. Any arriving customer, who finds the waiting room full, enters into the orbit of infinite space. The joint probability distribution of the number of customers in the waiting room, the orbit and the inventory level in system is obtained. System performance measures are computed and total expected cost rate is calculated. Optimal reorder quantity is obtained.

Model Formulation

- We consider a Supply Chain system consist of Distribution Center(DC), Retail vendor(RV) with service facility and inventory is maintained at both DC and RV nodes with the following operational assumptions.
- For every demand at the retailer node (RV) an item is supplied only after a exponential service time with parameter $\gamma_k : k = 0,1,\ldots, M$.
- The waiting space in the retailer node has maximum capacity N. An arriving customer seeing N customers in the system should leave the system.
- Inventory policy adopted at RV node is (s, S) type in which order for Q=S-s>s items are placed when the inventory level reaches the prefixed level s, and lead time is exponentially distributed with parameter $\mu(>0)$.
- Demand at RV node follows a Poisson process with parameter $\lambda(>0)$. This system is viewed as a Markov Decision Processes, in which the service rates γ_n are controlled at the service facility in RV node.
- At DC, items are packed as Q items in one pocket with maximum inventory level nQ (n pockets). The ordering policy at DC is of (0, nQ) type where the inventory level reach 0, instantaneous replenishment nQ=U items is made.
- Each item in inventory has the policy rate θ(>0), with exponentially distributed life time
- Impatient customers renege the system with rate α>0(exponential distributed)

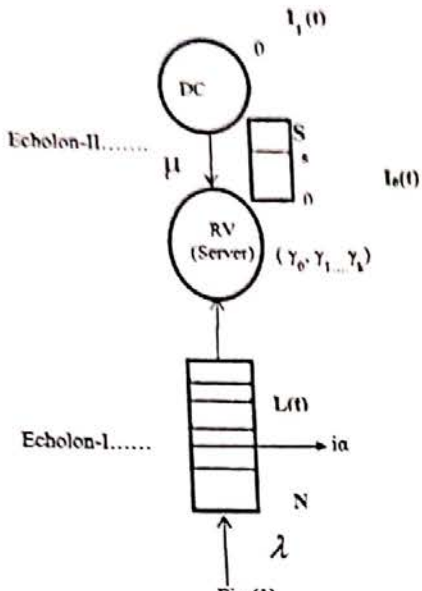

Fig (1)

Let I(t) and L(t) denote the on-hand inventory levels at retailer node and the number of customers in waiting room and X(t) denote the number of customer resides in the orbit at time t. Maximum capacity of the waiting room be N. Clearly {(I(t),L(t),X(t)): t≥0} is a Markov Process with state space E={,1,2.......S}×{0,1,2,.........N}×{0,1,2,.............}

We ignore the state of the system at DC, since the replenishment policy here is instantaneous. Since E is finite and all its states are recurrent non-null. $\{I(t), t \geq 0\}$ is an irreducible Markov process with state space E and it is also an Ergodic process. Hence, the limiting distribution exists and is independent of the initial state. In this article our objective is to find a policy that specifies the expected service rate adopted so as to minimize the long run expected cost rate.

The costs associated with the system operation have the following components:

h: inventory carrying cost per unit time.

c_1: cost per order.

c_2: waiting time cost per unit per unit time.

g: reneging cost per customer .

β_n:cost associated with using rateγ_n, $\beta_o = 0$

Analysis

System Analysis

Let $(I_0^R, L^R) = \{I_0^R(t); L^R(t) \geq 0\}$ denote the Markov process, when a stationary policy R is adopted. From our assumptions, it can be seen that the controlled process (I_0^R, L^R) is a finite state semi-Markov decision process. A policy R is called a stationary policy if it is randomized, time invariant and Markovian. Further, a process is said to be completely Ergodic if every stationary policy give rise to an irreducible Markov chain. From our assumptions it can be seen that for every stationary policy f, (I_0^f, L^f) is completely Ergodic. Since the action space is also finite, a stationary optimal policy exists. Hence we consider the class $\mathfrak{I}$ of all stationary policies.

