



Estd. 1989

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
 An International Open Free Access Peer Reviewed Research Journal of Mathematics
 website:- www.ultrascientist.org

New Class of Continuity in Nano Product Topology

¹M. LELLIS THIVAGAR and ²I. GEETHA ANTOINETTE

¹School of Mathematics, Madurai Kamaraj University, Madurai- 625 021. Tamilnadu, India.

²Jayaraj Annapackiam College for Women, Periyakulam, Theni - 625 601.
 Tamilnadu, India.

Corresponding Author Email : geetharegis@yahoo.co.in, E.mail: mlthivagar@yahoo.co.in
<http://dx.doi.org/10.22147/jusps-A/290706>

Acceptance Date 2nd June, 2017, Online Publication Date 2nd July, 2017

Abstract

The purpose of the study is to introduce a new class of continuous function among the nano product topology and study the behaviour of these functions. We characterise the properties of the new function. The impact of nano projection mapping between nano product topology is also considered.

Keywords and Phrases : Nano product topology, nano continuous function, nano projection mapping, nano quotient topology.

2010 MSC : 54C05

1 Introduction

Functions are means to link or connect two universes of same type or of different type. Lellis Thivagar *et al.*¹ introduced nano topological space with respect to a subset X of an finite universe which is defined interms of lower and upper approximations of X . He also introduce nano continuous functions, nano pre continuous functions. Here we define nano continuous functions between the product of nano topological spaces and study their behaviour.

2 Preliminaries

*Definition 2.1*⁴ : Let u be a non-empty finite set of objects called the universe and R be an equivalence relation on u named as the indiscernibility relation. Elements belonging to the same equivalence class are said to indiscernible with one another. The pair (u, R) is said to be the approximation space.

- (i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \cup\{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by x.
- (ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \cup\{R(x) : R(x) \cap X \neq \emptyset\}$
- (iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2⁴ : Let u be the universe, R be an equivalence relation on u and $\tau_R(X) = \{u, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq u$. Then $\tau_R(X)$ satisfies the following axioms

- (i) u and $\emptyset \in \tau_R(X)$.
- (ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
- (iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on u called the nanotopology on u with respect to X. We call $(u, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(x)$ are called as nano open sets.

Definition 2.3⁴ : If $(u, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq u$ and if $A \subseteq u$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by $Nint(A)$. That is $Nint(A)$ is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nanoclosed sets containing A and it is denoted by $Ncl(A)$. That is $Ncl(A)$ is the smallest nano closed set containing A.

Definition 2.4 Let u be a nonempty finite universe and $X \subseteq u$

- (NT₁) If $L_R(X) = \emptyset$ and $U_R(X) \neq u$ then $\tau_R(X) = \{u, \emptyset, U_R(X)\}$
- (NT₂) If $L_R(X) = U_R(X) = X$ then the nano topology $\tau_R(X) = \{u, \emptyset, L_R(X)\}$
- (NT₃) If $L_R(X) \neq \emptyset$ and $U_R(X) = u$ then $\tau_R(X) = \{u, \emptyset, L_R(X), B_R(X)\}$
- (NT₄) If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \emptyset$ and $U_R(X) \neq u$ then $\tau_R(X) = \{u, \emptyset, L_R(X), U_R(X), B_R(X)\}$ is the discrete nano topology on u
- (NT₅) If $L_R(X) = \emptyset$ and $U_R(X) = u$ then $\tau_R(X) = \{u, \emptyset\}$ the indiscrete nano topology on u

Definition 2.5 : Let $(u_1, \tau_{R_1}(X))$ and $(u_2, \tau_{R_2}(Y))$ are two nano topological spaces then the nano topology τ_R for the product $u_1 \times u_2 = u$ is called the nano product topology for $u_1 \times u_2$ and in this case the topological space $(u, \tau_R(X))$ is called the product space of nano topological spaces $(u_1, \tau_{R_1}(X))$ and $(u_2, \tau_{R_2}(Y))$

