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PERFORMANCE ANALYSIS OF TWO ECHELON PERISHABLE INVENTORY SYSTEM WITH LOST SALE

B. Amala Jasmine[1], K. Krishnan[2]
[1]Assistant Professor, PG & Research Department of Mathematics,
JeyarajAnnapackiam College for Women, Periyakulam Tamil Nadu, India -625513
[2]Assistant Professor, PG & Research Department of Mathematics,
Cardamom Planters' Association College, Bodinayakanur, Tamil Nadu, India -625513
[1] jasamala1407@gmail.com, [2] drkkmaths@gmail.com

***Abstract:** This paper presents a continuous review two echelon perishable inventory systems with two different items in stock, one is main product and other one is complement item for the main product. The operating policy at the lower echelon for the main product is (s, S) that is whenever the inventory level drops to 's'an order forQ = (S-s) items is placed, the ordered items are received after a random time which is distributed as exponential. It is also assumed that the main product is perishable in nature and it perishes with exponential rate $\gamma > 0$. The demands accruing during the stock-out period are assumed to be lost. The retailer replenishes the stock of main product from the supplier which adopts(0,M) policy. The complement product is replenished instantaneously from the local supplier.The joint probability disruption of the inventory levels of main product, complement item at retailer and the main product at supplier are obtained in the steady state case.Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of numerical examples, which provide insight into the behavior of the systemare presented.*
***Keywords:**Perishable inventory, Two-echelon,Positive lead time, Lost sale.*

1. Introduction

Study on multi-echelon systems are much less compared to those on single commodity systems. The determination of optimal policies and the problems related to a multi-echelon systems are, to some extent, dealt by Veinott and Wagner [18] and Veinott[19].Sivazlian [16] discussed the stationary characteristics of a multi commodity single period inventory system.The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain)when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw – materials, work-in process or finished goods.

The main objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature.

As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies Started with the work of Harris[9]. Clark and Scarf [7] had put forward

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the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size. One of the oldest papers in the field of continuous review multi-echelon inventory system is written by Sherbrooke in 1968.Hadley, G and Whitin, T. M.,[6], Naddor .E [14] analyses various inventory Systems.HP's(Hawlett Packard) Strategic Planning and Modeling(SPaM) group initiated this kind of research in 1977.

Sivazlian and Stanfel [17] analyzed a two commodity single period inventory system. Kalpakam and Arivarignan [10] analyzed a multi-item inventory model with renewal demands under a joint replenishment policy. They assumed instantaneous supply of items and obtain various operational characteristics and also an expression for the long run total expected cost rate. Krishnamoorthyet.al., [11] analyzed a two commodity continuous review inventory system with zero lead time. A two commodity problem with Markov shift in demand for the type of commodity required, is considered by Krishnamoorthy and Varghese [12]. They obtain a characterization for limiting probability distribution to be uniform. Associated optimization problems were discussed in all these cases. However in all these cases zero lead time is assumed.

Modeling of inventory for perishable items requires a characterization of the time to perishability. This time to perishability may be either deterministic or stochastic. The two most common models for perishability are outdatedness due to reaching expiry date (e.g., food items or medicine) and sudden perishability due to disaster (e.g., spoilage because of extreme weather conditions). The perishability due to outdatedness is typically modeled as a deterministic time to perishability and the perishability due to disaster is typically modeled as an exponential (or its discrete counterpart, geometric) time to perishability. This is because the memory-less property of these distributions often results in more tractable models. The treatment of lead time in the management of inventory for perishable items is also not trivial. One difficulty is that the items might perish during the delivery time. This might be addressed by assuming that the supplier supplies fresh items upon their delivery and changes their lead time accordingly.

Another challenge in the analysis of perishable items is that items that have not yet perished may have different remaining shelf-life. (Either because items have a unique shelf-life, or because batches with common shelf-lives arrived in different periods.) Thus, the information required to completely characterize the on-hand and on-order inventory includes not only the quantity of inventory but also the remaining shelf-lives of each unit in the inventory. This increase in information requirement makes the analysis of multi-echelon supply chains for perishable products especially challenging (because even for standard items, such an analysis often relies on dynamic programming and suffers from the curse of dimensionality). Therefore, some important theoretical contributions developed in the analysis of perishable items address the information required to characterize the inventory level process[15].

