ON (1,2)γ - OPEN SETS

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Abstract

The aim of the paper is to introduce (1,2)γ-open sets and (1,2)γ-continuous function in a bitopological space.

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1. Introduction

In a topological space a weaker form of open sets called pre-open sets were introduced by Mashhour et al. [6] in 1982. These open sets are used to define a new class of open sets, γ-open sets by Andrijevic [1] in 1987. The family of all the γ-open sets in a topological space is a topology in the same space. In 2004, separation axioms based on the γ-open sets were studied by Navalagi et al. [7]. Ekici [2] used γ-open sets to define a new class of spaces called γ-US spaces.

The concept of bitopological space was introduced by Kelly [3] in 1963. A new class of sets called (1,2)α-open sets was defined by Lellis Thivagar [4] in 1991. Lellis Thivagar et al. [5] introduced (1,2) semi-open set and (1,2) pre-open set in a bitopological space and the notion of (1,2) pre-continuous function was introduced by Lellis Thivagar et al. [5]. In this paper, the (1,2)γ-open set is defined as an extension of γ-open sets in bitopological spaces and (1,2)γ-continuous function is introduced.

2. Preliminaries

Definition 2.1

A subset A of a topological space X is called pre-open [6] if $A \subset \text{int}(\text{cl}(A))$. The complement of a pre-open set is defined to be pre-closed. The family of all pre-open sets of X is defined by PO(X).

Definition 2.2

A subset A of the topological space X is called γ-open [1] if $A \cap S$ is pre-open, for all $S \in PO(X)$. The complement of γ-open set is γ-closed.

Definition 2.3

If τ_1 and τ_2 are two topologies on a non-empty set X, then the triple (X, τ_1, τ_2) is called a bitopological space [3].

Definition 2.4

A subset A of a bitopological space X is called,

(i) $\tau_1\tau_2$-open [4] if $A \in \tau_1 \cup \tau_2$.

(ii) $\tau_1\tau_2$-closed [4] if $A^c \in \tau_1 \cup \tau_2$.

Remark 2.5

(i) The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-interior of A and is denoted by $\tau_1\tau_2$-int(A).

(ii) The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called $\tau_1\tau_2$-closure of A and is denoted by $\tau_1\tau_2$-cl(A).

Definition 2.6

A subset A of X is called (1,2) pre-open [5] if $A \subset \tau_1\text{-int}(\tau_1\tau_2\text{-cl}(A))$ and (1,2) pre-closed if its complement in X is (1,2) pre-open or equivalently, $\tau_1\text{-cl}(\tau_1\tau_2(A)) \subset A$.

Notation 2.7

(i) The family of all (1,2) pre-open sets of X is denoted by (1,2) PO(X).

(ii) (1,2) pre-closure of A denoted by (1,2) pre-cl(A) is the intersection of all (1,2) pre-closed sets containing A.

Remark 2.8

A subset A of X is (1,2) pre-closed [5] if and only if (1,2) pre-cl(A) = A.

Throughout this paper, by X and Y we mean bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) respectively.

Definition 2.9

A function $f : X \to Y$ is (1,2) pre-continuous [5] if $f^{-1}(V)$ is (1,2) pre-open set in X for each $\sigma_1\sigma_2$-open set V of Y.

3. (1,2)γ-open sets

In this section we define (1,2)γ-open sets in a bitopological space X.

Definition 3.1

A subset A of the bitopological space X is called (1,2)γ-open if $A \cap S$ is (1,2) pre-open, for all $S \in (1,2)PO(X)$.

The complement of (1,2)γ-open set is (1,2)γ-closed. The family of all (1,2)γ-open sets of X is denoted by (1,2)γO(X).

Example 3.2

Let X = {a, b, c, d}; τ_1 = {ϕ, {a, c}, X} and τ_2 = {ϕ, {b, d}, X}.

Then (1,2)γO(X) = {ϕ, {a}, {c}, {a, c}, {a, b, c}{a, c, d}, X}.

Remark 3.3

A subset A of X is (1,2)γ-closed if and only if (1,2)γcl(A) = A.

Remark 3.4

Every (1,2)γ-open set is (1,2) pre-open set. But the converse need not be true, in general. In Example 3.2, {a, b} is (1,2) pre-open but not (1,2)γ-open.

