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Optimal Inventory Control with Partial Backlogging in a Supply Chain: MDP Approach

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Abstract: In this paper, we consider a continuous review inventory control with partial backlogging at retailer in a two stage supply chain. The supply chain consists of a distribution center DC) and single retail-vendor (RV). Arrival of demands to a retailer node is assumed to be Poisson and lead times are exponentially distributed at retailer node and Distribution center with parameters μ_0 and μ_1 respectively. The items are supplied to the retailer in packs of fixed size. The system is formulated as a Markov Decision Process (MDP).A stationary cost structure is imposed and an optimal stationary policy is obtained by using Linear Programming technique. Numerical examples are provided to illustrate the problem.

Keywords: Markov Decision Processes, Supply Chain Management, Optimal inventory policy, Linear programming procedure, Two-stage Inventory control system.

1. Introduction

The study of Supply Chain Management (SCM) started in the late 1980s and has gained a growing level of interest from both companies and researchers over the past three decades. There are many definitions of Supply Chain management. A supply Chain may be defined as an integrated process wherein a number of various business entities(i.e. suppliers, manufacturers, distributors and retailers) work together in an effort to (i)acquire raw materials (ii) Convert these raw materials into specified final products and (iii) deliver these final products to retailers. The process and delivery of goods through this network needs efficient communication and transportation system. The supply chain is traditionally characterized by a forward flow of materials and products and backward flow of information.

One of the most important aspects of supply chain management is inventory control. Inventory control models are almost invariably stochastic optimization problems with objective function being either expected costs or expected profits or risks. In practice, a retailer may want an optimal decision which achieves a minimal expected cost or a maximal expected profit with low risk of deviating from the objective.

A complete review of SCM was provided by Benita M. Beamon (1998) [3].However, there has been increasing attention placed on performance, design and analysis of the supply chain as a whole. HP's (Hawlett Packard) Strategic Planning and Modelling (SPM) group

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initiated this kind of research in 1977. With-in manufacturing research, the supply chain concept grow largely out of two-stage multi-echelon inventory models, and it is important to note that considerable research in this area is based on the classic works of Clark and Scarf (1960)[5] and Sherbrooke,C.[9]

Recent developments in two-echelon models may be found in Q. M. He, and E. M. Jewkes (2000) [6], S. Axaster (1993) [1], Nahimas (1982) [7]. A continuous review (s, S) policy inventor system with positive lead times in two-echelon Supply Chain has been studied by K. Bakthavachalam C. Elango 2007 [2]. Ram Ganeshan studied a model of Managing Supply Chain inventories in multiple retailer, one warehouse and multiple supplier[8].

This paper deals with a simple supply chain that is modelled as system with a single warehouse, a distribution centre and single retailer (all retailers are identical in character), handling a single product. In order to avoid the complexity, at the same time without loss of generality, we assumed the Poisson demand pattern at retailer node. This restricts our study to design and analyse as the tandem network of inventory, which is the building block for the whole supply chain system.

The rest of the paper is organized as follows; the model formulation is described in section 2. In section 3, steady state analysis and MDP formulation are done. Section 4 deals with the LPP solution procedure for the system in steady state Numerical example and sensitivity analysis are provided in section 5. The last section 6 concludes the paper.

2. The Model Description

We consider a continuous review two-echelon inventory system in supply chain implementing partial backlogging at retailer node. A finished product is supplied from distribution centre to retailer who adopts (s,S) policy for maintaining inventory. The demand at retailer node follows a Poisson process with mean rate $\lambda>0$. Supply to the retailer in pockets of $Q=S-s$ items is administered with exponential lead time having parameter $\mu_0>0$. The replenishment of items in terms of pockets at distribution centre is made from a Ware house (WH) which apt one –for-one replenishment policy (M-1,M). The lead time for replenishment is exponentially distributed with parameter $\mu_1>0$ Demands occurring during the stock out periods are backlogged up to a specified quantity ‘b’ at retailer node(where b is the backorder level such that $Q>b+s$ (that is $Q-s>b>0$). In this model, the maximum inventory levels M and S are fixed and reorder level is assumed to be s such that $S-s=Q>b+s$ and $M=nQ$, $n\in N$.

