



SOLVING ASSIGNMENT PROBLEM BY USING TRAPEZOIDAL NEUTROSOPHIC NUMBER

K.Priya¹, and P.Raja Pushpam²

¹Department of Mathematics,
Idhaya college for women,
Affiliated to Bharathidasan University,
Kumbakonam. Tamil Nadu, India

²Department of Mathematics,
JayarajAnnappackiam College for Women(Autonomous),
Affiliated to Mother Teresa Womens' University,
Periyakulam.Theni District, Tamilnadu, India
happypriya18@gmail.com rajapshpm@gmail.com

Abstract: Assignment problem is one of the fundamental kind of Linear programming problem. In this paper, the cost value of the assignment problem are considered as Neutrosophic numbers. The approach is illustrated by a numerical example.

AMS Subject Classification: 90B06, 90C08, 90C70

Keywords: Assignment problem, Trapezoidal Neutrosophic numbers.

I. Introduction

A neutrosophic is a more general platform, which can be used to present uncertainty, imprecise, incomplete and inconsistent. F.Smarandache. [10] proposed in decision making process, the neutrosophic weighted aggregation operators (arithmetic and geometric operators) are adopted to aggregate the neutrosophic information related to each alternative. Thus, we can rank all alternatives and make the selection of the best one according to score-accuracy functions.

Any generalization of fuzzy set failed to handle problems with indeterminate or inconsistent information. To overcome this, H.Wang, F.Smarandache. Q. Zhang [14], introduced neutrosophic sets as a classical sets, fuzzy sets, and intuitionistic fuzzy sets as an extension of classical sets, fuzzy sets and intuitionistic fuzzy sets. The components of neutrosophic set, namely, truth membership degree, indeterminacy-membership degree, and falsity-membership degree, were suitable to represent indeterminacy and inconsistent information.

Though many researchers worked on assignment problem in fuzzy environment. R.E.Bellman and Zadeh [1] was introduced a Decision making in fuzzy environment. F.Smarandache., [10] proposed a neutrosophic logic, by adding another independent membership function named as indeterminacy-membership $I(x)$ along with truth-membership $T(x)$ and Falsity-membership functions $F(x)$.

R.Vidhya, R.Irene Hepzibah and A.Nagoorgani [13] and P.A.Thakre, D.S.Shelar [11] proposed a multi objective linear programming problems. Also, A.Thamaraiselvi. and R.Santhi. [12] discussed a new approach for Transportation problem in neutrosophic environment. S.L.Gauss [4] solved the transportation problem. M.OhEigeartaigh [8] discussed fuzzy transportation algorithm. Rittik Roy and Pintu Das [9] discussed Neutrosophic Linear Programming.

K.Kadhirvel, K.Balamurugan [5] solved a Assignment problem using Trapezoidal fuzzy number. S.Chanas and D.Kuchta [2] introduced a concept of optimal solution of the transportation problem with fuzzy cost co-efficients. R.R.Yager [15] proposed a procedure for ordering fuzzy subsets of the unit interval. DalbinnderKour and KajlaBasu [3] extended a Fuzzy programming Technique in real transportation problem.



The aim of this paper to obtain optimal cost in neutrosophic environment. This paper is well organized as follows. In section II the basic concept of neutrosophic set, neutrosophic number are reviewed. In section III, the solution of algorithm are developed for solving trapezoidal neutrosophic numbers. In section IV the numerical example and the results are interpreted. Finally, Section V concludes the paper with future work.

II. Preliminaries

Definition 2.1

Let X be a non empty set. Then a neutrosophic set \bar{A}^N of X defined as \bar{A}^N of X is defined as ,
 $\bar{A}^N = \{ \langle x, T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), F_{\bar{A}^N}(x) \rangle, x \in X, T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), F_{\bar{A}^N}(x) \in]^{-}0, 1^{+}[\}$ where $T_{\bar{A}^N}(x)$, $I_{\bar{A}^N}(x)$, are $F_{\bar{A}^N}(x)$ are truth membership function, an indeterminacy –membership function, and a falsity –membership function and there is no restriction on the sum of $T_{\bar{A}^N}(x)$, $I_{\bar{A}^N}(x)$, and $F_{\bar{A}^N}(x)$, so $^{-}0 \leq T_{\bar{A}^N}(x) + I_{\bar{A}^N}(x) + F_{\bar{A}^N}(x) \leq 3^{+}$ and $]^{-}0, 1^{+}[$ is a nonstandard unit interval.

