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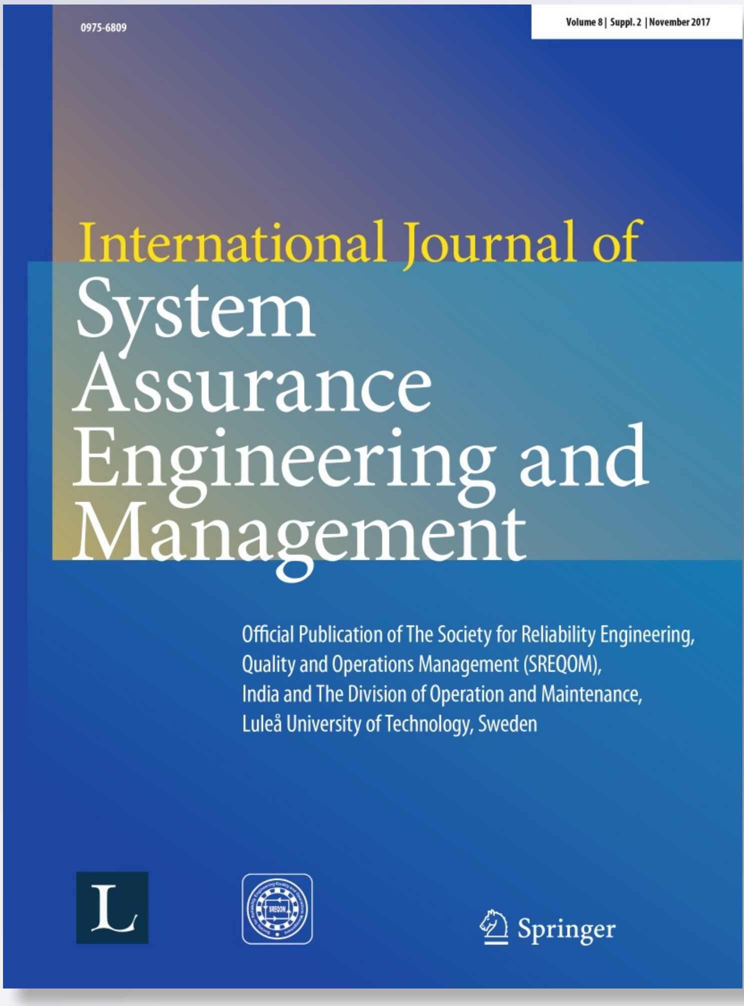

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ORIGINAL ARTICLE

A two-echelon supply chain coordination with quantity discount incentive for fixed lifetime product in a fuzzy environment

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Abstract Efficient and effective management of a supply chain is of great importance for the success of the digital economy. Coordination among the members of a supply chain play a vital role for its effective management. The members of the supply chain may agree to cooperate initially, but owing to the competition prevailing in business environments, they may be tempted to maximize their profits and deviate from any agreement. So, effective mechanism is essential to enforce coordination among the members in a supply chain. In today's competitive environment, the inventory managers are also interested in simple and easy procedures to apply them in their organizations. This paper investigates a single-manufacturer and a single-buyer two echelon supply chain model for a fixed lifetime product in a fuzzy cost environment with a quantity discount strategy as a coordination mechanism. Crisp models are developed under different scenarios (1) without coordination (2) with coordination and (3) system optimization. Fuzzy models are also formulated by representing the ordering cost and the holding cost of the manufacturer by trapezoidal fuzzy numbers. Signed distance method is adopted for defuzzification. Numerical results highlighting the sensitivity of various parameters are also elucidated.

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Keywords Supply chain coordination · Inventory · Quantity discount · Fixed lifetime product · Fuzzy numbers · Signed distance method

1 Introduction

Globalization has maximized business endeavors and ensured the stability of the economy of any nation. Challenges are in business but they make the business people more competitive and successful. Due to technological innovation and easy access to alternative products in today's market, customers move towards the product of their own choice. Supply chain is a complex system that involves different kinds of people (manufacturers cum suppliers, wholesalers, vendors, retailers, end customers etc.), resources required and available, information exchanged and activities performed. The present study concerns the two-echelon supply chain involving exclusively, the manufacturer and the buyer. Co-ordination between them is essential for the success of any business. Deviation of any one of them from the agreement for the sake of more profit will be detrimental to business. As success is the ultimate end of any business, enough care has to be taken in inventory management. A probe into the supply chain inventory models developed by various researchers will provide a clear understanding of the study undertaken.