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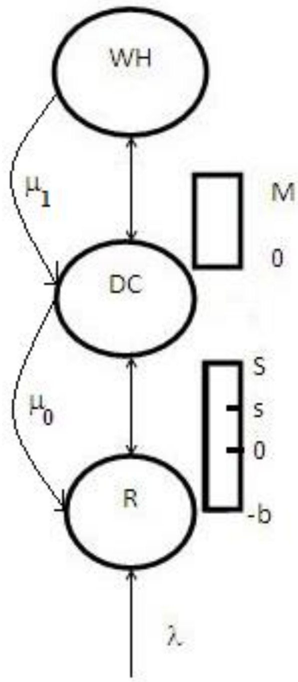

Figure1: Inventory Control with Partial Backlogging in Supply Chain

3. Analysis:

3.1 System Analysis

Let $I_0(t)$ and $I_1(t)$ denote the on hand inventory levels at retailer node and distribution centre respectively at time t. Then $I(t)=\{I_0(t), I_1(t): t \geq 0\}$ is a Markov process with state space $E=E_1 \times E_2$ where $E_1 = \{S,(S-1),...,s,(s-1),...,2,1,0,-1,-2,-3,...,-b\}$, $E_2 = \{nQ,(n-1)Q,...,Q,0\}$, where b is backorder level such that Q>b+s that is $0 \leq b \leq Q-s$(we use negative sign for backlogging quantity.

State transitions are given below:

(i) The arrival of a demand for an item at retailer node makes a state transition in the Markov Process from (j,q) to (j-1,q) with rate λ.

(ii) Replenishment of inventory at retailer node makes a state transition from (j,nQ) to

(j+Q, (n-1)Q) with rate $\mu_0>0$.

(iii) Replenishment of inventory at distribution centre node makes a state transition from

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(j, (n-1)Q) to (j, nQ) with rate $\mu_1>0$.

3.2 MDP Formulation:

Decision epochs: Random points of time at which a demand occurs for a single item at retailer node.

State Space: The state space E is partitioned as follows:

$E_1=\{S,(S-1),...,s,(s-1),...,2,1,0,-1,-2,-3,...,-b\}$, $E_2=\{nQ,(n-1)Q,...,Q,0\}$,

where b is backorder level such that Q>b+s that is$0\le b\le Q-s$(we use negative sign for backlogging quantity).

Action Set: The reordering decisions (0- no order;1- order; 2 –compulsory order) taken at each state of the system (j, q) $\in$ E. The compulsory order for S items is made when inventory level is zero. Let A_i (i =1, 2, 3) denotes the set of possible actions. Where, A_1 = {0}, A_2 = {0, 1}, A_3 = {2} and $A = A_1\cup A_2\cup A_3$.

The set of all possible actions are at $i\in$ E.

$$A_i=\begin{cases}\{0\}, & s+1\le j\le S\\ \{0,1\}, & 1\le j\le s\\ \{2\}, & j=0\end{cases},\quad A=\bigcup_{i\in E}A_i.$$

Transition Probability: $p_{(j,q)}^{(k,r)}(a)$ denote the transition probability from state (j, q) to state (k, r) when decision a is made at state (j, q).

$$R=\begin{matrix} nQ \\ (n-1)Q \\ (n-2)Q \\ (n-3)Q \\ \vdots \\ Q \\ 0 \end{matrix}\begin{pmatrix} A & B & 0 & 0 & \cdots & 0 & 0\\ C & D & B & 0 & \cdots & 0 & 0\\ 0 & C & D & B & \cdots & 0 & 0\\ 0 & 0 & C & D & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & \cdots & D & B\\ 0 & 0 & 0 & 0 & \cdots & C & E \end{pmatrix}$$