Definition 3.5

A set U in a bitopological space X is a (1,2)γ-neighborhood of a point $x \in X$ if U contains a (1,2)γ-open set V such that $x \in V \subset U$.

Definition 3.6

Let A be a subset of a space X. The union of all (1,2)γ-open subsets of X contained in A is called (1,2)γ-interior of A and is denoted by (1,2)γ-int(A).

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Definition 3.7

(1,2)γ-closure of A denoted by (1,2)γcl(A) is the intersection of all (1,2)γ-closed sets containing A.

Theorem 3.8

For a subset A of a space X, the following are true.

(i) (1,2)γ-int(A) is a (1,2)γ-open set.

(ii) (1,2)γ-int(A) is the largest (1,2)γ-open set contained in A.

(iii) A is (1,2)γ-open if and only if (1,2)γ-int(A) = A.

Theorem 3.9

Let A and B be two subsets of a space X, the following are true.

(i) (1,2)γ-int(ϕ) = ϕ.

(ii) (1,2)γ-int(X) = X.

(iii) $A \subset B \Rightarrow$ (1,2)γ-int(A) $\subset$ (1,2)γ-int(B).

(iv) (1,2)γ-int(A) $\cup$ (1,2)γ-int(B) $\subseteq$ (1,2)γ-int(A $\cup$ B).

(v) (1,2)γ-int(A $\cap$ B) = (1,2)γ-int(A) $\cap$ (1,2)γ-int(B).

(vi) (1,2)γ-int((1,2)γ)-int(A)) = (1,2)γ-int(A).

Theorem 3.10

Let A be a subset of a space X. Then x $\in$ (1,2)γ-cl(A) if and only if every (1,2)γ-open set U containing x intersects A.

Proof

It needs to prove x $\notin$ (1,2)γ-cl(A) if and only if there exists a (1,2)γ-open set U containing x such that U does not intersect A. If x is not an element of in (1,2)γ-cl(A), the set.

U = X(1, 2)γ-cl(A) is a (1,2)γ-open set containing x that does not intersect A.

Conversely, if there exist a (1,2)γ-open set U containing x which does not intersect A, then X\U is a (1,2)γ-closed set containing A. Since (1,2)γ-cl(A) is the smallest closed set containing A then (1,2)γ-cl(A) $\subset$ X\U. Therefore, x cannot be in (1,2)γ-cl(A).

Theorem 3.11

Let X be a bitopological space and A,B are two subsets of X. Then the following are true.

(i) $(1,2)\gamma\text{-cl}(\phi) = \phi$.

(ii) $A \subset (1,2)\gamma\text{-cl}(A)$.

(iii) $A \subset B \Rightarrow (1,2)\gamma\text{-cl}(A) \subset (1,2)\gamma\text{-cl}(B)$.

(iv) $(1,2)\gamma\text{-cl}(A \cup B) = (1,2)\gamma\text{-cl}(A) \cup (1,2)\gamma\text{-cl}(B)$.

(v) $(1,2)\gamma\text{-cl}(A \cap B) = (1,2)\gamma\text{-cl}(A) \cap (1,2)\gamma\text{-cl}(B)$.

(vi) $(1,2)\gamma\text{-cl}(1,2)\gamma\text{-cl}(A) = (1,2)\gamma\text{-cl}(A)$.

4. (1,2)γ-Continuous Functions

In this section the topological concept continuity is extended to a bitopological space using (1,2)γ-open sets.

Definition 4.1

A function $f : X \rightarrow Y$ is (1,2)γ-continuous if $f^{-1}(V)$ is (1,2)γ-open set in X for each $\sigma_1\sigma_2$-open in Y.

Example 4.2

Let $X = \{a, b, c, d\}$; $\tau_1 = \{\phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, X\}$ and $\tau_2 = \{\phi, \{b\}, \{d\}, \{b, d\}, \{a, d\}, \{b, c, d\}, \{a, b, d\}, X\}$. Let $Y = \{p, q, r\}$; $\sigma_1 = \{\phi, \{p\}, Y\}$ and $\sigma_2 = \{\phi, \{q\}, Y\}$. Define a function $f : X \rightarrow Y$ by $f(a) = p$, $f(c) = q$, $f(b) = f(d) = r$. Then f is (1,2)γ-continuous.