In the literature of stochastic inventory models, there are two differentassumptions about the excess demand unfilled from existing inventories:the backlog assumption and the lost sales assumption. The former ismore popular in the literature partly because historically the inventorystudies started with spare parts inventory management problems in militaryapplications, where the backlog assumption is realistic. Howeverin many other business situations, it is quite often that demand that cannot be satisfied on time is lost. This is particularly true in a competitive business environment. For example in many retail establishments, such as a supermarket or a department store, a customer chooses a competitive brand or goes to another store if his/her preferred brand is out of stock.

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All these papers deal with repairable items with batch ordering. A Complete review was provided by Benito M. Beamon[5]. Sven Axsater[1] proposed an approximate model of inventory structure in SC. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Anbazhagan and Arivarignan [2,3] have analyzed two commodity inventory system undervarious ordering policies. Yadavalliet. al., [20] have analyzed a model with joint ordering policy and varying order quantities. Yadavalliet. al., [21] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

In a very recent paper, Anbazhaganet. al. [4] considered analysis of two commodity inventory system with compliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item is also assumed when the items are out of stock. The above model is studied only at single location(Lower echelon). We extend the same in to multi-echelon structure (Supply Chain) for perishable product. The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 provides Numerical examples and sensitivity analysis.

2. Model

2.1.The Problem Description

The inventory control system considered in this paper is defined as follows. A finished perishable product is supplied from manufacturer to supplier which adopts (0,M) replenishment policy then the product is supplied to retailer who adopts (s,S) policy. The retailer also maintainan inventory of the complement product which has instantaneous replenishment from local supplier. The demand at retailer node follows an independent Poisson distributionwith rate λ_i (i = 1, 2) for main product and complement respectively. Demands accruing during the stock out periods of main product are assumed to be lost.The main product perishes with exponential rate $\gamma > 0$. The replacement of item in terms of product is made from supplier to retailer is administrated with exponential distribution having parameter $\mu > 0$.. The maximum inventory level at retailer node for main product is S, and the recorder point is s and the ordering quantity is Q(=S-s) items. The maximum inventory at supplier inM(=nQ) .

2.2.Notations and variables

We use the following notations and variables for the analysis of the paper.

Notations /variables	Used for
$[C]_{ij}$	The element of sub matrix at $(i,j)^{th}$ position of ***C***
0	Zero matrix
$\lambda_1,\ \lambda_2$	Mean arrival rate for Main& Compliment product at retailer
γ	Perishable rate for main product
μ	Mean replacement rate for main product at retailer node

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S, N	Maximum inventory level for main& Compliment product at retailer
s	reorder level for main product at retailer
M	Maximum inventory level for main product at supplier
H_m, H_c	Holding cost per item for main and compliment product at retailer
H_d	Holding cost per item for main product at distributor
O_r	Ordering cost per order for main product at retailer
O_c , O_m	Ordering cost per order for compliment and main product at retailer
I_m, I_c	Average inventory level for main and compliment product at retailer
I_d	Average inventory level for main product at retailer
R_d	Mean reorder rate for main productatsupplier.
R_c , R_m	Mean reorder rate for compliment and main productat retailer
S_m	Shortage rate for main product at retailer
T_m	Penalty rate for main product at retailer
$\sum_{i=Q}^{nQ} i$	Q + 2Q + 3Q + ... + nQ

3. Analysis

Let $I_m(t)$ and $I_c(t)$ denote the on hand Inventory levels of Main product, Compliment product at retailer and $I_d(t)$ denote the on hand inventory level of Main product at supplier at time t.

We define I (t) = $\{(I_m(t), I_c(t), I_d(t)) : t \geq 0\}$ as Markov process with state space E = { (i, j,k) | i = 0, ...S, j =1, 2, …, N, k=Q,2Q,.....nQ }. Since E is finite and all its states are aperiodic, recurrent, non- null and also irreducible. That is all the states are Ergodic. Hence the limiting distribution exists and is independent of the initial state.