2 Literature review

In real life, items such as medicinal products, food stuffs, provisions, chemicals, fresh products, etc. have fixed shelf life times. Perishability of such products is a pertinent issue globally. An ineffective inventory management of such products at each level of the supply chain increases the cost of the system. So, numerous researchers have worked on perishable inventory models. Hwang and Hahn (2000) have presented an optimal procurement policy for items with an inventory level-dependent demand rate and fixed lifetime. The investigation of Kanchana and Anulark (2006) were on the effect of product perishability and retailer's stockout policy on the total cost, net profit, service level and average inventory level in a two echelon inventory system and they proposed a periodic review inventory distribution model for dealing a fixed life perishable product.

Effective inventory management is one of the most significant task of a global manufacturing supply chain strategy. So, coordination mechanism such as quantity discount, credit option, sales rebate, etc. were incorporated in two echelon supply chain inventory models developed by various researchers. Monahan (1984), Jucker and Rosenblatt (1985), Lee and Rosenblatt (1986), Goyal and Gupta (1989) and Weng (1995) have focussed on quantity discount as a coordination mechanism. Luo (2007) developed a buyer–vendor coordination supply chain model with credit period incentive as a coordination mechanism. Sales rebate as a coordination mechanism was incorporated in the model developed by Wong et al. (2009).

Bannerjee (1986) proposed an integrated vendor–buyer (IVB) model where the demand rate of the buyer is a constant and the manufacturer's production schedule is to produce the same amount of inventory as the buyer orders each time. However this lot for lot policy would not be optimal if the manufacturer's setup cost was significantly larger than the buyer's ordering cost. So, the frequency of manufacturer's setup should not be the same as that of the buyer's ordering. Ben-Daya and Hariga (2004) proposed an integrated single vendor single buyer model with stochastic demand and variable lead time. Huang (2004) discussed an optimal policy for a single vendor single buyer integrated production inventory problem with process unreliability consideration. Lee (2005) developed an integrated inventory control model that comprised of IVB and integrated procurement production (IPP) systems making joint economic lotsizes of manufacturer's raw material ordering, production batch and buyer's ordering. The effectiveness of the quantity discount strategy in a single vendor and a single buyer supply chain for a fixed life time product was investigated by Duan et al. (2010). An integrated supply chain model for perishable items with trade credit policy under imprecise environment was presented by Singh and Singh (2012). Recently, a two level supply chain coordination with delay in payments for fixed lifetime products was highlighted by Duan et al. (2012). A production inventory model with probabilistic deterioration in two-echelon supply chain management was developed by Sarkar (2013). Taleizadeh et al. (2013) presented an economic order quantity (EOQ) model for perishable product with special sale and shortage. Recently Taleizadeh and Nematollahi (2014) have developed an inventory control problem for deteriorating items with backordering and financial considerations.

In today's challenging environment, the cost components involved in maintaining inventories are always likely to vary from one cycle to another and it is also not so easy to get the real statistical data especially for newly launched products. Hence it is difficult to find the probability distributions of these variables. As fuzziness is the closest possible approach to reality, the introduction of fuzzy set theory by Zadeh (1965) has drawn the attention of many researchers in different areas of science and technology including inventory control problems. In the classical newsboy problem, Ishii and Konno (1998) fuzzified the shortage cost to a L-fuzzy number. Maiti and Maiti (2006) have developed a fuzzy inventory model with two warehouses under different possible constraints. A two storage inventory model with fuzzy deterioration over a random planning horizon was discussed by Maiti et al. (2006). Using geometric programming approach, a pricing and marketing planning model was developed by Sadjadi et al. (2010) in which the selling price, marketing expenditure and lot size are considered as fuzzy numbers.

