APPLICATION OF SOFT SETS IN ROUGH TOPOLOGY

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Abstract

The aim of this paper is to apply soft sets in rough topology. An algorithm is developed in terms of soft rough topology to find the key factors in a soft set information system. It is used to analyze a real life problem.

Keywords: soft sets, rough topology and soft rough topology.

1. Introduction

Molodtsove [4] introduced soft sets in the year 1999 to deal with problems of incomplete information. Soft set is a completely generic mathematical tool for modeling uncertainties. The concept of soft sets enhanced the application potential of the different generalization of crisp sets due to additional advantage of parametrization tools. Molodtsove[5,6] investigated some applications of soft sets in information systems. It is also noted that the soft set theory has applications in different real life problems. Pawlak [7] introduced rough set theory in 1982. Nano topology was introduced by Lellis Thivagar et al.[2] in 2012 in terms of approximations and boundary region of a subset of an universe. The nano topology was used to analyze some real life problems by Lellis Thivagar et al.[3]. Mathew et al.[1] defined rough topology in 2012.The concept of soft rough topology is used information system.

2. Preliminaries

In this section we list some definitions and results which will be useful in the following sections.

Definition 2.1[4]

Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X where F is a mapping given by $F: A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2[7]

Let U be a non empty set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. The pair (U, R) is called the approximation space. Let Z be a subset of U.

i. The lower approximation of Z with respect to R is the set of all objects, which can be for certain classified as Z with respect to R and it is denoted by $L_R(Z)$. That is, $L_R(Z) = \bigcup_{z \in U} \{R(z) : R(z) \subseteq Z\}$ where $R(z)$ denotes the equivalence class determined by z.

ii. The upper approximation of Z with respect to R is the set of all objects, which can be possibly classified as Z with respect to R and it is denoted by $U_R(Z)$. That is $U_R(Z)=\bigcup_{z\in U}\{R(z):R(z)\bigcap Z\neq\phi\}$.

iii. The boundary region of Z with respect to R is the set of all objects, which can be classified neither as Z nor as not Z with respect to R and it is denoted by $B_R(Z)$. That is, $B_R(Z)=U_R(Z)-L_R(Z)$.

Definition 2.3[2]

Let U be the universe of objects and R be an equivalence relation on U. Then $\tau_R=\{U,\phi,L_R(Z),U_R(Z),B_R(Z)\}$ where $Z\subseteq U$ is called the nano topology on U.

Definition 2.4[1]

If τ_R is the rough topology on U with respect to U. then the set $\beta=\{U,L_R(Z),B_R(Z)\}$ is the basis for τ_R.

Definition 2.5[7]

A set is said to be a rough set if it has a non empty boundary region. If the boundary region is empty then the set is a crisp or exact set.

Definition 2.6[7]

The lower approximation of X denoted by X_L is the union of all granules which are entirely included in the set. That is, $X_L=\{x/R(x)\subset X\}$. Therefore, lower approximation of a set consists of all elements that surely belong to the set.

Definition 2.7[7]

The upper approximation of X denoted by X_U is the union of all granules with have non empty intersection with the set. That is, $X_U=\{x/R(x)\cap x\neq\phi\}$. Therefore,upper approximation of a set constitutes of all elements that possibly belong to the set.

Definition 2.8[1]

Let τ_L and τ_U be any two topologies which contain only exact subsets of X_L and X_U respectively. Then the pair $\tau=(\tau_L,\tau_U)$ is said to be a rough topology on the rough set $X=(X_L,X_U)$ and the pair (X,τ) is known as a rough topological space (in short, RTS). Also in a rough topology $\tau=(\tau_L,\tau_U)$, τ_L is known as the lower rough topology and τ_U is known as the upper topology on X.

Definition 2.9[1]

Let $A=(A_L,A_U)$ be any rough subset of a RTS (X,τ) where $X=(X_L,X_U)$ and $\tau=(\tau_L,\tau_U)$. Then A is said to be lower open if the lower approximation of A is in the lower rough topology. That is, $A_L\in\tau_L$. Also A is said to be upper open if the upper approximation of A is in the upper rough topology. That is, $A_U\in\tau_U$. A is said to be rough open if and only if A is lower rough open and upper rough open. That is, $A=(A_L,A_U)$ is rough open with respect to the RTS (X,τ) if and only if $A_L\in\tau_L$. and $A_U\in\tau_U$.