Definition 2.6 : Let $(u_1, \tau_{R_1}(X))$ and $(u_2, \tau_{R_2}(Y))$ are two nano topological spaces. $u_1/R_1 \times u_2/R_2$

R_2 is a partition of $u_1 \times u_2$. Suppose $X \times Y \in u_1 \times u_2$ then the lower, upper and boundary approximations of $X \times Y$ are defined as

$$L_R(X \times Y) = \bigcup_{(x,y) \in X \times Y} \{R(x, y) : R(x, y) \subseteq X \times Y\}$$

$$U_R(X \times Y) = \bigcup_{(x,y) \in X \times Y} \{R(x, y) : R(x, y) \cap X \times Y \neq \emptyset\}$$

$$B_R(X \times Y) = U_R(X \times Y) - L_R(X \times Y)$$

Definition 2.7 : Let $(u_1, \tau_{R_1}(X))$ and $(u_2, \tau_{R_2}(Y))$ are two nano topological spaces.

$u_1/R_1 \times u_2/R_2$ is a partition of $u_1 \times u_2$. Suppose $X \times Y \in u_1 \times u_2$ then

$\tau_R(X \times Y) = \{u_1 \times u_2, \emptyset, L_R(X \times Y), U_R(X \times Y), B_R(X \times Y)\}$ satisfies the following axioms

- (i) $u_1 \times u_2$ and $\emptyset \in \tau_R(X \times Y)$.
- (ii) The union of the elements of any subcollection of $\tau_R(X \times Y)$ is in $\tau_R(X \times Y)$.
- (iii) The intersection of the elements of any finite subcollection of $\tau_R(X \times Y)$ is in $\tau_R(X \times Y)$.

That is $\tau_R(X \times Y)$ is a topology on $u_1 \times u_2$ called the nano product topology on $u_1 \times u_2$ with respect to $X \times Y$. We call $(u_1 \times u_2, \tau_R(X \times Y))$ as the nano product topological space.

Definition 2.8⁴: Let $(u, \tau_R(X))$ and $(v, \tau_{R'}(Y))$ be nano topological spaces. Then the mapping $f: (u, \tau_R(X)) \rightarrow (v, \tau_{R'}(Y))$ is nano continuous on u if the inverse image of every nano open set in v is nano open in u .

3 Nano Continuity in Nano Product Space :

Definition 3.1 Let $(u_1 \times u_2, \tau_R(A \times B))$ and $(v_1 \times v_2, \tau_R(C \times D))$ are the nano product topological spaces with respect to $A \times B$ and $C \times D$. The function $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (v_1 \times v_2, \tau_R(C \times D))$ is said to be nano continuous if the inverse image of every nano open set in $v_1 \times v_2$ is nano open in $u_1 \times u_2$

Example 3.2 Let $u_1 = \{a, b, c\}$ with $u_1/R_1 = \{\{a\}, \{b\}, \{c\}\}$.

Let $u_2 = \{1, 2, 3, 4\}$ with $u_2/R_2 = \{\{1, 4\}, \{2, 3\}\}$.

$u_1/R_1 \times u_2/R_2 = \{\{(a, 1), (a, 4)\}, \{(a, 2), (a, 3)\}, \{(b, 1), (b, 4)\}, \{(b, 2), (b, 3)\}, \{(c, 1), (c, 4)\}, \{(c, 2), (c, 3)\}\}$

is a partition of $u_1 \times u_2$. Suppose $A \times B = \{(a, 1), (a, 3), (a, 4)\}$ then

$\tau_R(A \times B) = \{u_1 \times u_2, \emptyset, \{(a, 1), (a, 4)\}, \{(a, 1), (a, 2), (a, 3), (a, 4)\}, \{(a, 2), (a, 3)\}\}$

Let $v_1 = \{l, m, n\}$ with $v_1/R_1 = \{\{l\}, \{m, n\}\}$.

Let $v_2 = \{x, y\}$ with $v_2/R_2 = \{\{x\}, \{y\}\}$.