Denote by (k) the action of choosing rate $\mu_k(k=0,1,2,\ldots,N)$. Whenever $L(t)=0$ or $I_0(t)=0$, we must choose the rate μ_0. Based on the choice of actions, the state space E can be partitioned as follows:

$E_1=\{(i,q):0\le i\le S, q=0; i=0, 0\le q\le N\}$, $E_2=\{(i,q):1\le i\le S; 1\le q\le N\}$,

$E=E_1\cup E_2$.

Let A_j (j=1,2) represent the set of all possible actions of the system when it belongs to the set E_j.

Then we have $A_1=\{0\}$, $A_2=\{(k):1\le k\le N\}$ and $A=A_1\cup A_2$

A decision rule from the class f is equivalent to a function f: E→ A and is given by $f(i,q)=\{(k);(i,q)\in E,(k)\in A_j, j=1,2\}$.

For any fixed $f\in F$ and $(i,q),(j,r)\in E$, define

$$P_{iq}^f(j,r,t)=\Pr\{I_0^f(t)=j, L^f(t)=r \mid I_0^f(0)=i, L^f(0)=q\},\ (i,q),(j,r)\in E. \quad (1)$$

Then $P_{iq}(j,r,t)$ satisfies the Kolmogorov forward differential equations. As each policy, f, results in an irreducible Markov chain and action spaces are finite, $p^f(j,r)=\lim_{t\to\infty}P_{iq}(j,r,t)$ exist and is independent of the initial conditions.

Hence we have the balance equation (2)-(13) given below. The balance equations can also be obtained by using the fact that transition out of a state is equal to transition into a state. For example, let us consider a typical state is equal to transition into a state. For example, let us consider a typical state (j,r) that lies in the range $s+1\le j\le S-1; 1\le r\le N-1$. This state is represented in Eq.(12) below. When (j, r) is in this range, there is no order pending, and hence transition out of this state can be only due to either a demand or a service completion. This fact is reflected on the left-hand side of Eq.(12) A service completion in state $(j+1,r+1)$ will decrease both the inventory level and the number of customers by one unit, thus bringing it to state (j,r). State (j,r) can be reached from state $(j+1,r+1)$ when a customer arrives. These are the only two possible ways of reaching state (j,r) and are reflected on the right-hand side of Eq (12). The state (i,j) will transmit to (i-1,j) with state of transition iθ where $0\le\theta\le 1$

$$(\lambda+\mu)P^f(0,0)=\alpha P^f(0,1)+\gamma^f P^f(1,1) \quad (2)$$

$$(\lambda+\mu+j\alpha)P^f(0,j)=\lambda P^f(0,j-1)+(j+1)\alpha P^f(0,j+1)+\gamma^f P^f(1,j+1) \quad 1\le j\le N-1 \quad (3)$$

$$(\mu+N\alpha)P^f(0,N)=\lambda P^f(0,N-1) \quad (4)$$

For $1\le i\le s$

$$(\lambda+\mu)P^f(i,0)=\gamma^f P^f(i+1,1) \quad (5)$$

$(\lambda+\mu+(j-1)\alpha+\gamma^f)p^f(i,j)=\lambda p^f(i,j-1)+j\alpha p^f(i,j+1)+\gamma^f p^f(i+1,j+1), 1\le j\le N-1$ (6)

$(\mu+(N-1)\alpha+\gamma^f)p^f(i,N)=\lambda p^f(i,N-1)$

For $s+1\le i\le Q-1$ (7)

$\lambda p^f(i,0)=\gamma^f p^f(i+1,1)$

For $Q\le i\le S-1$, (8)

$(\lambda+(j-1)\alpha+\gamma^f)p^f(i,j)=\lambda p^f(i,j-1)+j\alpha p^f(i,j+1)+\gamma^f p^f(i+1,j+1), 1\le j\le N-1$ (9)