3. Algorithm

Step 1: Given a finite universe U. Input the soft set (F, B).

Step 2: Input the set A of choice parameters of Mr. Z which is subset of B.

Step 3: Define a suitable relation. According to that a finite set A of attributes is divided into two classes, C of condition attributes and D of decision attributes an equivalence relation R on U corresponding to C and a subset Z of U, represent the data as an information table, columns of which are labeled by attributes, rows be objects and entries of the table are attribute values.

Step 4: Find the lower approximation, upper approximation and the boundary region of Z with respect to R.

Step 5: Generate the soft rough topology τ_R on U and its basis β_R.

Step 6: Remove an attribute z from C and find the lower and upper approximations and the boundary region of Z with respect to the equivalence relation $C-(z)$.

Step 7: Generate the rough topology $\tau_{R-(z)}$ on U and its basis $\beta_{R-(z)}$.

Step 8: Repeat steps 6 and 7 for all attributes in C.

Step 9: Those attributes in C for which $\beta_{R-(z)} \neq \beta_R$ form the $Core(R)$.

4. Application of soft set in rough topology

In this section, we present application of soft sets in decision making problems with the help of rough topology. For this study the following problem is considered. Let $U = \{A_1, A_2, A_3, A_4, A_5, A_6\}$ be the set of six bikes in a showroom and B be the set of five attributes $\{B_1, B_2, B_3, B_4, B_5\}$ where B_1 denotes manufacturing retail price, B_2 denotes displacement, B_3 denotes milage, B_4 denotes kerb weight and B_5 denotes reserved fuel range.

features / bikes	MRP Rs.	displacement	Milage	kerb weight	reserved fuel range
Honda CD110 dream	47,318.00	109	65	109	1.30
Honda livo	58,963.00	109	60	111	2.00
Honda CB unicorn 160	75,339.00	162	62	134	1.30
TVS victor	56,347.00	109	72	113	2.00
TVS appache RTR 160	77,714.00	160	50	137	2.50
Bajaj discover 110	50,560.00	115	65	117	1.50

Table 4.1

Let (F,B) be a soft set representing "the best features in this set of bikes" given by $(F,B)=\{(B_1,\{A_2,A_3,A_4,A_5\}),(B_2,\{A_3,A_5,A_6\}),\ (B_3,\{A_1,A_3,A_4,A_6\}),\ (B_4,\{A_1,A_2\}),$ $(B_5,\{A_1,A_2,A_3,A_4,A_5\})\}$.

Suppose that the customer wants to find out the key attributes for the purchase of a bike consisting of only the parameters B_1,B_2,B_3,B_4 which is a subset $A=\{B_1,B_2,B_3,B_4\}$ of the set B. Here $U=\{A_1,A_2,A_3,A_4,A_5,A_6\}$ and manufacturing retail price B_1, displacement B_2, milage B_3 and kerb weight B_4 form the condition attributes. The problem is to study the selection of the best features in the bike in a particular showroom.

maximum retail price $\rightarrow$52,000 above $\rightarrow$ high and 52,000 below $\rightarrow$low; displacement $\rightarrow$ 109 above $\rightarrow$high and 109 below $\rightarrow$low; milage $\rightarrow$60 above $\rightarrow$high and 60 below $\rightarrow$low; kerb weight $\rightarrow$111 below $\rightarrow$high and 111 above $\rightarrow$low.

features / bikes	MRP (Rs.)	displacement	Milage	kerb weight
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Bajaj discover 110	50,560	115	65	117

Table 4.2

Solution

attributes / bikes	B_1^*	B_2	B_3^*	B_4	Decision
A_1	low	low	high	High	yes

A_2	high	low	low	High	no
A_3	high	high	high	Low	no
A_4	high	low	high	Low	no
A_5	high	high	low	Low	yes
A_6	low	high	high	Low	yes

Table 4.3

Case 1:

Let $Z=\{A_1,A_5,A_6\}$ be the set of selected bikes in the showroom. Let R be the equivalence relation on U with respect to the condition attributes. The family of equivalence classes corresponding to R is given by $U/I(R)=\{\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\}\}$. The lower and upper approximations of Z with respect to R are given by $L_R(Z)=\{A_1,A_5,A_6\}$ and $U_R(Z)=\{A_1,A_5,A_6\}$. Therefore, $B_R(Z)=\phi$. The soft rough topology on U is $\tau_R=\{U,\phi,\{A_1,A_5,A_6\}\}$ and its basis $\beta_R=\{U,\{A_1,A_5,A_6\}\}$. If the attribute '$B_1$' is removed from the set if condition attributes then we obtain $U/I(R-\{B_1\})=\{\{A_1\},\{A_2\},\{A_3,A_6\},\{A_4\},\{A_5\}\}$. $L_{R-\{B_1\}}(Z)=\{A_1,A_5\}, U_{R-\{B_1\}}(Z)=\{A_1,A_3,A_5,A_6\}$ and $B_{R-\{B_1\}}(Z)=\{A_3,A_6\}$, hence we get $\tau_{R-\{B_1\}}=\{U,\phi,\{A_1,A_3,A_5,A_6\},\{A_1,A_5\},\{A_3,A_6\}\}$ and its basis $\beta_{R-\{B_1\}}=\{U,\{A_1,A_5\},\{A_3,A_6\}\}$ $\neq\beta_R$. If the attribute 'B_2' is removed from the set of condition attributes then we obtain $U/I(R-\{B_2\})=\{\{A_1\},\{A_2\},\{A_3,A_4\},\{A_5\},\{A_6\}\}$. Then we get $L_{R-\{B_2\}}(Z)=\{A_1,A_5,A_6\}$, $U_{R-\{B_2\}}(Z)=\{A_1,A_5,A_6\}$ and $B_{R-\{B_2\}}(Z)=\phi$, hence we get the topology $\tau_{R-\{B_2\}}=\{U,\phi,\{A_1,A_5,A_6\}\}$ and its basis $\beta_{R-\{B_2\}}=\{U,\{A_1,A_5,A_6\}\}=\beta_R$. If the attribute '$B_3$' is removed, then we obtain $U/I(R-\{B_3\})=\{\{A_1\},\{A_2\},\{A_3,A_5\},\{A_4\},\{A_6\}\}$. Then we get $L_{R-\{B_3\}}(Z)=\{A_1,A_6\}$, $U_{R-\{B_3\}}(Z)=\{A_1,A_3,A_5,A_6\}$ and $B_{R-\{B_3\}}(Z)=\{A_3,A_5\}$, hence we get the topology $\tau_{R-\{B_3\}}=\{U,\phi,\{A_1,A_6\},\{A_1,A_3,A_5,A_6\},\{A_3,A_5\}\}$ and its basis $\beta_{R-\{B_3\}}=\{U,\{A_3,A_5\}\}\neq\beta_R$. If the attribute '$B_4$' is removed from the set of condition attributes then we obtain $U/I(R-\{B_4\})=\{\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\}\}$. Then we get $L_{R-\{B_4\}}(Z)=\{A_1,A_5,A_6\}, U_{R-\{B_4\}}(Z)=\{A_1,A_5,A_6\}$ and $B_{R-\{B_4\}}(Z)=\phi$. Hence we get the topology $\tau_{R-\{B_4\}}=\{U,\phi,\{A_1,A_5,A_6\}\}$ and its basis $\beta_{R-\{B_4\}}=\{U,\{A_1,A_5,A_6\}=\beta_R$. if $M=\{B_1,B_3\}$ then we obtain $U/I(r)=\{\{A_1,A_6\},\{A_2,A_5\},\{A_3,A_4\}\}$, $L_r(Z)=\phi$ and $U_r(Z)=\{A_1,A_2,A_5,A_6\}$ where r is the equivalence relation on M is given by $\beta_M=\{U,\{A_1,A_2,A_5,A_6\}\}\neq\beta_{R-\{B_1\}}\neq\beta_{R-\{B_3\}}$. Therefore, Core $(R)=\{B_1,B_3\}$.