$\nu_1/R_1 \times \nu_2/R_2 = \{\{(l, x)\}, \{(l, y)\}, \{(m, x), (n, x)\}, \{(m, y), (n, y)\}\}$ is a partition of $\nu_1 \times \nu_2$.

Suppose $C \times D = \{(l, x), (l, y), (m, y)\}$ then $\tau_R(C \times D) = \{\nu_1 \times \nu_2, \emptyset, \{(l, x), (l, y)\}, \{(l, x), (l, y)(m, y), (n, y)\}, \{(m, y), (n, y)\}\}$. Define the function f as $f(a, 1) = (l, x); f(a, 2) = (m, y); f(a, 3) = (n, y); f(a, 4) = (l, y); f(b, 1) = f(b, 2) = f(b, 3) = f(b, 4) = (m, x); f(c, 1) = f(c, 2) = f(c, 3) = f(c, 4) = (n, x)$. The open set of $\nu_1 \times \nu_2$ are $\nu_1 \times \nu_2, \emptyset, \{(l, x), (l, y)\}, \{(l, x), (l, y)(m, y), (n, y)\}$ and $\{(m, y), (n, y)\}$. The inverse image of these open sets are $f^{-1}\{(l, x), (l, y)\} = \{(a, 1), (a, 4)\}; f^{-1}\{(m, y), (n, y)\} = \{(a, 2), (a, 3)\}; f^{(-1)}\{(l, x), (l, y)(m, y), (n, y)\} = \{(a, 1), (a, 2), (a, 3), (a, 4)\}; f^{-1}(\emptyset) = \emptyset$ and $f^{-1}\{u_1 \times u_2\} = \nu_1 \times \nu_2$. Inverse image of the open sets are open in $u_1 \times u_2$. Therefore the function is continuous.

Theorem 3.3: A function $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (\nu_1 \times \nu_2, \tau_R(C \times D))$ is nano continuous if and only if the inverse image of every nano closed set in $\nu_1 \times \nu_2$ is nano closed in $u_1 \times u_2$

Proof: Let f be nano continuous and G be a nano closed set in $\nu_1 \times \nu_2$. Then $(\nu_1 \times \nu_2) - G$ is a nano open set in $\nu_1 \times \nu_2$. Since f is nano continuous $f^{-1}(\mathcal{Y}_1 \times \mathcal{Y}_2) - G$ is nano open in $u_1 \times u_2$. That is $u_1 \times u_2 - (f^{-1}(G))$ is nano open in $u_1 \times u_2$. Therefore $f^{-1}(G)$ is nano closed in $u_1 \times u_2$. Thus inverse image of every nano closed set in $\nu_1 \times \nu_2$ is nano closed in $u_1 \times u_2$. Conversely, let inverse image of every nano closed set is nano closed. Let F be a nano open set in $\nu_1 \times \nu_2$ then $(\nu_1 \times \nu_2) - F$ is nano closed in $\nu_1 \times \nu_2$. Then $f^{-1}(\nu_1 \times \nu_2 - F)$ is nano closed in $u_1 \times u_2$, that is $u_1 \times u_2 - f^{-1}(F)$ is nano closed in $u_1 \times u_2$. Hence $f^{-1}(F)$ is nano open in $u_1 \times u_2$. Thus f is nano continuous.

Theorem 3.4: A function $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (\nu_1 \times \nu_2, \tau_R(C \times D))$ is nano continuous if and only if $f(Ncl(H)) \subseteq Ncl(f(H))$ for every subset H of $u_1 \times u_2$.