But it is difficult to apply neutrosophic set theories in real life problems directly. So, Wang introduced single-valued neutrosophic set as a subset of neutrosophic set and the definition as follows.

Definition 2.2

Let X be a non empty set. Then a single valued neutrosophic set \bar{A}_s^N of X is defined as
 $\bar{A}_s^N = \{ \langle x, T_{\bar{A}_s^N}(x), I_{\bar{A}_s^N}(x), F_{\bar{A}_s^N}(x) \rangle, x \in X, T_{\bar{A}_s^N}(x), I_{\bar{A}_s^N}(x), F_{\bar{A}_s^N}(x) \in [0, 1]$ for each $x \in X$ and $0 \leq T_{\bar{A}_s^N}(x) + I_{\bar{A}_s^N}(x) + F_{\bar{A}_s^N}(x) \leq 3$

Definition 2.3

Let $r_{\bar{a}}, s_{\bar{a}}, t_{\bar{a}} \in [0, 1]$ be any real numbers A single valued neutrosophic number,
 $\bar{a} = \langle (a_1, b_1, c_1, d_1); r_{\bar{a}}, (a_2, b_2, c_2, d_2); s_{\bar{a}}, (a_3, b_3, c_3, d_3); t_{\bar{a}} \rangle$ is represented as a special single valued neutrosophic set on the set of real numbers R , and the truth membership function $\mu_{\bar{a}}: R \rightarrow [0, r_{\bar{a}}]$, a indeterminacy membership function $\nu_{\bar{a}}: R \rightarrow [\mu_{\bar{a}}, 1]$, and a falsity-membership function $\lambda_{\bar{a}}: R \rightarrow [t_{\bar{a}}, 1]$ is given by

$$\mu_{\bar{a}}(x) = \begin{cases} f_{\mu l}(x), & a_1 \leq x \leq b_1 \\ r_{\bar{a}}, & b_1 \leq x \leq c_1 \\ f_{\mu r}(x), & c_1 \leq x \leq d_1 \\ 0, & \text{otherwise} \end{cases}$$

$$\nu_{\bar{a}}(x) = \begin{cases} f_{\nu l}(x), & a_2 \leq x < b_2 \\ s_{\bar{a}}, & b_2 \leq x < c_2 \\ f_{\nu r}(x), & c_2 \leq x < d_2 \\ 1, & \text{otherwise} \end{cases}$$

$$\lambda_{\bar{a}}(x) = \begin{cases} f_{\lambda l}(x), & a_3 \leq x < b_3 \\ t_{\bar{a}}, & b_3 \leq x < c_3 \\ f_{\lambda r}(x), & c_3 \leq x < d_3 \\ 1, & \text{otherwise} \end{cases}$$

respectively, where $r_{\bar{a}}, s_{\bar{a}}$, and $t_{\bar{a}}$ denote the maximum truth-membership degree, minimum indeterminacy degree falsity-membership degree respectively. where the functions

$f_{\mu l}(x): [a_1, b_1] \rightarrow [0, r_{\bar{a}}]$, $f_{\nu r}: [c_2, d_2] \rightarrow [s_{\bar{a}}, 1]$,

$f_{\lambda r}: [c_3, d_3] \rightarrow [t_{\bar{a}}, 1]$ are continuous and non decreasing, and satisfy the conditions:

$f_{\mu l}(a_1) = 0$, $f_{\mu l}(b_1) = r_{\bar{a}}$, $f_{\nu r}(c_2) = s_{\bar{a}}$, $f_{\nu r}(d_2) = 1$, $f_{\lambda r}(c_3) = t_{\bar{a}}$ and $f_{\lambda r}(d_3) = 1$; the functions

$f_{\mu r}: [c_1, d_1] \rightarrow [0, r_{\bar{a}}]$, $f_{\nu l}(x): [a_2, b_2] \rightarrow [s_{\bar{a}}, 1]$ and $f_{\lambda l}: [a_3, b_3] \rightarrow [t_{\bar{a}}, 1]$ are continuous and non increasing, and satisfy the conditions:

$f_{\mu r}(b_2) = 0$, $f_{\mu r}(d_1) = r_{\bar{a}}$, $f_{\nu l}(c_2) = s_{\bar{a}}$, $f_{\nu l}(d_2) = 1$, $f_{\lambda l}(c_3) = t_{\bar{a}}$, and $f_{\lambda l}(d_3) = 1$; and a_1, d_1 are called the mean interval and the lower and upper limits of the general neutrosophic number \bar{a} for the truth-membership function, respectively.

$[b_2, c_2]$, c_2 and d_2 are called the mean interval and the lower and upper limits of the general neutrosophic number \bar{a} for the indeterminacy-membership function, respectively.



$[b_3, c_3]$, a_3 and d_3 are called the mean interval and the lower and upper limits of the general neutrosophic number \tilde{a} for the falsity- membership function, respectively.

$r_{\tilde{a}}$, $s_{\tilde{a}}$ and $t_{\tilde{a}}$ are called the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree, respectively.

Obviously, the single valued neutrosophic numbers are a generalization of the intuitionistic fuzzy numbers. Thus, the neutrosophic number may express more uncertainty information than the intuitionistic fuzzy number.

For some specific values of the parameters $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2, a_3, b_3, c_3, d_3, r_{\tilde{a}}, s_{\tilde{a}}$ and $t_{\tilde{a}}$, we can be further constructed some particular forms of neutrosophic number.

Definition 2.4

Arithmetic operators:

Let $\tilde{a} = \langle (a_1, b_1, c_1, d_1); r_{\tilde{a}}, s_{\tilde{a}}, t_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (a_2, b_2, c_2, d_2); r_{\tilde{b}}, s_{\tilde{b}}, t_{\tilde{b}} \rangle$ be the two single valued trapezoidal neutrosophic numbers and $\gamma \neq 0$ then,

$$1. \tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); r_{\tilde{a}} \wedge r_{\tilde{b}}, s_{\tilde{a}} \vee s_{\tilde{b}}, t_{\tilde{a}} \vee t_{\tilde{b}} \rangle$$

$$2. \tilde{a} - \tilde{b} = \langle (a_1 + d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); r_{\tilde{a}} \wedge r_{\tilde{b}}, s_{\tilde{a}} \vee s_{\tilde{b}}, t_{\tilde{a}} \vee t_{\tilde{b}} \rangle$$

Definition 2.5

Let $\tilde{a} = \langle (a_1, b_1, c_1, d_1); r_{\tilde{a}}, s_{\tilde{a}}, t_{\tilde{a}} \rangle$ is a single valued neutrosophic number. Then

$$S(\tilde{a}) = \frac{1}{16} (a_1 + b_1 + c_1 + d_1) \times (2 + r_{\tilde{a}} - s_{\tilde{a}} - t_{\tilde{a}}) \text{ and}$$

$$A(\tilde{a}) = \frac{1}{16} (a_1 + b_1 + c_1 + d_1) \times (2 + r_{\tilde{a}} - s_{\tilde{a}} + t_{\tilde{a}}) \text{ are called the score and accuracy degrees of } \tilde{a} \text{ respectively.}$$

III. Fuzzy assignment problem

There are n jobs for a factory and the factory has n machines to process the jobs. A job $i (=1, 2, \dots, n)$ when processed by the machine $j (=1, 2, \dots, n)$ is assumed to incur a cost C_{ij} . Let C_{ij} be the fuzzy cost of assigning i th machine to the j th job. Let y_{ij} be the decision variable denoting the assignment of the i th machine to j th job. Determine an assignment of jobs to machines so as to minimize the overall cost.