Based on the extension principle, Liu (2012) has presented a solution to the fuzzy integrated production and marketing planning model. Fuzzy EOQ models for deteriorating items and for imperfect items with inspection errors and shortages were developed by Saha and Chakrabarti (2012) and Liu and Zheng (2012) respectively. Yadav et al. (2013) have discussed the retailers optimal policy under trade credit and inflation in a fuzzy environment. Notable researchers Yao and Lee (1999), Yao et al. (2003), Maiti (2011), Kumar et al. (2012) and Mahata and Goswami (2013) have worked on developing fuzzy inventory models by fuzzifying one or more parameters and keeping the remaining as crisp parameters. Recently, Samal and Pratihar (2014) have investigated an inventory model which provided an optimum value of EOQ under uncertainties owing to fuzziness in demand and various costs like ordering cost, holding cost and backordering cost. Numerous defuzzification methods such as centroid method, graded mean integration representation method, signed distance method, extension principle method, etc., have been suggested in the literature. Ouyang and Chang (2001) used the centroid method to defuzzify the fuzzy total cost. Graded mean integration representation method was adopted by Dutta and Chakraborty (2005) to find the optimal order quantity in a single-period inventory model with discrete fuzzy random demand involving imprecise probabilities. Yao and Chiang (2003) developed an EOQ model with the total demand and the unit carrying cost being triangular fuzzy numbers in which the signed distance and the centroid defuzzification methods are used. Chiang et al. (2005) also adopted the signed distance method to defuzzify the fuzzy total cost. Researchers Bjork (2009) and Yao et al. (2003), etc. have shown that signed distance method is better than other methods of defuzzification.

The focus of this paper is on a two echelon supply chain model consisting of a single manufacturer and a single buyer dealing with a fixed life time product. A manufacturing system is considered in which the manufacturer produces the product at a finite constant rate P. In order to avoid shortages it is assumed that the production rate is greater than the demand rate. Initially the order size of the buyer is Q_0 and that of the manufacturer is an integer multiple of the buyer. In order to reduce the setup cost, ordering cost and inventory holding costs, the manufacturer influences the buyer to change his order size by a factor K($K > 1$) proposing an order size dependent quantity discount offer which ultimately decreases his inventory costs and earns him additional savings. The effectiveness of the quantity discount strategy is analyzed by developing crisp models under three different scenarios. To study the effect of uncertainties on inventory decisions, fuzzy models are also formulated by representing the ordering cost and the holding cost of the manufacturer as fuzzy trapezoidal numbers. Signed distance method is adopted for defuzzification in each model owing to its efficiency.

The paper is designed as follows. In Sect. 3, the assumptions and notations to be used throughout the paper are introduced. Section 4 deals with the crisp models: (1) a model without system coordination (2) a model with system coordination and (3) a model for system optimization. The effectiveness of the quantity discount strategy is also analyzed. In Sect. 5, inventory models with fuzzy cost components are developed. In Sect. 6, a numerical example is provided and sensitivity analysis is carried out to analyze the impact of various crisp parameters on the optimal solution. The proposed model serves as a pioneering work to analyze the effectiveness of the quantity discount strategy in a two-echelon supply chain management under fuzzy environment.

3 Assumptions and notations

The following assumptions and notations are used in developing the crisp inventory model.

3.1 Assumptions

1. Demand rate is known and constant
2. Production rate is known and constant, $P > D$.
3. The lead time is zero and the replenishment rate is infinite.
4. Shortages are not allowed.
5. A single item is considered.
6. The items ordered by the buyer are received new and fresh (Hwang and Hahn 2000) that is, their age equals zero. To be more realistic, if the items received have positive age denoted by l, as l is exogenous, replacing the product lifetime L by $L = L - l$ in the following analysis, the general case leads to the same conclusions.