Proof: Let f be nano continuous and $H \subseteq u_1 \times u_2$ then $f(H) \subseteq \nu_1 \times \nu_2$. Since f is nano continuous and $Ncl f(H)$ is nano closed in $\nu_1 \times \nu_2$, $f^{-1}(Ncl(f(H)))$ is nano closed in $\nu_1 \times \nu_2$. Since $f(H) \subseteq Ncl(f(H)), (H) \subseteq f^{-1}(Ncl(f(H)))$. Since $Ncl(H)$ is the smallest nano closed set containing H , we get $f(Ncl(H)) \subseteq Ncl(f(H))$. conversely let $f(Ncl(H)) \subseteq Ncl(f(H))$ for every subset H of $u_1 \times u_2$. Let G is nano closed in $\nu_1 \times \nu_2$. Since $f^{-1}(G) \subseteq u_1 \times u_2, f(Ncl(f^{-1}(G))) \subseteq Ncl(f(f^{-1}(G))) = Ncl(G)$. That is $Ncl(f^{-1}(G)) \subseteq f^{-1}(Ncl(G)) = f^{-1}(G)$, since G is nano closed. Thus $Ncl(f^{-1}(G)) \subseteq f^{-1}(G)$, but $f^{-1}(G) \subseteq Ncl(f^{-1}(G))$. Therefore $Ncl(f^{-1}(G)) = f^{-1}(G)$. Hence $f^{-1}(G)$ is nano closed in $u_1 \times u_2$ for every nano closed set G in $\nu_1 \times \nu_2$. Thus f is nano continuous.

Theorem 3.5: A function $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (\nu_1 \times \nu_2, \tau_R(C \times D))$ is nano continuous if

and only if $Ncl(f^{-1}(G)) \subseteq f^{-1}(Ncl(G))$ for every subset G of $\nu_1 \times \nu_2$.

Proof: Let f be nano continuous and $G \subseteq \nu_1 \times \nu_2$ then $Ncl(G)$ is a nano closed set in $\nu_1 \times \nu_2$. Therefore $f^{-1}(Ncl(G))$ is nano closed in $u_1 \times u_2$. Hence $Ncl[f^{-1}(Ncl(G))] = f^{-1}(Ncl(G))$. Since $G \subseteq Ncl(G)$, $f^{-1}(G) \subseteq f^{-1}(Ncl(G))$. Therefore $Ncl(f^{-1}(G)) \subseteq Ncl(f^{-1}(Ncl(G))) = f^{-1}(Ncl(G))$. Hence $Ncl(f^{-1}(G)) \subseteq f^{-1}(Ncl(G))$. Conversely, let $Ncl(f^{-1}(G)) \subseteq f^{-1}(Ncl(G))$ for every G of $\nu_1 \times \nu_2$. If G is nano closed in $\nu_1 \times \nu_2$, then $Ncl(G) = (G)$. By our assumption $Ncl(f^{-1}(G)) \subseteq f^{-1}(Ncl(G)) = f^{-1}(G)$. Since $f^{-1}(G) \subseteq Ncl(f^{-1}(G))$. Therefore $Ncl(f^{-1}(G)) = f^{-1}(G)$. That is $f^{-1}(G)$ is nano closed in $u_1 \times u_2$ for every nano closed set G of $\nu_1 \times \nu_2$. Therefore f is nano continuous.

Theorem 3.6: A function $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (\nu_1 \times \nu_2, \tau_R(C \times D))$ is nano continuous if and only if $f^{-1}(Nint(G)) \subseteq Nint(f^{-1}(G))$ for every subset G of $\nu_1 \times \nu_2$.

Proof: Let f be nano continuous and $G \subseteq \nu_1 \times \nu_2$ then $Nint(G)$ is nano open in $\nu_1 \times \nu_2$. Therefore $f^{-1}(Nint(G))$ is nano open in $u_1 \times u_2$. Hence $Nint[f^{-1}(Nint(G))] = f^{-1}(Nint(G))$. Since $Nint(G) \subseteq G$, $f^{-1}(Nint(G)) \subseteq f^{-1}(G)$. Therefore $Nint[f^{-1}(Nint(G))] \subseteq Nint(f^{-1}(G))$. Hence $f^{-1}(Nint(G)) \subseteq Nint(f^{-1}(G))$. Conversely, let $f^{-1}(Nint(G)) \subseteq Nint(f^{-1}(G))$ for every subset G of $\nu_1 \times \nu_2$. If G is open in $\nu_1 \times \nu_2$ then $Nint(G) = (G)$ and also $f^{-1}(Nint(G)) \subseteq Nint(f^{-1}(G))$. That is $f^{-1}(G) \subseteq Nint(f^{-1}(G))$. Thus $f^{-1}(G)$ is nano open in $u_1 \times u_2$ for every subset G of $\nu_1 \times \nu_2$.