The infinitesimal generator matrix of this process **C** = $(a(i, j, k, :l, m, n))_{(i,j,k)(l,m,n) \in E}$ can be obtained from the following arguments.

- The arrival of a demand or perish or an itemfor main product at retailer make a state transition in the Markov process from (i, j, k) to (i-1, j-1, k) with the intensity of transition λ_1.
- The perish or an itemfor main product at retailer make a state transition in the Markov process from (i, j, k) to (i-1, j, k) with the intensity of transition iγ.
- The arrival of a demand for compliment product at retailer make a state transition in the Markov process from (i, j, k) to (i, j-1, k) with the intensity of transition $\lambda_2 > 0$.
- The replacement of inventory atretailer make a state transition in the Markov process from (i, j, k) to (i+Q, j, k-Q) or (i, j, Q) to (i+Q, j, nQ) with the intensity of transition $\mu > 0$.

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The infinitesimal generator **C** is given by

$$\mathbf{C}=\begin{bmatrix} A & B & 0 & \cdots & 0 & 0 \\ 0 & A & B & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & B \\ B & 0 & 0 & \cdots & 0 & A \end{bmatrix}$$

The entries of C are given by

$$[C]_{pq}=\begin{cases} A & p=q; \quad q=nQ, \quad (n-1)Q,\ldots Q \\ B & p=q+Q; \quad q=(n-1)Q,\ldots.Q \\ B & p=q-(n-1)Q \quad q=nQ \\ 0 & otherwise \end{cases}$$

Where the entries of the matrices are given by

$$[A]_{ij}=\begin{cases} A_1 & i=j; \quad i=1,2,\ldots N \\ A_2 & i+1=j; \quad i=1,2,\ldots(N-1) \\ A_2 & i-(N-1)=j; \quad i=N \\ 0 & otherwise \end{cases} \qquad [B]_{ij}=\begin{cases} B_1 & i=j; i=1,2,\ldots,N \\ 0 & otherwise \end{cases}$$

The elements in the sub matrices of A and B are

$$[A_1]_{ij}=\begin{cases} -(\lambda_1+i\gamma+\lambda_2) & if\ \ i=j\ :i=S,S-1,\ldots\ldots.(s+1) \\ -(\lambda_1+i\gamma+\lambda_2+\mu) & if\ \ i=j\ :i=s,s-1,\ldots\ldots.1 \\ -(\lambda_2+\mu) & if\ \ i=j\ :i=0 \\ 0 & \text{otherwise} \end{cases}$$

$$[A_2]_{ij}=\begin{cases} \lambda_2 & if\ \ i=j \quad :i=S,S-1,\ldots\ldots.1,0 \\ \lambda_1+i\gamma & if\ \ i+1=j\ :i=S,S-1,\ldots\ldots.1 \\ 0 & \text{otherwise} \end{cases}$$

$$[B_1]_{ij}=\begin{cases} \mu & if\ \ i+Q=j\ :i=s,s-1,\ldots\ldots.1,0 \\ 0 & \text{otherwise} \end{cases}$$

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3.1.Steady State Analysis

The structure of the infinitesimal matrix C, reveals that the state space E of the Markov process { I (t) : t ≥ 0 } is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$$\prod_{i,j}^{k} = \lim_{t\to\infty} Pr\{(I_m(t), I_c(t), I_d(t) = (i,j,k))\}$$

where $\prod_{i,j}^{k}$ *is* the steady state probability that the system be in state *(i, j, k*).

Let $\prod = \{\prod_{i,j}^{nQ}, \prod_{i,j}^{(n-1)Q}, \ldots\ldots\ldots \prod_{i,j}^{Q}\}$denote the steady state probability distribution. For each (

(i, j, k) ,$\prod_{i,j}^{k}$ *can* be obtained by solving the matrix equation $\prod$ C = 0.