3.2 Notations

D	Annual demand of the buyer, units/year.
P	Manufacturer's production rate, units/year.
L	Lifetime of the product.
A_1	Manufacturer's production setup cost per batch, \$/ batch.
A_2	Buyer's ordering cost per order, \$/ order.
h_1	Manufacturer's holding cost.
h_2	Buyer's holding cost.
p_2	Delivered unit price paid by the buyer.
Q_0	Buyer's economic order quantity, units/order.
m	Manufacturer's order multiple in the absence of any coordination.
m^*	Optimal value of m.
K	Buyer's order multiple under coordination.
n	Manufacturer's order multiple under coordination.
n^*	Optimal value of n.
d(K)	Discount factor to the buyer if he orders KQ_0 per unit time.
TCB	Total annual cost of the buyer.
$TC_M(m)$	Total cost of the manufacturer without coordination.
$\overline{TC_M}(n)$	Total cost of the manufacturer with coordination.

4 Crisp inventory models

4.1 Model formulation for the system without coordination

The inventory model for a single manufacturer and a single buyer supply chain for a single fixed lifetime product is designed similar to that as in Lee (2005): The manufacturer produces the product in batches at a finite rate P($P > D$) so as to avoid shortages. The finished goods are delivered to the buyer periodically. In the absence of any coordination, the optimal ordering quantity of the buyer is $Q_0 = \sqrt{\frac{2DA_2}{h_2}}$ and therefore the total annual cost of the buyer is $TCB = \sqrt{2DA_2h_2}$. To meet the buyer's demand at fixed interval's of time $t_0 = Q_0/D$, the production lot size of the manufacturer is mQ_0, where m is a positive integer. The operations of manufacturer's production and the delivery of its finished goods is considered to be synchronous. During the production uptime, the manufacturer's on hand finished goods are gradually increasing with a rate P and it depletes by a quantity Q_0 for every time interval Q_0/D. Therefore the on hand inventory appears as a sawtooth pattern during production uptime. While during the production downtime the manufacturer's finished goods inventory is flat if no replenishment has occurred, and it will be depleted by a quantity Q_0 when a quantity Q_0 is delivered to the buyer. Figures 1 and 2 represents the inventory of the buyer and the time weighted inventory of the manufacturer respectively.

From Fig. 2, the manufacturer's average inventory of finished goods

$$
\begin{aligned}
&= \frac{D}{mQ_0}[Area\ of\ EACD - Area\ of\ EAB \\
&\quad - Accumulated\ buyer\ consumption] \\
&\quad = \left[mQ_0\left(\frac{Q_0}{P} + (m-1)\frac{Q_0}{D}\right) - \frac{1}{2}(mQ_0)(mQ_0/P) \right.\\
&\qquad \left. - \frac{m(m-1)}{2}Q_0\frac{Q_0}{D}\right] \Big/ \frac{mQ_0}{D} \\
&\quad = \frac{Q_0}{2}\left[(m-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right]
\end{aligned}
$$

Hence the total cost for the manufacturer is given by

$$
\begin{aligned}
TC_M(m) &= \frac{DA_1}{mQ_0} + \frac{h_1Q_0}{2}\left[(m-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] \\
&= \frac{A_1}{m}\sqrt{\frac{Dh_2}{2A_2}} + h_1\sqrt{\frac{DA_2}{2h_2}}\left[(m-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right]
\end{aligned}
$$

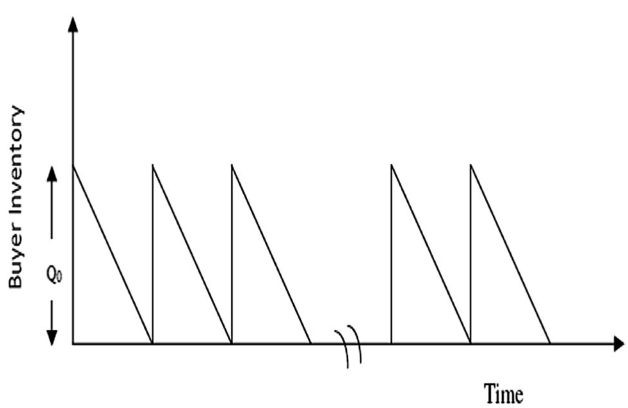

Fig. 1 Buyer's inventory

So, the manufacturer's problem without coordination can be formulated as follows: $Min\ TC_M(m)$ subject to the constraints

$$
\begin{cases} mt_0 \le L, \\ m \ge 1 \end{cases} \tag{4.1}
$$

where $mt_0 \le L$ ensures that items remain fresh before they are used by the buyer.