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The entries of the block partition matrix R can be written as

$$[R]_{q\times r}=\begin{cases} A & if \quad q=nQ & ; \quad r=q \\ B & if \quad q=nQ,(n-1)Q,(n-2)Q,...,2Q,Q; & r=q-Q \\ C & if \quad q=(n-1)Q,(n-2)Q,...,2Q,Q,0 & ; \quad r=q+Q \\ D & if \quad q=(n-1)Q,(n-2)Q,...,2Q,Q & ; \quad r=Q \\ E & if \quad q=0 & ; \quad r=q \\ 0 & \quad otherwise \end{cases}$$

The sub matrices A, B, C, D and E are given by

$$[A]_{j\times q}=\begin{cases} \lambda & if \quad q=j-1, \quad j=S,S-1,S-2,...,2,1. \\ \lambda & if \quad q=j-1, \quad j=0,-1,-2,...,-(b-1). \\ -\lambda & if \quad q=j, \quad j=S,S-1,S-2,...,(s+1). \\ -(\lambda+\mu_0) & if \quad q=j, \quad j=s,s-1,s-2,...,2,1. \\ -(\lambda+\mu_0) & if \quad q=j, \quad j=0,-1,-2,...,-(\mathrm{b}-1). \\ \mu_0 & if \quad q=j, \quad j=0 \\ 0 & \quad otherwise. \end{cases}$$

$$[B]_{j\times q}=\begin{cases} \mu_0 & if \quad q=j+Q, \quad j=s,s-1,s-2,...,2,1,0,-1,-2,...,-b. \\ 0 & \quad otherwise \end{cases}$$

$$[C]_{j\times q}=\begin{cases} \mu_1 & if \quad q=j \quad , \quad j=S,S-1,S-2,...,s,s-1,s-2,...,2,1,0,-1,-2,...,-b. \\ 0 & \quad otherwise \end{cases}$$

$$[D]_{j\times q}=\begin{cases} \lambda & if \quad q=j-1, \quad j=S,S-1,S-2,...,2,1. \\ \lambda & if \quad q=j-1, \quad j=0,-1,-2,...,-(b-1). \\ -(\lambda+\mu_1) & if \quad q=j, \quad j=S,S-1,S-2,...,(s+1). \\ -(\lambda+\mu_0+\mu_1) & if \quad q=j, \quad j=s,s-1,s-2,...,2,1. \\ -(\lambda+\mu_0+\mu_1) & if \quad q=j, \quad j=0,-1,-2,...,-(\mathrm{b}-1). \\ -(\mu_0+\mu_1) & if \quad q=j, \quad j=0 \\ 0 & \quad otherwise. \end{cases}$$

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$$[E]_{j\times q}=\begin{cases}\lambda & if \quad q=j-1, \quad j=S,S-1,S-2,...,2,1.\\ \lambda & if \quad q=j-1, \quad j=0,-1,-2,...,-(b-1).\\ -(\lambda+\mu_1) & if \quad q=j, \quad j=S,S-1,S-2,...,(s+1).\\ -(\lambda+\mu_1) & if \quad q=j, \quad j=0,-1,-2,...,-(b-1).\\ -\mu_1 & if \quad q=j, \quad j=0\\ 0 & otherwise.\end{cases}$$

The steady state balance equations are

$$\upsilon^{nQ}A+\upsilon^{(n-1)Q}C=0$$

$$\upsilon^{iQ}B+\upsilon^{(i-1)Q}D+\upsilon^{(i-2)Q}C=0 \quad ; i=n,n-1,n-2,.......2$$

$$\upsilon^{Q}B+\upsilon^{0}E=0$$

and the normalizing equation $\sum_{(j,q)\in E}\upsilon_j^q=1$

Solving the above equations we get the steady state probability distribution $\{\upsilon_j^q\}$.

Cost function:

$c_{(j,q)}(a)$ denote the cost occurred in the system when action 'a' is taken at state (j, q).