The system of equations may be written as follows

i.	$\prod_j^{nQ} A_1 + \prod_i^{nQ} A_2 + \prod_j^{Q} B_1 = 0$	*i = 1; j = S*
ii.	$\prod_j^{nQ} B_1 + \prod_j^{(n-1)Q} A_1 + \prod_i^{(n-1)Q} A_2 = 0$	*i = 1; j = S*
iii.	$\prod_j^{(n-1)Q} B_1 + \prod_j^{Q} A_1 + \prod_i^{Q} A_2 = 0$	*i = 1; j = S*
iv.	$\prod_j^{nQ} A_2 + \prod_{i-1}^{nQ} A_1 + \prod_{i-1}^{Q} B_1 = 0$	*i = S, S-1,……..,2*
v.	$\prod_{i-1}^{nQ} B_1 + \prod_i^{(n-1)Q} A_2 + \prod_{i-1}^{(n-1)Q} A_1 = 0$	*i = S, S-1,……..,2*
vi.	$\prod_{i-1}^{(n-1)Q} B_1 + \prod_i^{Q} A_2 + \prod_{i-1}^{Q} A_1 = 0$	*i = S, S-1,……..,2*

By solving the above system of equations, together with normalizing condition $\sum_{(i,j,k)\in E} \prod_{i,j}^{k} = 1$,

the steady probability of allthe system states are obtained.

4. Operating Characteristic

In this section we derive some important system performance measures.

4.1. Average inventory Level

The event I_m, I_c *and* I_d denote the average inventory level for main product, complement product at retailer and main product at distributor respectively,

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(i) $I_m = \sum_{k=Q}^{nQ} \sum_{j=1}^{N} \sum_{i=0}^{S} i.\prod_{i,j}^{k}$ $\quad I_c = \sum_{k=Q}^{nQ} \sum_{i=0}^{S} \sum_{j=1}^{N} j.\prod_{i,j}^{k}$ $\quad I_d = \sum_{i=0}^{S} \sum_{j=1}^{N} \sum_{k=Q}^{nQ} k.\prod_{i,j}^{k}$

4.2. Mean Reorder Rate

Let R_m, R_c, *and* $R_d$be the mean reorder rate for main product, complement product at retailer and main product at distributorrespectively,

(i) $R_m = (\lambda_1 + (s+1)\gamma) \sum_{k=Q}^{nQ} \sum_{j=1}^{N} \prod_{s+1,j}^{k}$

(ii) $R_c = (\lambda_1 + \lambda_2) \sum_{k=Q}^{nQ} \sum_{i=0}^{S} \prod_{i,1}^{k}$ $\quad R_d = \mu \sum_{i=0}^{S} \sum_{j=1}^{N} \prod_{i,j}^{Q}$

4.3. Shortage rate

Shortage occur at retailer only for main product. LetS_m be the shortage rate at retailer for main product, then

(i) $S_m = \lambda_1 \sum_{k=Q}^{nQ} \sum_{j=1}^{N} \prod_{0,j}^{k}$

5. Cost Analysis

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time. The long run expected cost rate *TC(s, Q)*is given by

$$TC(s, Q) = I_m.\, H_m + I_c.H_c + I_d.H_d + R_m.\, O_m + R_c.\, O_c + R_d.O_d + S_m.\, T_m$$

Although we have a not proved analytically the convexity of the cost function *TC(s, Q)*our experience with considerable numberof numerical examples indicate that *TC(s, Q)*for fixed '*S*' appears to be convex in s. In some cases it turned out to be increasing function of s. For large number case of *TC(s, Q)*revealed a locally convex structure. Hence we adopted the numerical search procedure to determine the optimal value of '*s*'

6. Numerical Example and Sensitivity Analysis

6.1.Numerical Example

In this section we discuss the problem of minimizing the structure. We assume $H_c \le H_m \le H_d$, i.e, the holding cost for compliment product is at retailer node is less than that of main product at retailer node and the holding cost of main product is less than that of main product at distributor node. Also $O_c \le O_m \le O_d$ the ordering cost at retailer node for compliment product is less than that of main product. Also the ordering cost at the distributor is greater thanthat of compliment product at retailer node.