Theorem 1 *Let m^* be the optimum value of m obtained from (4.1). If $L^2 \ge \frac{2A_2}{Dh_2}$ then*

$$
m^* = \min\left\{\left\lceil\sqrt{\frac{A_1h_2}{A_2h_1(1-\frac{D}{P})} + \frac{1}{4}} - \frac{1}{2}\right\rceil, \left\lceil\frac{L}{\sqrt{\frac{2A_2}{Dh_2}}}\right\rceil\right\} \tag{4.2}
$$

where $\lceil x \rceil$ is the least integer greater than or equal to x, $L^2 \ge \frac{2A_2}{Dh_2}$ is ensures that $m^* \ge 1$.

Proof Since

$$
\frac{d^2TC_M(m)}{dm^2} = \frac{2A_1}{m^3}\sqrt{\frac{Dh_2}{2A_2}} > 0
$$

$TC_M(m)$ is strictly convex in m. Let m_1^* be the optimum of $\min_{m \ge 1} TC_M(m)$ then

$$
\begin{aligned}
m* &= \max\left\{\min\left\{m | TC_M(m) \le TC_M(m+1)\right\}, 1\right\} \\
&= \max\left\{\min\left\{m(m+1) \ge \frac{A_1h_2}{A_2h_1(1-\frac{D}{P})}\right\}, 1\right\} \\
&= \left\lceil\sqrt{\frac{A_1h_2}{A_2h_1(1-\frac{D}{P})} + \frac{1}{4}} - \frac{1}{2}\right\rceil \ge 1
\end{aligned}
$$

using $t_0 = \sqrt{\frac{2A_2}{Dh_2}}$ in (4.1), we get $m\sqrt{\frac{2A_2}{Dh_2}} \le L$

Set $m_2^* = \left\lceil\frac{L}{\sqrt{\frac{2A_2}{Dh_2}}}\right\rceil$. Since $L^2 \ge \frac{2A_2}{Dh_2}, m_2^* \ge 1$ is true. Since $TC_M(m)$ is convex in m, if $m_1^* \le m_2^*, m^* = m_1^*$ else $m^* = m_2^*$. So if $L^2 \ge \frac{2A_2}{Dh_2}, m^* = \min\{m_1^*, m_2^*\}$.

Remark 1 For the system without coordination, the production lot size of the manufacturer is $m^*\sqrt{2DA_2/h_2}$ units each year with an interval of $m^*\sqrt{2DA_2/h_2}/D$ throughout that time. The minimised total cost is $TC_M(m^*)$.

4.2 Model formulation for the system with coordination

If the current order size of the buyer is changed by a factor K(K > 1), the manufacturer agrees to offer a quantity discount at a discount factor $d(K)$ so that the order quantity of the manufacturer and the buyer are nKQ_0 (n is a positive integer) and KQ_0 respectively. The total cost $\overline{TC_M}(n)$ of the manufacturer is given by

