SOLVING FUZZY LINEAR PROGRAMMING PROBLEM USING MATLAB

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Abstract : The linear programming problem is one of the most popular approach used in Operation research and Decision making. So the research into this area is very useful for the society. In real life situation uncertainty arises in decision making problem due to either lack of knowledge or inherent vagueness. Different Ranking Techniques will be used for defuzzification. In this paper, fuzzy linear programming problem can be transform to crisp linear programming problem by average ranking technique and MATLAB coding has been applied to find an optimal solution. Numerical example is provided to illustrate the method.

IndexTerms - Fuzzy linear programming, Triangular fuzzy numbers, Ranking, Optimal solution, MATLAB

I. INTRODUCTION

In a fuzzy decision making problem, the concept of maximizing decision was introduced by Bellman and Zadeh [5] (1970). It is concerned with the optimization of a linear function while satisfying a set of linear equality and/ or inequality constraints or restrictions. In the present practical situations, the available information in the system under consideration are not exact, therefore fuzzy linear programming was introduced. Fuzzy set theory has been applied to many disciplines such as control theory, management sciences, mathematical modeling and industrial applications. Campos and Verdegay [8] proposed a method to solve LP problems with fuzzy coefficients in both matrix and right hand side of the constraint.
Cadenas and Verdegay [7] solved a LP problem in which all its elements are defined as fuzzy sets. Buckley and Feuring [6] proposed a method to find the solution for a fully fuzzified linear programming problem by changing the objective function into a multi objective LP problem. Ganesan and Veeramani [10] proposed an approach for solving FLP problem involving symmetric trapezoidal fuzzy numbers without converting it into crisp LP problems. Jimenez et al. [12] developed a method using fuzzy ranking method for solving LP problems where all the coefficients are fuzzy numbers . Allahviranloo et al. [1] solved fuzzy integer LP problem by reducing it into two crisp integer Amit Kumar et al. [3, 4] proposed a method for solving the FLP problems by using fuzzy ranking function in the fuzzy objective function. Jayalakshmi and Pandian[11] proposed a bound and decomposition method to find an optimal
fuzzy solution for fully fuzzy linear programming (FFLP) problems .

2. Preliminaries

2.1 Definition

Let A be a classical set $\mu_A(x)$ be a real valued function defined from R into [0, 1]. A fuzzy set A^* with the function $\mu_A(x)$ is defined by $A^* = \{(x, \mu_A(x)) : x \varepsilon A \, and \; \mu_A(x) \varepsilon [0,1]\}$. The function $\mu_A(x)$ is known as the membership function of A^*.

2.2 Definition:

Given a fuzzy set A defined on X and any number $\alpha \varepsilon [0,1]$, the $\alpha -$cut, α_A, is the crisp set $\alpha_A = \{x / A(x) \geq \alpha\}$.

2.3 Definition:

Given a fuzzy set A defined on X and any number $\alpha\varepsilon[0,1]$, the strong $\alpha-$cut, $\alpha+_A$, is the crisp set $\alpha+_A=\{x/A(x)>\alpha\}$.

2.4 Definition:

A fuzzy number $\tilde{A}$ in R is said to be a triangular fuzzy number if its membership function $\mu_{\tilde{A}}:R\to[0,1]$ has the following characteristics.

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & a_1\le x\le a_2\\ 1 & x=a_2\\ \dfrac{a_3-x}{a_3-a_2} & a_2\le x\le a_3\\ 0 & otherwise\end{cases}$$

2.5 Definition

Let $(a_1,a_2,a_3)\, and\, (b_1,b_2,b_3)$ be two triangular fuzzy numbers. Then

1) $(a_1,a_2,a_3)\oplus(b_1,b_2,b_3)=(a_1+b_1,a_2+b_2,a_3+b_3)$
2) $k(a_1,a_2,a_3)=(ka_1,ka_2,ka_3)\ for\ k\ge 0$
3) $k(a_1,a_2,a_3)=(ka_3,ka_2,ka_1)\ for\ k<0$
4) $(a_1,a_2,a_3)\otimes(b_1,b_2,b_3)=\begin{cases}(a_1b_1,a_2b_2,a_3b_3) & for\ a_1\ge 0\\ (a_1b_3,a_2b_2,a_3b_3) & for\ a_1<0,a_3\ge 0\\ (a_1b_3,a_2b_2,a_3b_1) & for\ a_3<0\end{cases}$