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The results we obtained in the steady state case may be illustrated through the following numerical example,

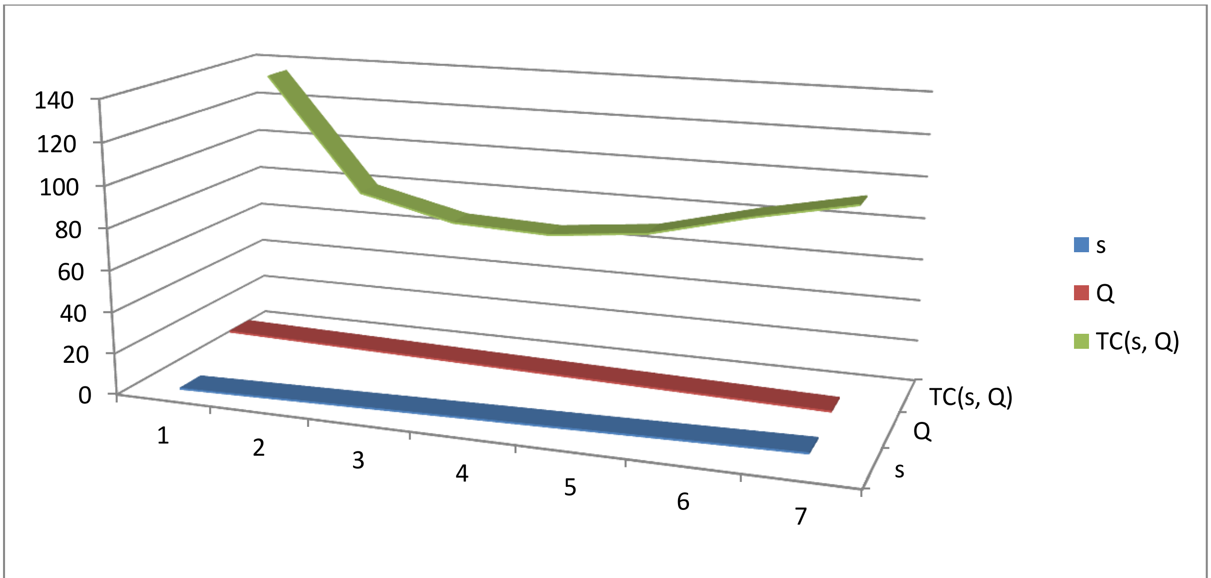
S =16, N=15, M = 80, $\lambda_1 = 3$, $\lambda_2 = 2$, γ = 2, $\mu = 3$ $H_c = 1.1$, $H_m = 1.2$, $H_d = 1.3$ $O_c = 2.1$, $O_m = 2.2$, $O_d = 2.3$ $T_m = 3.1$

The cost for different reorder level are given by

S	1	2	3	4*	5	6	7
Q	15	14	13	12*	11	10	9
TC(s, Q)	129.97132	75.5903	64.679	62.7863*	67.3499	78.8514	88.3332

Table:1. Total expected cost rate as a function s and Q

For the inventory capacity S, the optimal reorder level s* and optimal cost *TC(s, Q)* are indicated by the symbol *. The Convexity of the cost function is given in the graph.



6.2.Sensitivity Analysis

Below tables are represented a numerical study to exhibit the sensitivity of the system on the effect of varying different parameters.

$$\lambda_1 \& \mu, \lambda_2 \& \mu, H_c \& H_m; H_m \& H_d, O_m \& O_c, O_m \& O_d;$$

For the following cost structure S =16, s = 4, N=15, M = 80, $\lambda_1 = 3$, $\lambda_2 = 2$, $\mu = 3$ $H_c = 1.1$, $H_m = 1.2$, $H_d = 1.3$, $O_c = 2.1$, $O_m = 2.2$, $O_d = 2.3$ $T_m = 3.1$.