$(N-1)\alpha+\gamma^f)p^f(i,N)=\lambda p^f(i,N-1)$

$\lambda p^f(i,0)=\gamma^f p^f(i+1,1)+\mu p^f(i-Q,0)$ (10)

$(\lambda+\gamma^f+(j-1)\alpha)p^f(i,j)=\lambda p^f(i,j-1)+j\alpha p^f(i,j+1)+\gamma^f p^f(i+1,j+1)$ (11)

$+\mu p^f(i-Q,j), 1\le j\le N-1$

$(\gamma^f+(N-1)\alpha)p^f(i,N)=\lambda p^f(i,N-1)+\mu p^f(i-Q,N)$ (12)

$\lambda p^f(S,0)=\mu p^f(s,0)$ (13)

$(\lambda+(j-1)\alpha+\gamma^f)p^f(S,j)=\lambda p^f(S,j-1)+j\alpha p^f(S,j+1)+\mu p^f(s,j),\quad 1\le j\le N-1$ (14)

$((N-1)\alpha+\gamma^f)p^f(S,N)=\lambda p^f(S,N-1)+\mu p^f(s,N)$ (15)

The above set of equations together with the condition (16)

$\sum_{(i,j)\in E} p^f(i,j)=1$

determine the steady-state probabilities uniquely. (17)

System Performance Measures

1. $P^f(j,r)$ also gives the long-run fraction of time the system is in the state (j, r), the average inventory level is given by

$$\bar{I}^f=\sum_{j=1}^{s} j\sum_{r=0}^{N} p^f(j,r)$$

(18)

2. The expected cost due to different service rates utilized is $\bar{\Gamma}_{f}=\sum_{(j,r)\in E}\Gamma_{f(r)}p^f(j,r)$, where

$\Gamma_{f(r)}=\beta_n \text{ if } f(r)=n$

(19)

3. The reorder rate is given by $\bar{\alpha}_1^f=\gamma^f\sum_{r=0}^{N} p^f(s+1,r)$

(20)

4. The mean waiting time is given by $\bar{\alpha}_2^{f} = \sum_{r=1}^{N} \frac{r}{\gamma_r^{f}} \sum_{j=0}^{S} P^{f}(j,r) + \frac{1}{\mu} \sum_{m=1}^{[N/S]} \sum_{r=1}^{mS} m \sum_{j=0}^{S} P^{f}(j,r)$ (21)

5. The balking rate is given by $\bar{\alpha}_3^{f} = \lambda \sum_{j=0}^{S} P^{f}(j,N)$ (22) The long-run expected cost rate when policy f is adopted is given by

$$C^{f} = h\bar{I}^{f} + c_1\alpha_1^{f} + c_2\alpha_2^{f} + g\alpha_3^{f} + \bar{\Gamma}_I^{f} \qquad (23)$$

Where in the steady state, for a given policy f,

$\bar{I}^{f}$ is the average inventory level, α_1^{f} is the expected reorder rate, α_2^{f} is the average waiting time for a customer; α_3^{f} is the expected balking rate, and $\bar{\Gamma}_I^{f}$ is the expected cost per unit time associated with using the different rates.

Our objective is to find an optimal policy f^* for which $C^{f^*} \le C^{f}$ for every f.

Hence the average cost rate of the system is given by

$$C^{f} = h\sum_{j=1}^{S} j \sum_{r=0}^{N} P^{f}(j,r) + c_1\gamma^{f} \sum_{r=0}^{N} P^{f}(s+1,r) + c_2 \sum_{r=1}^{N} \frac{r}{\gamma_r^{f}} \sum_{j=0}^{S} P^{f}(j,r)$$
$$+ \frac{c_2}{\mu} \sum_{m=1}^{[N/S]} \sum_{r=1}^{mS} m \sum_{j=0}^{S} P^{f}(j,r) + g\lambda \sum_{j=0}^{S} P^{f}(j,N) + \sum_{(j,r)\in E} \Gamma_{f(r)} P^{f}(j,r) \qquad (24)$$

Linear programming problem

LPP Formulation

Let us define the variables D (j, r, k) as D (j, r,k)= Pr [decision is k | state is (j,r)].