Fuzzy assignment problem is given by,

$$\text{Min } Z = \sum_{i=1}^n \sum_{j=1}^n C_{ij} y_{ij}$$

Subject to

$$\sum_{i=1}^n y_{ij} = 1 \text{ for } j=1, 2, \dots, n$$

$$\sum_{j=1}^n y_{ij} = 1 \text{ for } i=1, 2, \dots, n, y_{ij} \in [0, 1]$$

$$\text{where } y_{ij} = \begin{cases} 1 & \text{if the } i \text{th machine is assigned to } j \text{th job} \\ 0 & \text{if the } i \text{th machine is not assigned to that } j \text{th job} \end{cases}$$

Procedure

Step 1: Check the assignment problem is balanced or not

a) if balanced go to step 2

b) if not balanced go to step 3

Step 2: Use Hungarian assignment problem method and get optimal solution.

Step 3: Adding dummy row or column with cost value as zero to make balance one and go to step 2.

IV. Numerical example

Consider the problem of assigning 4 jobs to 4 persons. The assignment cost are given as neutrosophic numbers.



	Job 1	Job 2	Job 3	Job 4
A_1	$\langle(18,19,20,21);0.8,0.6,0.3\rangle$	$\langle(7,8,9,10);0.4,0.6,0.8\rangle$	$\langle(16,17,18,19);0.5,0.6,0.2\rangle$	$\langle(17,18,19,20);0.6,0.5,0.4\rangle$
A_2	$\langle(11,12,13,14);0.2,0.1,0.2\rangle$	$\langle(9,10,11,12);0.5,0.7,0.4\rangle$	$\langle(8,9,10,11);0.5,0.7,0.5\rangle$	$\langle(16,17,18,19);0.5,0.6,0.2\rangle$
A_3	$\langle(10,11,12,13);0.6,0.3,0.6\rangle$	$\langle(17,18,19,20);0.6,0.5,0.4\rangle$	$\langle(1,2,3,4);0.5,0.4,0.8\rangle$	$\langle(17,18,19,20);0.6,0.5,0.4\rangle$
A_4	$\langle(16,17,18,19);0.5,0.6,0.2\rangle$	$\langle(11,12,13,14);0.2,0.1,0.2\rangle$	$\langle(18,19,20,21);0.8,0.6,0.3\rangle$	$\langle(9,10,11,12);0.5,0.7,0.4\rangle$

After doing step 1, problem is solved by Hungarian method. The assignment schedule is as follows.

	Job 1	Job 2	Job 3	Job 4
A_1	$\langle(2,6,10,14);0.2,0.7,0.8\rangle$	$\langle(0,0,0,0);0.5,0.3,0.5\rangle$	$\langle(6,8,10,12);0.4,0.6,0.3\rangle$	$\langle(7,9,11,13);0.4,0.6,0.8\rangle$
A_2	$\langle(0,0,0,0);0.5,0.3,0.5\rangle$	$\langle(-2,0,2,4);0.5,0.7,0.5\rangle$	$\langle(0,0,0,0);0.5,0.3,0.5\rangle$	$\langle(5,7,9,11);0.5,0.7,0.5\rangle$
A_3	$\langle(0,4,8,12);0.2,0.7,0.8\rangle$	$\langle(13,15,17,19);0.5,0.5,0.8\rangle$	$\langle(0,0,0,0);0.5,0.3,0.5\rangle$	$\langle(13,15,17,19);0.5,0.5,0.8\rangle$
A_4	$\langle(-2,2,6,10);0.2,0.7,0.5\rangle$	$\langle(-1,1,3,5);0.2,0.7,0.4\rangle$	$\langle(6,8,10,12);0.5,0.7,0.4\rangle$	$\langle(0,0,0,0);0.5,0.3,0.5\rangle$

Examine the rows and columns successively until a row / column with exactly one marked zero.