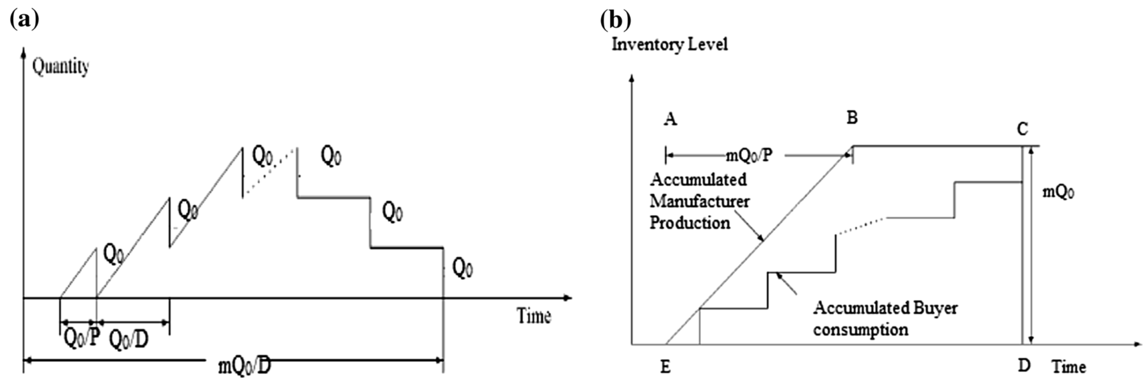

Fig. 2 Manufacturer's inventory

$$\overline{TC_M}(n) = \frac{DA_1}{nKQ_0} + \frac{KQ_0h_1}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + p_2Dd(K) \tag{4.3}$$

The problem with coordination can be formulated as follows:

$Min\,\overline{TC_M}(n)$

subject to the constraints

$$\begin{cases} nKt_0 \le L \\ \dfrac{DA_2}{KQ_0} + \dfrac{KQ_0h_2}{2} - \sqrt{2DA_2h_2} \le p_2Dd(K) \\ n \ge 1 \end{cases} \tag{4.4}$$

The first constraint ensures the freshness of the items before they are sold by the buyer whereas the second constraint is the retailer's participation constraint. (i.e) The buyer's cost under coordination cannot exceed that in the absence of any coordination.

Theorem 2 *Let m^* and n^* be the optimum value of m and n obtained from (4.1) and (4.4) respectively then*

$$\overline{TC_M}(n^*) \le TC_M(m^*) \tag{4.5}$$

Proof If the buyer changes his order size by a factor K($K > 1$) it increases his inventory cost and so the manufacturer offers him a compensation $p_2Dd(K)$. This compensation takes the smallest value when the second constraint is an equation. So if $\overline{TC_M}(n)$ is minimized then

$$\frac{DA_2}{KQ_0} + \frac{KQ_0h_2}{2} - \sqrt{2DA_2h_2} = p_2Dd(K)$$

and hence

$$d(K) = \frac{\frac{DA_2}{KQ_0} + \frac{KQ_0h_2}{2} - \sqrt{2DA_2h_2}}{p_2D} \tag{4.6}$$

putting K $= 1$ in (4.6) we get

$$d(1) = \frac{\frac{DA_2}{Q_0} + \frac{Q_0h_2}{2} - \sqrt{2DA_2h_2}}{p_2D} = 0$$

Hence (4.4) is equivalent to (4.1) if K $= 1$. i.e (4.1) is a special case of (4.4) and so (4.5) holds.

Remark 2 The result of the above theorem is shown graphically in Fig. 3. As the optimal total cost under coordination is less than that without coordination, the manufacturer will be benefitted by motivating the buyer to order KQ_0 units every time.

Next the manufacturer's and the buyer's optimal order quantity will be determined as follows. Substituting (4.6) into (4.3) we get,

$$\overline{TC_M}(n) = \frac{D}{KQ_0}\left[\frac{A_1}{n} + A_2\right] + \frac{KQ_0}{2}\left[h_1\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + h_2\right] - \sqrt{(2DA_2h_2)}$$

Since d(K) is convex in K, $\overline{TC_M}(n)$ is obviously convex in K. Let K^* be the minimum of $\overline{TC_M}(n)$, then

$$K^*(n) = \sqrt{\frac{2\left(\frac{DA_1}{nQ_0} + \frac{DA_2}{Q_0}\right)}{Q_0h_1\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + Q_0h_2}} \tag{4.8}$$

substituting (4.8) and $t_0 = \sqrt{\frac{2A_2}{Dh_2}}$ in $nKt_0 \le L$ we get $n^2\left(\frac{A_1}{n} + A_2\right) \le \frac{L^2Q_0^2h_2}{4A_2}\left[h_1\left((n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right)+h_2\right]$

Then the first constraint of (4.4) becomes

$$g(n) = -A_2n^2 + n\left[\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right) - A_1\right] + \frac{L^2D}{2} \times \left[h_2 - h_1 + \frac{2Dh_1}{P}\right] \ge 0 \tag{4.9}$$