System Performance Measures

1. **Mean Reorder Rates:**

Consider the event β_i of reorders at node i (i=0, 1). Observe that β_0 denote the event that the inventory level at retailer node reaches s, whereas the β_1 an event that the inventory level at distribution centre reaches the level M-Q (i.e. n-1 packets). The mean reorder rate at retailer node is given by

$$\beta_0=\lambda\sum_{q=0}^{nQ}{}^{*}v_{s+1}^{q}$$

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The mean reorder rate at the distribution centre is given by

$$\beta_0 = \mu_0 \sum_{j=0}^{s} \sum_{q=0}^{(n-1)Q} {}^{*} v_j^q$$

2. **Mean Inventory Levels:**

Let $\overline{I}_i$ denote the mean inventory level in the steady state at node i (i=0, 1).

Thus the mean inventory level at retailer node is given by

$$\overline{I}_0 = \sum_{q=0}^{nQ} {}^{*} \left(\sum_{j=0}^{S} j \cdot v_j^q \right)$$

The mean inventory level at distribution centre is given by

$$\overline{I}_1 = \sum_{j=0}^{S} \left(\sum_{q=0}^{nQ} {}^{*} q \cdot v_j^q \right).$$

3. **Mean Shortage Rates:**

Let α_i denote the shortage rate at node i (i=0, 1). Thus the shortage rate at retailer node is given by

$$\alpha_0 = \lambda \sum_{q=0}^{nQ} {}^{*} v_{-b}^q$$

The demand at distribution centre (Q items) s satisfied by local purchase during the stock out periods; hence the shortage rate at distribution centre α_1 is given by

$$\alpha_1 = \mu_0 \sum_{q=0}^{nQ} {}^{*} v_j^0.$$

Cost Analysis:

The long-run expected cost rate when policy f is adopted is given by

$$C(s,Q) = \mathrm{h}_0 \overline{\mathrm{I}}_0 + h_1 \overline{\mathrm{I}}_1 + k_0 \beta_0 + k_1 \beta_1 + g_0 \alpha_0 + g_1 \alpha_1$$

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where k_0 and k_1 are setup costs (ordering cost) regardless of order size; h_0 and h_1 be holding cost per unit item per unit time at retailer and distributor nodes respectively; g_i is the shortage cost at node i (i=0, 1) for unit shortage per unit time.

Hence the average cost rate of the system is given by

$$\mathrm{C}(s,Q) = h_0\left(\sum_{q=0}^{nQ}{}^{*}\left(\sum_{j=0}^{S} j\cdot v_j^q\right)\right) + h_1\left(\sum_{j=0}^{S}\left(\sum_{q=0}^{nQ}{}^{*} q\cdot v_j^q\right)\right) + k_0\left(\lambda\sum_{q=0}^{nQ}{}^{*} v_{s+1}^q\right)$$
$$+ k_1\left(\mu_0\sum_{j=0}^{s}\sum_{q=0}^{(\mathrm{n}-1)Q}{}^{*} v_j^q\right) + g_0\left(\lambda\sum_{q=0}^{nQ}{}^{*} v_{-b}^q\right) + g_1\left(\mu_0\sum_{q=0}^{nQ} v_j^0\right)$$

When policy R is adopted. From our assumptions, it can be seen that the controlled process $(I_0{}^R, I_1{}^R)$ is a finite state semi-Markov decision process. A policy R is called a stationary policy if it is randomized, time invariant and Markovian. Further, a process is said to be completely ergodic if every stationary policy give rise to an irreducible Markov chain. From our assumptions it can be seen that for every stationary policy f, $(I_0{}^f, I_1{}^f)$ is completely ergodic. Since the action space is also finite, a stationary optimal policy exists. Hence we consider the class f of all stationary policies.

Our objective is to find an optimal policy f^* for which $C^{f^*} \le C^f$ for every f.