Remark 4.3

Every (1,2)γ-continuous function is (1,2) pre-continuous.

Theorem 4.4

Let $f : X \rightarrow Y$ be a function. Then the following are equivalent.

(i) f is (1,2)γ-continuous.

(ii) The inverse image of each $\sigma_1\sigma_2$-closed set in Y is (1,2)γ-closed in X.

(iii) $(1,2)\gamma\text{-cl}[f^{(-1)}(V)] \subseteq f^{(-1)}[\sigma_1\sigma_2\text{-cl}(V)]$, for every $V \subseteq Y$.

(iv) $f[(1,2)\gamma\text{-cl}(U)] \subseteq \sigma_1\sigma_2\text{-cl}[f(U)]$, for every $U \subseteq X$.

Proof

(i) ⇒ (ii) Let $F \subseteq Y$ be a $\sigma_1\sigma_2$-closed set. Since $f : X \to Y$ is a $(1,2)\gamma$-continuous m $f^{-1}[Y - F] = X - f^{-1}(F)$ and this set is $(1,2)\gamma$-open. Therefore, $f^{-1}(F)$ is $(1,2)\gamma$-closed in X.

(ii) ⇒ (iii) Since $\sigma_1\sigma_2$-cl(V) is $\sigma_1\sigma_2$-closed for every $V \subseteq Y$, $f^{(-1)}[\sigma_1\sigma_2\text{-cl}(V)]$ is $(1,2)\gamma$-closed.

Therefore, $f^{(-1)}[\sigma_1\sigma_2\text{-cl}(V) = (1,2)\gamma\text{-cl}[f^{(-1)}(\sigma_1\sigma_2\text{-cl}(V))] \supseteq (1,2)\gamma\text{-cl}[f^{-1}(V)]$.

(iii) ⇒ (iv) Let $U \subseteq X$ and $f(U) = V$. Then $f^{(-1)}[\sigma_1\sigma_2\text{-cl}(V)] \supseteq (1,2)\gamma\text{-cl}[f^{-1}(V)]$.

Thus $f^{-1}(\sigma_1\sigma_2\text{-cl}(f(U)) \supseteq (1,2)\gamma\text{-cl}[f^{-1}(f(U))] \supseteq f[(1,2)\gamma\text{-cl}(U)]$.

(iv) ⇒ (ii) Let $W \subseteq Y$ be a $\sigma_1\sigma_2$-closed set, $U = f^{-1}(W)$, then $f[(1,2)\gamma\text{-cl}(U)] \subseteq \sigma_1$ $\text{cl}[f(U)] = f^{(-1)}[f(1,2)\gamma\text{-cl}[f(f^{-1}(W))] = \sigma_1\sigma_2\text{-cl}(W) = W$.

Thus $(1,2)\gamma\text{-cl}(U) \subseteq f^{(-1)}[f((1,2)\gamma\text{-cl}(U))] \subseteq f^{(-1)}(W) = U$. So U is $(1,2)\gamma$-closed.

(ii) ⇒ (i) Let $V \subseteq Y$ be a $\sigma_1\sigma_2$-open set, then $Y - V$ is $\sigma_1\sigma_2$-closed.

Then $f^{-1}(Y - V) = X - f^{-1}(V)$ is $(1,2)\gamma$-closed in X and hence $f^{-1}(V)$ is $(1,2)\gamma$-open in

Remark 4.5

If $f : X \to Y$ is a function, and one of the following holds, then f is (1 continuous.

(i) $f^{(-1)}[(1,2)\gamma\text{-int}(B)] \subseteq (1,2)\gamma\text{-int}(B)] \subseteq (1,2)\gamma\text{-int}[f^{-1}(B)]$ for each $B \subseteq Y$.

(ii) $(1,2)\gamma\text{-cl}[f^{(-1)}(B)] \subseteq f^{(-1)}[(1,2)\gamma\text{-cl}(B)]$ for each $B \subseteq Y$.

(iii) $f[(1,2)\gamma\text{-cl}(A)] \subseteq (1,2)\gamma\text{-cl}[f(A)]$ for each $A \subseteq X$.

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