Substituting (4.8) and $Q_0 = \sqrt{\frac{2DA_2}{h_2}}$ into $\overline{TC_M}(n)$, we get

$$\overline{TC_M}(n) = \left(2\left[DA_1h_1\left(1-\frac{D}{P}\right) + \frac{1}{n}\left(DA_1(h_2-h_1) + \frac{2D^2h_1A_1}{P}\right) + n\left(DA_2h_1\left(1-\frac{D}{P}\right)\right) - DA_2h_1\left(1-\frac{2D}{P}\right) + DA_2h_2\right]\right)^{1/2} - \left(2DA_2h_2\right)^{1/2} \tag{4.10}$$

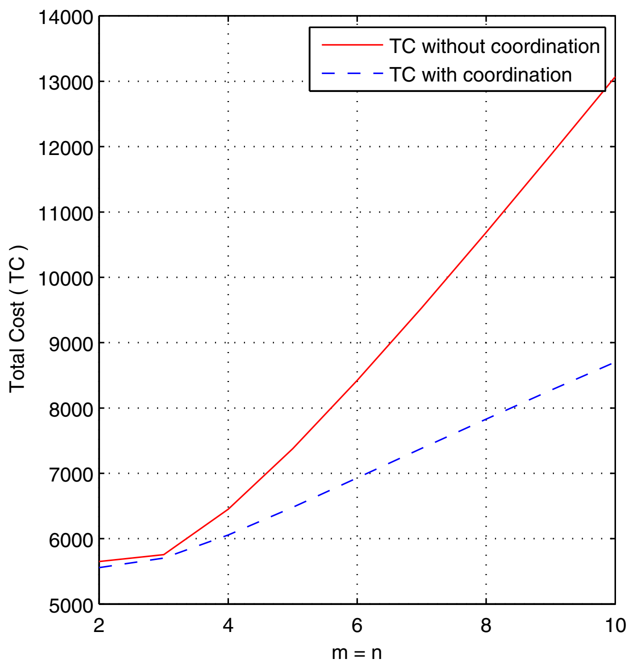

Fig. 3 The total cost curve

Since $\sqrt{x}$ is a strictly increasing function for $x \geq 0$, the problem given by (4.4) becomes

$$Min\ \widetilde{TC_M}(n) = \left[DA_1h_1\left(1-\frac{D}{P}\right)+\frac{1}{n}\left(DA_1(h_2-h_1)+\frac{2D^2h_1A_1}{P}\right)\right.$$
$$\left.+n\left(DA_2h_1\left(1-\frac{D}{P}\right)\right)-DA_2h_1\left(1-\frac{2D}{P}\right)+DA_2h_2\right]$$

subject to the constraints

$$\begin{cases} g(n) \geq 0 \\ n \geq 1 \end{cases} \tag{4.11}$$

which is a nonlinear programming problem. Since $\widetilde{TC_M}''(n) = 2A_1(h_2-h_1)/n^3 + 4D^2h_1A_1/n^3P, \ > 0$ when $h_2 \geq h_1$, $\widetilde{TC_M}(n)$ is convex when $h_2 \geq h_1$ and concave otherwise. Also since $g''(n) = -2A_2 < 0$, g(n) is strictly concave.

Proposition 1 *Let n_1^* be the minimum of $\widetilde{TC_M}(n)$ for $n \geq 1$ then*

$$n_1^* = \begin{cases} \left[\sqrt{\dfrac{A_1}{A_2}\left(\dfrac{h_2-h_1}{h_1}+\dfrac{2D}{P}\right)+\dfrac{1}{4}}-\dfrac{1}{2}\right] & \dfrac{A_1}{A_2}\left(\dfrac{h_2-h_1}{h_1}+\dfrac{2D}{P}\right) \geq 2 \\ 1 & otherwise \end{cases} \tag{4.12}$$