For any fixed $f \in F$ and $(\mathrm{j},q),(\mathrm{k},r) \in E$, define

$$P_{jq}^f(\mathrm{k},r,t) = \Pr[I_0{}^f(t)=k, I_1{}^f(t)=r \mid I_0{}^f(0)=j, I_1{}^f(0)=q],\ (\mathrm{j},q),(\mathrm{k},r)\in E$$

Then $P_{jq}(\mathrm{k},r,t)$ satisfies the Kolmogorov forward differential equations. As each policy, f, results in an irreducible Markov chain and action spaces are finite,

$P^f(\mathrm{k},r) = \lim P_{jq}(\mathrm{k},r,t)$ exists and is independent of the initial conditions.

The condition

$\sum_{(\mathrm{j,k})\in E} P^f(\mathrm{j,k}) = 1$, determine the steady-state probabilities uniquely

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4. Linear programming problem

4.1 LPP Formulation

Let us define the variables D(j, q, a) as

D(j, q, a)= Pr [decision is a \ state is (j, q)].

Then for any stationary policy f, we have D(j, q, a) =0 or 1.Suppose D(j, q, a) were continuous variable (instead of integers), then the semi-Markov decision problem can be reformulated as a linear programming problem . For this purpose we consider the class of all randomized, time – invariant Markovian policies for which the probability functions D(j, r, k) satisfy

$$0 \le D(j,q,a) \le 1$$

and

$$\sum_{a \epsilon A} D(j,q,a) = 1, 0 \le j \le S, 0 \le q \le M$$

The linear programming problem is best expressed in terms of the variables y(j, q, a), which are defined as

$y(j, q, a) = D(j, q, a)P^f(j,q)$, (1)

As y(j, q, a)=Pr[state is (j ,a) and decision is a],for any given f, we have

$p^f(j,q) = \sum_{a \epsilon A} y(j,q,a)(j,q) \epsilon E$ (2)

Expressing $P^f(j, q)$ in terms of y(j, q, a) , we obtain the following linear programming problem:

Minimize

$$C(s,Q) = h_0\left(\sum_{a\in\{0,1,2\}} \sum_{q=0}^{nQ}{}^{*}\left(\sum_{j=0}^{S} j \cdot v_j^q(a)\right)\right) + h_1\left(\sum_{a\in\{0,1,2\}} \sum_{j=0}^{S}\left(\sum_{q=0}^{nQ}{}^{*} q \cdot v_j^q(a)\right)\right) + k_0\left(\sum_{a\in\{0,1,2\}} \lambda \sum_{q=0}^{nQ}{}^{*} v_{s+1}^q(a)\right)$$
$$+ k_1\left(\sum_{a\in\{0,1,2\}} \mu_0 \sum_{j=0}^{s} \sum_{q=0}^{(n-1)Q}{}^{*} v_j^q(a)\right) + g_0\left(\sum_{a\in\{0,1,2\}} \lambda \sum_{q=0}^{nQ}{}^{*} v_{-b}^q(a)\right) + g_1\left(\sum_{a\in\{0,1,2\}} \mu_0 \sum_{q=0}^{nQ} v_j^0(a)\right)$$

The constraints of the linear programming problem are as follows:

a) $y(j,q,a) \ge 0$ $(j,q) \in E, a \in A_i$, i=0,1,2. (3)

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b) $\sum_{(j,q)\in E} P^f(j,q) = 1$and

$\sum_{i=0}^{2}\sum_{(j,q)\in E_i}\sum_{k\in A_i} y(j,r,k) = 1$

c) $\sum_{i=0}^{2}\sum_{(j,q)\in E}\sum_{a\in A_i} y(j,q,a) = 1,$ (4)

d) The remaining constraints are the balance equations

As we can see from the lemma below solving the linear programming problem gives the optimal solution when the $y(j,r,k)$are constrained to be integers.

The optimal solution of the above linear programming problem yields a deterministic policy.