Case 2:

Consider $Z=\{A_2,A_3,A_4\}$ be the set of unselected bikes in a showroom. Let R be the equivalence relation on U with respect to the condition attributes. The family of equivalence classes corresponding to R is given by $U/I(R)=\{\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\}\}$. The lower and upper approximations of Z with respect to R are given by $L_R(Z)=\{A_2,A_3,A_4\}$, $U_R(Z)=\{A_2,A_3,A_4\}$. Therefore $B_R(Z)=\phi$. The soft rough topology on U is $\tau_R=\{U,\phi,\{A_2,A_3,A_4\}\}$ and its basis $\beta_R=\{U,\{A_2,A_3,A_4\}\}$. If the attribute '$B_1$' is removed from the set if condition attributes then we obtain $U/I(R-\{B_1\})=\{\{A_1\},\{A_2\},\{A_3,A_6\},\{A_4\},\{A_5\}\}$, $L_{R-\{B_1\}}(Z)=\{A_2,A_4\}$, $U_{R-\{B_1\}}(Z)=\{A_2,A_3,A_4,A_6\}$ and $B_{R-\{B_1\}}(Z)=\{A_3,A_6\}$, hence we get $\tau_{R-\{B_1\}}=\{U,\phi,\{A_2,A_3,A_4,A_6\},\{A_2,A_4\},\{A_3,A_6\}\}$ and its basis $\beta_{R-\{B_1\}}=\{U,\{A_2,A_4\},\{A_3,A_6\}\}\neq\beta_R$. If the attribute '$B_2$' is removed from the set of condition attributes then we obtain $U/I(R-\{B_2\})=\{\{A_1\},\{A_2\},\{A_3,A_4\},\{A_5\},\{A_6\}\}$. Then we get $L_{R-\{B_2\}}(Z)=\{A_2,A_3,A_4\}$, $U_{R-\{B_2\}}(Z)=\{A_2,A_3,A_4\}$ and $B_{R-\{B_2\}}(Z)=\phi$, hence we get the topology $\tau_{R-\{B_2\}}(Z)=\{U,\phi,\{A_2,A_3,A_4\}\}$ and its basis $\beta_{R-\{B_2\}}=\{U,\{A_2,A_3,A_4\}\}=\beta_R$. If the attribute '$B_3$' is removed, then we obtain $U/I(R-\{B_3\})=\{\{A_1\},\{A_2\},\{A_3,A_5\},\{A_4\},\{A_6\}\}$. Then we get $L_{R-\{B_3\}}(Z)=\{A_2,A_4\}$, $U_{R-\{B_3\}}(Z)=\{A_2,A_3,A_4,A_5\}$ and $B_{R-\{B_3\}}(Z)=\{A_3,A_5\}$, hence we get the topology $\tau_{R-\{B_3\}}(Z)=\{U,\phi,\{A_2,A_4\},\{A_2,A_3,A_4,A_5\},\{A_3,A_5\}\}$ and its basis $\beta_{R-\{B_3\}}=\{U,\{A_2,A_3,A_4,A_5\},\{A_3,A_5\}\neq\beta_R$. If the attribute '$B_4$' is removed from the set of condition attributes, then we obtain $U/I(R-\{B_4\})=\{\{A_1\},\{A_2\},\{A_3\},\{A_4\},\{A_5\},\{A_6\}\}$. Then we get $L_{R-\{B_4\}}(Z)=\{A_2,A_{3,}A_4\}$, $U_{R-\{B_4\}}(Z)=\{A_2,A_3,A_4\}$ and $B_{R-\{B_4\}}(Z)=\phi$. Hence we get the topology $\tau_{R-\{B_4\}}=\{U,\phi,\{A_2,A_3,A_4\}\}$ and its basis $\beta_{R-\{B_4\}}=\{U,\{A_2,A_3,A_4\}\}=\beta_R$. If $M=\{B_1,B_3\}$ then we obtain $U/I(r)=\{\{A_1,A_6\},\{A_2,A_5\},\{A_3,A_4\}\}$, $L_R(Z)=\{A_{3,}A_4\}$, $U_R(Z)=\{A_2,A_3,A_4,A_5\}$ and $\beta_r(Z)=\{A_2,A_5\}$ where r is the equivalence relation to M is given by $\beta_M=\{U,\{A_2,A_5\},\{A_3,A_4\}\}\neq\beta_{R-\{B_1\}}\neq\beta_{R-\{B_3\}}$. Therefore, Core $(R)=\{B_1,B_3\}$.

Observation

From both cases we conclude that 'manufacturing retail price' and 'milage' are the key attributes necessary to decide whether a bike is to be bought.

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