Table:2 Effect on Replenishment rate& Demand rates $\mu \backslash \lambda_1$

$\mu \backslash \lambda_1$	**1**	**2**	**3**	**4**	**5**
1	46.754448	109.9654	190.3959	272.4966	354.8979
2	66.75968	83.89493	137.827	212.5022	292.6373
3	87.004528	104.8954	125.788	174.3123	243.6096
4	103.894856	132.886	147.3295	171.9827	218.7193
5	119.20896	158.053	178.6075	194.5684	222.0951

Table:3 Effect on Replenishment rate& Demand rates $\mu \backslash \lambda_{2,}$

$\mu \backslash \lambda_{2,}$	**1**	**3**	**5**	**7**	**9**
1	274.988688	194.4675	234.6323	320.7495	403.7051
3	271.628656	167.6407	178.724	238.783	296.9803
5	270.928528	161.355	164.9565	218.3542	270.3033
7	270.624952	158.5594	158.7321	209.092	258.1966
9	270.459904	156.9776	155.1857	203.7984	251.2692

Table 4: Effect on Holding cost $(H_m \backslash \mathrm{H}_{d,})$

$H_m \backslash \mathrm{H}_{d,}$	**1.1**	**2.1**	**3.1**	**4.1**	**5.1**
1.1	174.437744	176.7137	178.8124	180.9132	183.013
2.1	178.80408	180.9049	182.9974	185.1065	187.2042
3.1	182.99736	185.0982	187.1958	189.2966	191.3974
4.1	187.18804	189.281	191.3891	193.4868	195.5876
5.1	191.38132	193.4795	195.5803	197.6801	199.7809

Table 5: Effect on Ordering Cost $(H_c \backslash \mathrm{H}_{m,})$

$H_c \backslash \mathrm{H}_{m,}$	**2.1**	**2.2**	**2.3**	**2.4**	**2.5**
2.1	174.437744	175.0315	175.5291	176.0782	176.6232
2.2	175.03148	175.58	176.1261	166.2825	177.2202
2.3	175.62792	176.1763	176.722	177.268	177.8161
2.4	176.224256	176.1763	177.319	177.8639	178.413
2.5	176.820592	177.3664	177.9149	178.4609	179.0069

Table 6 : Effect on Penalty Cost $(O_m \,\&\, \mathrm{O}_d)$

$O_m \backslash \mathrm{O}_d$	**3.1**	**3.2**	**3.3**	**3.4**	**3.5**
3.1	174.437744	175.032	175.5291	176.0782	176.6232
3.2	175.03148	175.5801	176.1261	166.2825	177.2202
3.3	175.62792	176.176	176.722	177.268	177.8161
3.4	176.224256	176.176	177.319	177.8639	178.413
3.5	176.820592	177.3668	177.9149	178.4609	179.0069

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Table 7: effect on Holding cost & ordering cost(O_m & $O_{d;}$)

$O_m \backslash O_{d;}$	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**
1.1	172.843008	173.2879	173.706	174.1262	174.5442
1.2	173.529928	173.9556	174.3685	174.7897	175.2067
1.3	174.195528	174.6139	175.032	175.4522	175.8702
1.4	174.858424	175.2764	175.6976	176.1146	176.5327
1.5	175.521424	175.9389	176.3601	176.7782	177.1962

It is observed that from the table,the total expect cost *TC(s, Q)*increases with the other cost increases.In some cases the cost increses exponentially but in some case it increses gradually depending upon the cost parameter.

Conclusion

This paper deals with a two echelon perishable Inventory system with two products namely main and compliment product. The demand at retailer node follows independent Poisson with rate λ_1 for main product λ_2 for compliment product.If the demand occur for the perishablemain product then it is also the demand for the compliment product. But the compliment product demand do not disturb the main product.The structure of the chain allows vertical movement of goods from to supplier to Retailer. If there is no stock for main product at retailer the demand is refused and it is treated as lost sale. The model is analyzed within the framework of Markov processes. Joint probability distribution of inventory levels at DC and Retailer for both products are computed in the steady state. Various system performance measures are derived and the long-run expected cost is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values on the total expected cost rate.It would be interesting to analyze the problem discussed in this paper by relaxing the assumption of exponentially distributed lead-times to a class of arbitrarily distributed lead-times using techniques from renewal theory and semi-regenerative processes. Once this is done, the general model can be used to generate various special eases.

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