From equations (1) and (2), we have

$$D(j, r, k)=\frac{y(j,M,k)}{\sum_{k=0}^{K} y(j,r,k)} \quad (5)$$

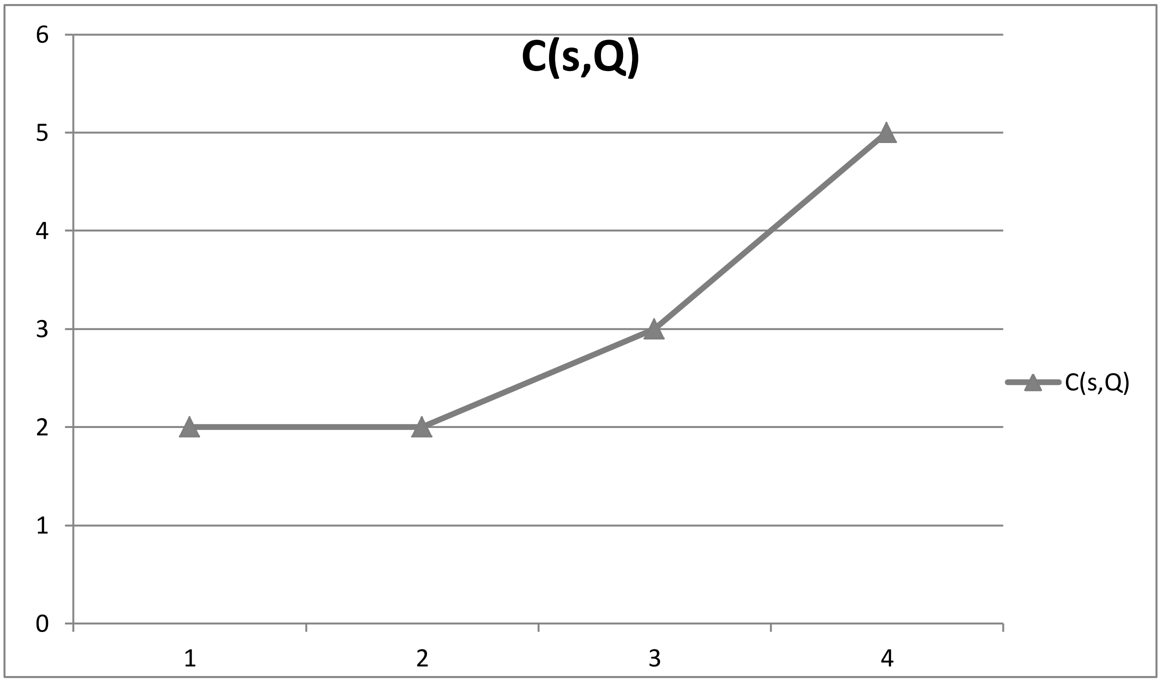
Since the decision problem is completely ergodic, every basic feasible solution to the above linear programming problem has the property that for each(j,r)∈ E,D(j,r,k) is 1 for exactly one value of k and zero for all other values of k. Thus ,given the amount of inventory on-hand and the number of customers in the system ,we have to choose the service rate γ_k for which D(j,r,k) is1.Hence any basic feasible solution of the linear programming yields a deterministic policy.

5. Numerical illustration and discussion:

Total excepted cost rate C(s, Q) as a function s and Q is given in the following table

s	Q	C(s,Q)
0	7	8.0002
1	6	7.8571
2	5	7.4543
*3	*4	*6.4932
4	3	7.6702
5	2	7.7945
6	1	7.9820

The graphical representation of the long run expected cost rate C(s*,Q*) is given below:



6. Conclusion:

Analysis of inventory control in Supply Chain has been obtained by many researchers in the recent past. But most of them are in the direction of determining. The optimal inventory level when a fixed policy is implemented. We, in this article studied the problem in a new approach. For a given Supply Chain structure (two-stage), the ordering quantities are controlled with MDP, and the optimal ordering policy is founded. For both DC and Retailer only partial backlogging is considered for study, one may extend this full backlogging in future.

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