2.6 Definition:

Let $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ be in F(R), then

1) $\tilde{A}<\tilde{B}\ if\ and\ only\ if\ \ R(\tilde{A})<R(\tilde{B}).$
2) $\tilde{A}>\tilde{B}\ if\ and\ only\ if\ \ R(\tilde{A})>R(\tilde{B}).$
3) $\tilde{A}=\tilde{B}\ if\ and\ only\ if\ \ R(\tilde{A})=R(\tilde{B}).$
4) $\tilde{A}-\tilde{B}\ if\ and\ only\ if\ \ R(\tilde{A})-R(\tilde{B})=0.$

A triangular fuzzy number $\tilde{A}=(a_1,a_2,a_3)\ \varepsilon F(R)$ is said to be positive if $R(\tilde{A})>0$ and denoted by $\tilde{A}>0$. Also if $R(\tilde{A})>0$ then $\tilde{A}>0$ and if $R(\tilde{A})=0$, then $\tilde{A}=0$. If $R(\tilde{A})=R(\tilde{B})$, then the triangular numbers $\tilde{A}$ and $\tilde{B}$ are said to be equivalent and is denoted by $\tilde{A}=\tilde{B}$.

2.7 Definition:

Minimize (or Maximize) $\tilde{z}=\tilde{c}^T\tilde{x}$
Subject to,
$\tilde{A}\otimes\tilde{x}\{\le,\approx,\ge\}\tilde{b},\tilde{x}\ge 0$ and are integers.

Where the cost vector $\tilde{c}^T = (\tilde{c}_j)_{1\times n}$, $\tilde{A} = (\tilde{a}_{ij})_{m\times n}, \tilde{x} = (\tilde{x}_j)_{n\times 1}$ *and* $\tilde{b} = (\tilde{b}_i)_{m\times 1}$ *and* $\tilde{a}_{ij}, \tilde{x}_j, \tilde{b}_i, \tilde{c}_j \varepsilon F(R)$ *for all* $1 \le j \le n$ *and* *for all* $1 \le i \le m.$

3. Defuzzification:

We mentioned defuzzyfication in the previous section as a method of assigning a real number to a fuzzy subset of the real number. In this section we will look at some methods of defuzzication. The operation of defuzication is a function, which we now call 'defuzz', mapping fuzzy subsets of the real numbers into the real numbers. We will restrict the discussion to continuous fuzzy subsets or to discrete fuzzy subsets of the reals.

Defuzzification is very important in the fuzzy controller. We do not discuss the fuzzy controller in this book, but its internal method of processing information is very similar to the fuzzy reasoning methods discussed in this chapter 14. In both systems the final conclusion turns out to be a fuzzy subset of the real's, like the one in figure 14.3.if this final conclusion is to be communicated to a machine (set new speed, new voltage, etc.) it must be defuzzified because a machine will not understand a complete fuzzy set. So if the final conclusion is fuzzy set $\bar{B}$, Then defuzzy ($\bar{B}$) will be sent to the machine.

Let $\bar{A} = (a_1 / a_2 / a_3)$ where a_3 - $a_2 = a_2$ - a_1, or $\bar{A}$ is a symmetric triangular fuzzy number. Then many would agree that defuzzy ($\bar{A}$) $= a_2$ is a reasonable defuzzification. But what if $\bar{A}$ is not symmetric? We now present some popular defuzzification methods through the following examples

3.1 The proposed average ranking method:

The average defuzzifier δ =defuzzy($\bar{A}$) is

$$\delta = \int_{-2}^{6} (x\bar{A}(x))dx \div (\text{area}), \text{..........................(1)}$$

Where "area" is the area of the triangular fuzzy number. To evaluate δ we need the function $y = f_1(x)$ *and* $y = f_2(x)$ for the left and right side of $\bar{A}$.