Theorem 3.7: Let $(u_1 \times u_2, \tau_R(A \times B))$ and $(\nu_1 \times \nu_2, \tau_R(C \times D))$ are the nano product topological space with respect to $A \times B$ and $C \times D$ then for any function $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (\nu_1 \times \nu_2, \tau_R(C \times D))$ the following are equivalent

- (i) f is nano continuous
- (ii) The inverse image of every nano closed set in $\nu_1 \times \nu_2$ is nano closed in $u_1 \times u_2$
- (iii) $f(Ncl(H)) \subseteq Ncl(f(H))$ for every subset H of $u_1 \times u_2$.
- (iv) $f^{-1}(Nint(H)) \subseteq Nint(f^{-1}(G))$ for every subset G of $\nu_1 \times \nu_2$.

4 Nano Homeomorphism in Nano Product Space :

Here we introduce the notion of a topological mapping between two topological spaces $u_1 \times u_2$ and $\nu_1 \times \nu_2$ and the properties are discussed.

Definition 4.1 Let $(u_1 \times u_2, \tau_R(A \times B))$, $(\nu_1 \times \nu_2, \tau_R(C \times D))$ are the nano product topological

space with respect to $A \times B$ and $C \times D$. The function $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (v_1 \times v_2, \tau_R(C \times D))$ is said to be nano homeomorphism if

- (i) f is 1-1 and onto
- (ii) f is nano-continuous
- (iii) f is nano-open

Example 4.2 Let $u_1 = \{a, b, c\}$ with $u_1/R_1 = \{\{a\}, \{b, c\}\}$.

Let $u_2 = \{1, 2, 3\}$ with $u_2/R_2 = \{\{1, 2\}, \{3\}\}$.

$u_1/R_1 \times u_2/R_2 = \{\{(a, 1), (a, 2)\}, \{(a, 3)\}, \{(b, 1), (b, 2), (c, 1), (c, 2)\}, \{(b, 3), (c, 3)\}\}$ is a partition of $u_1 \times u_2$. Suppose $A \times B = \{(a, 1), (a, 2), (b, 1)\}$ then

$$\tau_R(A \times B) = \{u_1 \times u_2, \emptyset, \{(a, 1), (a, 2)\}, \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}, \{(b, 1), (b, 2), (c, 1), (c, 2)\}\}$$

Let $v_1 = \{x, y, z\}$ with $v_1/R_1 = \{\{x\}, \{y, z\}\}$. Let $v_2 = \{l, m, n\}$ with $v_2/R_2 = \{\{l, m\}, \{n\}\}$. $v_1/R_1 \times v_2/R_2 = \{\{(x, l), (x, m)\}, \{(x, n)\}, \{(y, l), (y, m), (z, l), (z, m)\}, \{(y, n), (z, n)\}\}$ is a partition of $v_1 \times v_2$. Suppose $C \times D = \{(x, l), (x, m), (y, l)\}$ then

$$\tau_R(C \times D) = \{v_1 \times v_2, \emptyset, \{(x, l), (x, m)\}, \{(x, l), (x, m), (y, l), (y, m), (z, l), (z, m)\}, \{(y, l), (y, m), (z, l), (z, m)\}\}. \text{ Define the function } f \text{ as } f(a, 1) = (x, l); f(a, 2) = (x, m); f(a, 3) = (x, n); f(b, 1) = (y, l); f(b, 2) = (y, m); f(b, 3) = (y, n); f(c, 1) = (z, l); f(c, 2) = (z, m); f(c, 3) = (z, n). \text{ The function is a homeomorphism.}$$

Theorem 4.3 Let $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (v_1 \times v_2, \tau_R(C \times D))$ be a 1-1 and onto function. The function f is a nano homeomorphism if and only if f is nano closed and nano continuous.