Then for any stationary policy f, we have D (j, r, k) =0 or 1.Suppose D(j, r, k) were continuous variable (instead of integers), then the semi-Markov decision problem can be reformulated as a linear programming problem. For this purpose we consider the class of all randomized, time – invariant Markovian policies for which the probability functions

D(j, r, k) satisfy $0 \le D(j,r,k) \le 1$ and $\sum_{k\in A_i} D(j,r,k) = 1, 0 \le r \le N, 0 \le j \le S; i = 1,2$

The linear programming problem is best expressed in terms of the variable $y(j,r,k)$, which are defined as

$$y(j,r,k) = D(j,r,k)P^{f}(j,r) \qquad (25)$$

As y(j, r, k)=Pr [state is (j,r) and decision is k],for any given f, we have

$$P^{f}(j,r) = \sum_{k\in A} y(j,r,k) \quad (j,r)\in E \qquad (26)$$

Expressing $p^{f}(j,r)$ in terms of $y(j,r,k)$ in (22) we obtain the following linear programming problem:

Minimize

$$C = h\sum_{k=1}^{K} j\sum_{j=1}^{S}\sum_{r=1}^{N} y(j,r,k) + h\sum_{j=1}^{S} jy(j,0,0) + c_1\sum_{k=1}^{K}\sum_{r=1}^{N}\gamma_k y(s+1,r,k) + g\lambda y(0,N,0) + c_2\sum_{k=1}^{K}\sum_{r=1}^{N}\frac{r}{\mu_k}\sum_{j=0}^{S} y(j,r,k)$$

$$+c_2\sum_{k=1}^{K}\frac{1}{\mu}\sum_{m=1}^{N}\sum_{r=1}^{mS} m\sum_{j=0}^{S} y(j,r,k) + g\lambda\sum_{k=1}^{K}\sum_{j=1}^{S} y(j,N,k) + \sum_{(j,r)\in E, r\neq 0}\sum_{k=1}^{K}\mu_k y(j,r,k) \qquad \text{.....(27)}$$

The constraints of the linear programming problem are as follows:

a) From (25), we have

$$y(j,r,k) \geq 0 \quad (j,r) \in E_l, k \in A_l, l = 1,2 \qquad (28)$$

b) Since $\sum_{(j,r)\in E} p^{f}(j,r) = 1$ we have from (26)

$$\sum_{l=1}^{2}\sum_{(j,r)\in E_l}\sum_{k\in A_l} y(j,r,k) = 1 \qquad (29)$$

c) The remaining constraints are the balance equations. After discarding the last equation as redundant in the set of equations, we have

$$\lambda y(S,0,0) = \mu\sum_{j=0}^{S} y(j,0,0), \qquad (30)$$

$$\sum_{k=1}^{K}(\lambda+\gamma_k)y(S,r,k) = \mu\sum_{k=1}^{K}\sum_{j=0}^{S} y(j,r,k) + \lambda y(S,r-1,k), 1 \leq r \leq N-1, \qquad (31)$$

$$\sum_{k=1}^{K}\gamma_k y(S,N,k) = \mu\sum_{k=1}^{K}\sum_{j=0}^{S} y(j,N,k) + \sum_{k=1}^{K}\lambda y(S,N-1,k), \qquad (32)$$

$$\lambda y(j,0,0) = \sum_{k=1}^{K}\gamma_k y(j+1,1,k), \quad s+1 \leq j \leq S-1, \qquad (33)$$

$$\sum_{k=1}^{K}(\lambda+\gamma_k)y(j,r,k) = \sum_{k=1}^{K}\gamma_k y(j+1,r+1,k) + \sum_{k=1}^{K}\lambda y(j,r-1,k),$$