No unmarked zero is left. Hence assignment schedule is A_1 goes to job 2, A_2 goes to job1, A_3 goes to job 3, A_4 goes to job 4.

Hence the minimum total neutrosophic cost is $\langle(28,32,36,40);0.2,0.7,0.8\rangle$.

V. Conclusion

In this paper the assignment cost has been considered as neutrosophic numbers which are more realistic and general in nature. It is always possible to get an optimal solution for a given fuzzy assignment problem whether maximize or minimize objective function. This approach will be effective in assignment problems involving data. In future the approach of assignment problem may be extended to neutrosophic logic.

References

- [1] R.E.Bellman, and L.A.Zadeh, "Decision making in a fuzzy environment", *Management Science*, 17, 1970, 141-164,
- [2] S.Chanas and D. Kuchta , "A concept of the optimal solution of the transportation problem with fuzzy cost coefficients" , *Fuzzy sets and systems*. Vol 82. 1996,pp. 299-305.
- [3] DalbinderKour and KajlaBasu, "Application of Extended Fuzzy Programming Technique to a real life Transportation Problem in Neutrosophic environment, Neutrosophic Sets andSystems", Vol. 10, 2015, 74-87(P).



AIJRSTEM is a refereed, indexed, peer-reviewed, multidisciplinary and open access journal published by
International Association of Scientific Innovation and Research (IASIR), USA
(An Association Unifying the Sciences, Engineering, and Applied Research)

- [4] S.L.Gauss, "On solving the transportation problem" Journal of operation Research Society, Vol.41, 1990.pp.291-297.
- [5] K.Kadhirvel, K.Balamurugan, "Method for solving Hungarian Assignment Problems using Triangular and Trapezoidal Fuzzy Number", International journal of Engineering Research and Applications, Vol.2(5), 2012.pp. 399-403.
- [6] P.Majumdr and S.K.Samanta "On similarity and entropy of neutrosophic sets", J.Intell .Fuzzy syst. Vol.26(3),2014, pp.1245-1252.
- [7] Matteo Brunelli, JozsefMezei, "How different are ranking methods for fuzzy numbers?A numerical study" 0888-613X/\$, 54(2013),pp.627-639 doi:10.1016/j.ijar 2013.01.009.
- [8] M.OhEigartaigh, "A fuzzy transportation algorithm" Fuzzy sets and systems, Vol.8,1982,no.3,pp-235-243.
- [9] Rittik Roy and Pintu Das, "A Multi-Objective Production Planning Problem Based on NeutrosophicLinear Programming approach", J.Intern.. Fuzzy Mathematical Archive, Vol. 8, No. 2, 2015, 81-91,ISSN: 2320 –3242 (P), 2320 –3250 (online).
- [10] F.Smarandache, "Neutrosophic set, a generalization of the intuitionistic fuzzy sets", Int. J.PureAppl.Math.24: 2005, 287-297.
- [11] P.A.Thakre, D.S. Shelar and S.P.Thakre "Solving fuzzy Linear Programming problem as multi-objective linear programming problem" Journal of Engineering and Technology, Vol.2(5) 2009,p.82-85.
- [12] A.Thamaraiselvi. and R.Santhi., "A new Approach for Optimization of real life transportation problem Neutrosophic environment", Vol 2016,Article ID 5950747, 2016, 9 pages .
- [13] R.Vidhya, R.Irene Hepzibah. and A.Nagoorgani. "A Neutrosophic Multi Objective Linear Programming Problems". ISSN 0973-1768 Vol.13,Number 2(2017) 265-271.
- [14] H.Wang, F.Smarandache,Q.Zhang. and R.Sunderraman., "Single valued neutrosophic sets.Multispace and multistructure" vol 4, 2010, pp.410-413.
- [15] R.R.Yager, "A procedure for ordering fuzzy subsets of the unit interval" Information Sciences, Vol.24, 1981, pp.143-161.