Proof: Let f be a nano homeomorphism. Then f is nano continuous. Let H be nano closed subset of $(u_1 \times u_2, \tau_R(A \times B))$. Then $(u_1 \times u_2 - H)$ is nano open in $u_1 \times u_2$. Since the function f is nano open $f(u_1 \times u_2 - H)$ is nano open in $v_1 \times v_2$. That is $v_1 \times v_2 - f(H)$ is nano-open in $v_1 \times v_2$. Hence $f(H)$ is nano closed in $v_1 \times v_2$. Therefore f is nano-closed. Conversely, suppose f is nano-closed and nano-continuous. Let G be a nano-open set in $u_1 \times u_2$. Then $(u_1 \times u_2 - G)$ is nano closed in $u_1 \times u_2$. Since f is nano-closed, $f(u_1 \times u_2 - G) = v_1 \times v_2 - f(G)$ and it is closed in $v_1 \times v_2$. Therefore $f(G)$ is nano-open in $v_1 \times v_2$. Thus f is nano-open and hence f is a nano homeomorphism.

Theorem 4.4 Let $f: (u_1 \times u_2, \tau_R(A \times B)) \rightarrow (v_1 \times v_2, \tau_R(C \times D))$ be a 1-1 and onto function. The function f is a nano homeomorphism if and only if $f(Ncl(A)) = Ncl[f(A)]$ for every subset A of $u_1 \times u_2$.

Proof: Suppose f is a nano homeomorphism then f is nano continuous and nano closed. Since f is nano continuous, $f(Ncl(A)) \subseteq Ncl[f(A)]$ for $A \subseteq u_1 \times u_2$. Since f is nano closed and $Ncl(A)$ is nano closed, $f(Ncl(A))$ is nano closed in $v_1 \times v_2$. Hence we get $Ncl(f(Ncl(A))) = f(Ncl(A))$. Since $A \subseteq Ncl(A)$, $f(A) \subseteq f(Ncl(A))$ and hence $Ncl(f(A)) \subseteq Ncl[f(Ncl(A))] = f(Ncl(A))$. Therefore

$Ncl(f(A)) \subseteq f(Ncl(A))$. Hence $f(Ncl(A)) = Ncl[f(A)]$. Conversely, suppose $f(Ncl(A)) = Ncl[f(A)]$ for every subset A of $u_1 \times u_2$, then f is nano continuous. If A is a nano closed set in $u_1 \times u_2$, $Ncl(A) = A$ which gives $f(Ncl(A)) = f(A)$. Hence $Ncl(f(A)) = f(A)$. Therefore $f(A)$ is nano closed in $v_1 \times v_2$, forevery A nano closed set in $u_1 \times u_2$. Therefore f is nano closed and nano continuous, hence f is a nano homeomorphism.

5. Projection mapping in Nano product Space :

In this section we discuss the method of finding Nano quotient topology using nano projection mapping.

Definition 5.1 : Let $(u_1 \times u_2, \tau_R(X \times Y))$ be a nano product topological space. The mapping $p : u_1 \times u_2 \rightarrow (u_1 \times u_2)/r$ where r is an equivalence relation on $u_1 \times u_2$ defined by $p(x, y) = r(x, y)$ for all $(x, y) \in u_1 \times u_2$ is called nano projection of $u_1 \times u_2$ onto $(u_1 \times u_2)/r$. Since each $(x, y) \in u_1 \times u_2$ belongs to exactly one equivalence class, p is well defined and surjective.

Let $\tau(p) = \{G \subseteq (u_1 \times u_2)/r : p^{-1}(G) \in \tau_R(X \times Y)\}$. $\tau(p)$ is a nano topology on $(u_1 \times u_2)/r$ and it is called as nano quotient topology.

Example 5.2 Let $u_1 = \{a, b, c\}$ with $u_1/R_1 = \{\{a\}, \{b\}, \{c\}\}$.