$s+1 \leq j \leq S-1;\ 1 \leq r \leq N-1$ (34)

$$\sum_{k=1}^{K}\gamma_k y(j,N,k) = \sum_{k=1}^{K}\lambda y(j,N-1,k),$$

$$\sum_{j=1}^{N}\gamma_k y(j,M,k) = \sum_{j=1}^{N}\lambda y(j,N-1,k), s+1 \leq j \leq S-1 \qquad (35)$$

$$(\lambda+\mu)y(j,0,0)=\sum_{k=1}^{K}\gamma_k y(j+1,1,k),\ 1\le j\le s \tag{36}$$

$$\sum_{k=1}^{K}(\lambda+\mu+\mu_k)y(j,r,k)=\sum_{k=1}^{K}\gamma_k y(j+1,r+1,k)+\sum_{k=1}^{K}\lambda y(j,r-1,k),$$

$1\le j\le s;\ 1\le r\le N\text{-}1$ (37)

$$\sum_{k=1}^{K}(\mu+\gamma)y(j,N,k)=\sum_{k=1}^{K}\lambda y(j,N-1,k),\ 1\le j\le s \tag{38}$$

$$(\lambda+\mu)y(0,0,0)=\sum_{k=1}^{K}\gamma_k y(1,1,k), \tag{39}$$

$$\sum_{k=1}^{K}(\lambda+\mu)y(0,r,k)=\sum_{k=1}^{K}\gamma_k y(1,r+1,k)+\sum_{k=1}^{K}\lambda y(0,r-1,k),\ 1\le r\le N\text{-}1 \tag{40}$$

As we can see from the lemma below solving the linear programming problem gives the optimal solution when the $y(j,r,k)$'s are constrained to be integers.

The optimal solution of the above linear programming problem yields a deterministic policy. From equations (25) and (26), we have (41)

$$D(j,r,k)=\frac{y\ j,r,k}{\sum_{k=0}^{K}y\ j,r,k}$$

Since the decision problem is completely ergodic, every basic feasible solution to the above linear programming problem has the property that for each $(j,r)\in E$, D(j,r,k) is 1 for exactly one value of k and zero for all other values of k. Thus given the amount of inventory on-hand and the number of customers in the system, we have to choose the service rate μ_k for which D (j,r,k) is1.Hence any basic feasible solution of the linear programming yields a deterministic policy.

Numerical illustration and discussion

In this section, we illustrate the method described in section 4 through numerical examples. We use the simplex package in TORA for solving the linear programming problem. In all cases the computational time was less than 10 s on a PENTIUM 3.0 alpha machine.

It is our conjecture that the service rate to be employed depends only on the number of customers in the system and not on the inventory level. This conjecture can be proved for the zero-lead time case assuming that the same μ is used for all states. Then the expected cost rate is,

$$C(\mu)=\frac{c_1}{S[m+\alpha(1)/\lambda]}+h\frac{S+1}{2}+c_2\sum_{k=1}^{K}\sum_{j=1}^{S}kmP(j,k)+g\sum_{j=0}^{S}P(j,N)+\mu(\gamma)\sum_{j=1}^{S}\sum_{r=1}^{N}P(j,r), \tag{42}$$

Where m=1/ μ and

$$\alpha(1) = \frac{1-(\lambda/\gamma)}{1-(\lambda/\gamma)^{N}} \tag{43}$$

For $0 \le j \le S, 1 \le r \le N,$

$$P(j,r) = \left[\frac{\lambda}{\gamma}\right]^{r-1} P(j,0) \tag{44}$$

For $0 \le j \le S,$

$$P(j,0) = \frac{1}{S}\frac{1-(\lambda/\gamma)}{1-(\lambda/\gamma)^{N+1}} \tag{45}$$

In the above equations, $\eta(\gamma)$, an increasing function of γ, is the cost associated with service rate γ. By differentiating C(γ) with respect to γ, it is easy to verify that the stationary value of γ is independent of the inventory level. When different service rates are used for different states we were unable to prove the conjecture. However, from the numerous numerical examples we ran, we found that the service rate is insensitive to changes in the inventory level and replenishment rates. Furthermore, service rates also seem to be independent of ordering costs, inventory carrying costs and balking costs.