3.2 Algorithm:

Step -1: Defuzzify the fuzzy quantities of the problem to crisp by using the proposed average ranking techniques

Step -2 :Solve using MATLAB code

3.3 Numerical Example:

Consider the following fully fuzzy linear programming problem

Maximize $\tilde{z} \approx (1,2,3) \otimes \tilde{x}_1 + (2,3,4) \otimes \tilde{x}_2$

Subject to,

$$(0,1,2) \otimes \tilde{x}_1 + (1,2,3) \otimes \tilde{x}_2 \approx (1,10,27);$$
$$(1,2,3) \otimes \tilde{x}_1 + (0,1,2) \otimes \tilde{x}_2 \approx (2,11,28);$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0.$$

Solution:

Formulate the chosen problem in to the following fuzzy LPP as

Maximize $\tilde{z} \approx (1,2,3) \otimes \tilde{x}_1 + (2,3,4) \otimes \tilde{x}_2$

Subject to,

$$(0,1,2)\otimes\tilde{x}_1+(1,2,3)\otimes\tilde{x}_2\approx(1,10,27);$$
$$(1,2,3)\otimes\tilde{x}_1+(0,1,2)\otimes\tilde{x}_2\approx(2,11,28);$$
$$\tilde{x}_1,\tilde{x}_2\geq 0.$$

$$(1,2,3):\ \delta=\int_1^3 (x\tilde{A}(x))\,dx \div area$$
$$\delta=\int_1^2 x\,f_1(x)\,dx+\int_2^3 x\,f_2(x)\,dx$$
$$f_1(x)=\frac{x-a}{b-a},\ f_2(x)=\frac{c-x}{c-b}$$
$$f_1(x)=\frac{x-1}{2-1}=\frac{x-1}{1}=x-1$$
$$f_2(x)=\frac{3-x}{3-2}=\frac{3-x}{1}=3-x$$

$$\delta=\int_1^2 x(x-1)\,dx+\int_2^3 x(3-x)\,dx$$
$$=\int_1^2 (x^2-x)\,dx+\int_2^3 (3x-x^2)\,dx$$
$$=\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_1^2+\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_2^3$$
$$=\left[\left(\frac{2^3}{3}-\frac{2^2}{2}\right)-\left(\frac{1}{3}-\frac{1}{2}\right)\right]+\left[\left(\frac{3(3)^2}{2}-\frac{3^3}{3}\right)-\left(\frac{3(2)^2}{2}-\frac{2^3}{3}\right)\right]$$
$$=[(2.666-2)+0.1666]+[(13.5-9)-(6-2.666)]$$
$$=0.8326+1.166$$
$$=1.9986$$
$$Area=\frac{hb}{2}=\frac{(1)(2)}{2}=1$$
$$(1,2,3):\quad \delta=1.9986\div 1$$
$$\delta=1.9986$$

Similarly

$$(2,3,4):\quad \delta=\int_2^3 x(x-2)\,dx+\int_3^4 x(4-x)\,dx\div\frac{(1)(2)}{2}$$
$$\delta=1.667$$
$$(0,1,2):\quad \delta=\int_0^1 x(x)\,dx+\int_1^2 x(2-x)\,dx\div\frac{(1)(2)}{2}$$
$$\delta=1.00133$$
$$(1,10,27):\quad \delta=\int_1^{10} x\left(\frac{x-1}{9}\right)dx+\int_{10}^{27} x\left(\frac{27-x}{17}\right)dx\div\frac{(1)(26)}{2}$$
$$\delta=12.666$$
$$(2,11,28):\quad \delta=\int_2^{11} x\left(\frac{x-2}{9}\right)dx+\int_{11}^{28} x\left(\frac{28-x}{17}\right)dx\div\frac{(1)(26)}{2}$$
$$\delta=13.666$$

We get

$$\begin{aligned} &Max \quad 1.9986\tilde{x}_1 + 1.667\tilde{x}_2 \\ &subject\ to \quad 1.00133\tilde{x}_1 + 1.9986\tilde{x}_2 \le 12.66 \\ &\qquad 1.9986\tilde{x}_1 + 1.00133\tilde{x}_2 \le 13.66 \end{aligned}$$

3.4 MATLAB Code

```
f=[1.9986;1.667]
Aeq = [1.00133 1.9986;1.9986 1.00133]
Beq=[12.66;13.66]
lb=[0;0]
[x,fval,exitflag,output,lambda]=linprog(f,[],[],Aeq,Beq,lb)
```

Result:
x_1= 4.8881,x_2 = 3.8854

fval = 16.2464

3.5 Conclusion

Method	x_1	x_2	z
Existing method	4.335	3.333	19.665
Proposed method	4.8881	3.8854	16.2464

4. Conclusion

In this paper, fuzzy linear programming problem with triangular fuzzy number has been transformed into crisp linear programming problem using the proposed average ranking technique and MATLAB coding has been applied to find an optimal solution by comparing the result of the proposed method and existing method. The result shown that it is better to use the proposed method to solve fuzzy linear programming problem.

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