Let $u_2 = \{1, 2, 3\}$ with $u_2/R_2 = \{\{1\}, \{2, 3\}\}$.

$u_1/R_1 \times u_2/R_2 = \{\{(a, 1)\}, \{(a, 2), (a, 3)\}, \{(b, 1)\}, \{(b, 2), (b, 3)\}, \{(c, 1)\}, \{(c, 2), (c, 3)\}\}$ is a partition of $u_1 \times u_2$. Suppose $A \times B = \{(a, 2), (b, 2)\}$ then

$\tau_R(A \times B) = \{u_1 \times u_2, \emptyset, \{(a, 2), (a, 3), (b, 2), (b, 3)\}\}$. Let r be an equivalence relation on $u_1 \times u_2$ and $(u_1 \times u_2)/r = \{\{(a, 1)\}, \{(a, 2), (a, 3)\}, \{(b, 1), (c, 1)\}, \{(b, 2), (b, 3), (c, 2), (c, 3)\}\}$.

Let $p : (u_1 \times u_2) \rightarrow (u_1 \times u_2)/r$ as $p((a, 1)) = \{(a, 1)\}$; $p((a, 2)) = \{(a, 2), (a, 3)\} = p(a, 3)$; $p((b, 1)) = \{(b, 1), (c, 1)\} = p(c, 1)$; $p((b, 2)) = p((b, 3)) = \{(b, 2), (b, 3), (c, 2), (c, 3)\} = p((c, 2)) = p((c, 3))$ then $\tau(p) = \{u_1 \times u_2, \emptyset\}$

Remark 5.3 :

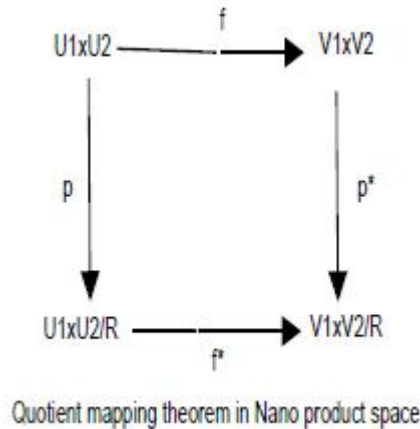
1. If the equivalence relations R and r are equal then the nano topology defined on $u_1 \times u_2$ and τp are one at the same.
2. If the equivalence relations R and r are different then $\tau(p) \subseteq \tau_R(X \times Y)$
3. In general the canonical projection p is continuous and surjective but in nano topology it is surjective, not continuous.
4. If the equivalence relations R and r are equal then p is continuous.

Theorem 5.4 (Quotient mapping theorem in Nano product space)

Let $(u_1 \times u_2, \tau_R(A \times B)), (v_1 \times v_2, \tau_R(C \times D))$ be nano product topological spaces. If $f :$

$(u_1 \times u_2, \tau_R(A \times B)) \rightarrow (v_1 \times v_2, \tau_{R'}(C \times D))$ is continuous and relation preserving then the function $f^* : (u_1 \times u_2)/R \rightarrow (v_1 \times v_2)/R'$ defined as $f^*(R[x, y]) = R'(f^*[x, y])$ is continuous.

Proof : The projection mappings $p : u_1 \times u_2 \rightarrow (u_1 \times u_2)/R$ and $p^* : v_1 \times v_2 \rightarrow (v_1 \times v_2)/R'$ are continuous. Since f, f^*, p, p^* are surjective, we have $f^* \circ p = p^* \circ f$. Since p^* and f are continuous $p^* \circ f$ is continuous. Therefore $f^* \circ p$ is also continuous. Hence f^* is continuous.



Conclusion

Here we introduced the nano continuous mapping, nano homeomorphism, nano projection mapping in nano product topology and discussed their properties. This can be further extended to introduce some weak nano continuous functions and the real life application.

Acknowledgement

The research work is funded by University Grants Commussion (UGC-FDP).

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