The only three parameters that have appreciable effect on the service rates are the arrival rate, the waiting time costs and the number of customers in the system.

Typical numerical results, from one of the examples we studied, are summarized in Tables 1-2, where n is the number of customers in the system. The first two tables show the effect of changes in waiting time costs .In the above two tables, we use γ_1=1, γ_2=2, γ_3=3, γ_4=4.

Two immediate conclusions can be drawn from the computational results in Tables 1-2:

(1) As the cost of waiting, c_2, increases, larger service rates are utilized for larger values of n. For examples, in Table 1,when c_2=1, γ_1=1is used for all n values, expect for n=9,10, where γ_2=1 is utilized .When c_2=10 ,γ_1=1is not used at all and γ_2, γ_3, γ_4 are used for respectively $1 \le n \le 3, 4 \le n \le 6$ and $7 \le n \le 10$

(2) As the arrival rate λ is increasing, larger service rates are utilized for larger values of n. We note that for a service facility, a large number of customers (n) waiting is equivalent to poor service. Therefore the conclusions above are quite important as they show that the model can be used to decrease the average waiting time (providing good service) by selecting the appropriate service rates in a cost effective way.

Tables 1

Optimal rates for c_1=50, h=0.1, g=2, $\mu_1 = 1, \mu_2 = 2, \mu_3 = 3, \mu_4 = 4, S = 25, s = 6, N = 10, \lambda = 7, \gamma = 0.3$

C_2	γ_1=1	γ_2=2	γ_3=3	γ_4=4
1	$1 \le n \le 8$	$9 \le n \le 10$	–	–
2	$1 \le n \le 7$	$8 \le n \le 10$	–	–
3	$1 \le n \le 6$	$7 \le n \le 10$	–	–
4	$1 \le n \le 5$	$6 \le n \le 9$	n=10	–
5	$1 \le n \le 4$	$5 \le n \le 8$	$9 \le n \le 10$	–
6	$1 \le n \le 3$	$4 \le n \le 7$	$8 \le n \le 9$	n=10
7	$1 \le n \le 3$	$4 \le n \le 6$	$7 \le n \le 8$	$9 \le n \le 10$
8	$1 \le n \le 2$	$3 \le n \le 5$	$6 \le n \le 7$	$8 \le n \le 10$
9	$n = 1$	$2 \le n \le 4$	$5 \le n \le 7$	$8 \le n \le 10$
10	–	$1 \le n \le 3$	$4 \le n \le 6$	$7 \le n \le 10$

Tables 2

Optimal rates for c_1=50, h=0.1, g=2, $\mu_1 = 1, \mu_2 = 2, \mu_3 = 3, \mu_4 = 4, S = 25, s = 6, N = 10, \lambda = 7, \gamma = 0.3$

C_2	γ_1=1	γ_2=2	γ_3=3	γ_4=4
1	$1 \le n \le 9$	n=10	–	–
2	$1 \le n \le 8$	$9 \le n \le 10$	–	–
3	$1 \le n \le 7$	$8 \le n \le 9$	n=10	–
4	$1 \le n \le 6$	$7 \le n \le 8$	$9 \le n \le 10$	–
5	$1 \le n \le 4$	$5 \le n \le 6$	$7 \le n \le 10$	–
6	$1 \le n \le 4$	$5 \le n \le 6$	$7 \le n \le 9$	n=10
7	$1 \le n \le 4$	n=5	$6 \le n \le 8$	$9 \le n \le 10$
8	$1 \le n \le 3$	n=4	$5 \le n \le 7$	$8 \le n \le 10$
9	$1 \le n \le 2$	–	$3 \le n \le 7$	$8 \le n \le 10$
10	n=1	n=2	$3 \le n \le 6$	$7 \le n \le 10$

In Table1, the costs associated with the different rates are linear functions of the rate ($\mu_i = \gamma_i$ and $\mu_i = 2\gamma_i$, respectively). In Table 2 where there is a steeper increase in a cost for changing the rate from γ_1 to γ_2 ($\mu_1 = \gamma_1 = 1, \mu_2 = 3$), the number of states for which rate is γ_2 is smaller than that in Table 1. For example when c_2=9, there is no state in which rate γ_2 is employed and when c_2=1, 7,8,10, there is only one state in which rate γ_2 is employed.

The case where $\gamma_n = n\gamma$ (n is an integer) can be interpreted as a system in which n service are used each serving at a rate γ. Hence, the problem of determining the number of servers to be employed as a function of the state of the system can be solved by using our model by simply taking $\gamma_n = n\gamma$

Conclusions and future research

Analysis of inventory control at service facilities is fairly recent. Most of the earlier work in this direction has been on the determination of ordering policies or on finding optimal stocking levels for a given policy. We approach the problem in a different manner. Given an ordering policy, we determine the service rates to be employed as a function of the number of customers in the queue and the amount of inventory on hand so that the long-run expected cost rate is minimized. The rationale is that quite often it is possible to control the service rate by changing the number of servers or by using a faster or slower server, whereas there may be constraints such as limited storage size and type of vendors that make it difficult to change the stocking level or the frequency of ordering. As such, determination of optimal service rates is an important problem in the service industry.

In our problem, we use an (s, S) ordering policy we used the tools of Markov decision processes to analyze the problem and linear programming to determine the optimal service rates.

The main contribution of the paper is the determination of the control policy that indicates the specific optimal service rate to be used for every possible state of the system. We made two interesting observations.

One is that the joint probability distribution of the number of customers in the waiting room, the orbit and the inventory level in system is obtained. System performance measures are computed and total expected cost rate is calculated. The second is that the parameters which influence the service rates are the customer arrival rates and waiting time costs.

References

1. S. Axsater, Simple solution for a class of two-echelon inventory problems, operations research, 38(1)(1990), 64-69.
2. Benita and M. Beamon, Supply Chain design and Analysis: Models and methods, International Journal of Production Economics. Vol. 55, No. 3(1998), 281 294
3. O. Berman, K. P Sapna , Inventory Management at service Facilities for systems with Arbitrarily Distributed Service Times, Stochastic Models, vol. 16,pp.343-360.(2000)
4. E. Cinlar, Introduction to Stochastic Processes, Prentice Hall, Englewood Cliffs, NJ, 1975.
5. J. Clark and H. Scarf, Optimal policies for a multi-Echelon Inventory problem, Management science, 6(4)(1960),475-490.

6. Elango, A continuous review perishable inventory system at service facilities, Ph.D., Thesis, Madurai Kamaraj University, 2001.
7. S. C. Graves, The Applications of queueing theory to continuous perishable inventory systems, Management science, 28(1982), 400-406.
8. S.C. Graves, The multi-Echelon inventory model for repairable item with one-for-one replenishment, Management science, 31(10)(1985), 1247-1256.
9. K. Krishnan and C. Elango, A continuous review (s, S) policy with positive lead times in Supply Chain, Proceedings of National Conference on Intelligent Optimization Modelling (2005), 245-253.
10. K. Moinzadeh and H.L. Lee, Batch size and stocking levels in multi-echelon repairable systems, Management Science, 32(12)(1986), 1567-1581 .
11. O.Berman. and E. Kim, Stochastic Inventory Policies for Inventory Management of Service Facilities, Stchastic Models, vol.15,pp.695-718(1999).
12. Mine, H. and Osaki S., Markov Decision Processes, American Elsevier Publishing Company Inc, New York (1970).
13. Puterman, M.L., Markov Decision Processes: Discrete Stochastic Dynamic Programming, Wiley Interscience Publications Inc., (2005).
14. Tijms, H. C. A First Course in Stochastic Models, John Wiley & Sons Inc., (2003).
15. Hilal Mohamed AlHamadi, N.Sangeetha, B.Sibakkumar,Optimal Control of Service parameter for a perishable inventory system maintained at service facility with impatient customers(2014).