Proof Since n_1^* is the minimum of $\widetilde{TC_M}(n)$ for $n \geq 1$, the following inequality holds: $\widetilde{TC_M}(n_1^*) \leq min\,(\widetilde{TC_M}(n_1^*-1), \widetilde{TC_M}(n_1^*+1))$ $\widetilde{TC_M}(n_1^*) - \widetilde{TC_M}(n_1^*-1) \leq 0 \Longrightarrow$

$$\left(n_1^*-\frac{1}{2}\right)^2 \leq \frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)+\frac{1}{4} \tag{4.13}$$

Similarly, $\widetilde{TC_M}(n_1^*) - \widetilde{TC_M}(n_1^*+1) \leq 0 \Longrightarrow$

$$\left(n_1^*+\frac{1}{2}\right)^2 \geq \frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)+\frac{1}{4} \tag{4.14}$$

Hence if $\frac{A_1}{A_2}(\frac{h_2-h_1}{h_1}+\frac{2D}{P})+\frac{1}{4} < 0$, $\widetilde{TC_M}(n_1^*) \leq \widetilde{TC_M}(n_1^*+1)$ for any given n, so $n_1^* = 1$. If $\frac{A_1}{A_2}(\frac{h_2-h_1}{h_1}+\frac{2D}{P})+\frac{1}{4} \geq 0$, by (4.13) and (4.14),

$$\sqrt{\frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)+\frac{1}{4}}-\frac{1}{2} \leq n_1^* \leq \sqrt{\frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)+\frac{1}{4}}+\frac{1}{2}$$

Therefore, $n_1^* = \left[\sqrt{\frac{A_1}{A_2}(\frac{h_2-h_1}{h_1}+\frac{2D}{P})+\frac{1}{4}}-\frac{1}{2}\right]$
If $0 < \frac{A_1}{A_2}(\frac{h_2-h_1}{h_1}+\frac{2D}{P}) < 2$ then $n_1^* = 1$ so (4.12) holds.

Proposition 2 *Let $n_{2(1)}^*$ and $n_{2(2)}^*$ be solutions of the quadratic equation $g(n) = 0$ then the following two conclusions hold.*

(i) If $\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)^2+2A_2L^2D(h_2-h_1+\frac{2Dh_1}{P}) < 0$ or $\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)^2+2A_2L^2D(h_2-h_1+\frac{2Dh_1}{P}) \geq 0$ and $n_{2(1)}^* < 1$ then $g(n) < 0$ for $n \geq 1$

(ii) If $\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)^2+2A_2L^2D(h_2-h_1+\frac{2Dh_1}{P}) \geq 0$ and $n_{2(1)}^* \geq 1$, then

  (a) if $n_{2(2)}^* \geq 1$, $g(n) \geq 0$ for $\left[n_{2(2)}^*\right] \leq n \leq \left[n_{2(1)}^*\right]$

  (b) if $n_{2(2)}^* < 1$ and $n_{2(1)}^* \geq 1$, $g(n) \geq 0$ for $1 \leq n \leq \left[n_{2(1)}^*\right]$

Proof

$$n_{2(1)}^* = \frac{\sqrt{\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)^2+2A_2L^2D(h_2-h_1+\frac{2Dh_1}{P})}+\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)}{2A_2}$$

$$n_{2(2)}^* = \frac{-\sqrt{\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)^2+2A_2L^2D(h_2-h_1+\frac{2Dh_1}{P})}+\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)}{2A_2}$$

Because g(n) is a quadratic equation, the following conclusions hold.

(1) If $\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)^2+2A_2L^2D\left(h_2-h_1+\frac{2Dh_1}{P}\right)<0$ then $g(n)<0$ for every n.

(2) If $\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right)-A_1\right)^2+2A_2L^2D\left(h_2-h_1+\frac{2Dh_1}{P}\right)\geq 0$ then $n^*_{2(1)}$ and $n^*_{2(2)}$ are the real solutions of g(n) = 0

In view of $n\geq 1$,

(i) if $n^*_{2(1)}<1$ then $g(n)<0$ for $n\geq 1$.

(ii) if $n^*_{2(2)}\geq 1$ then $g(n)\geq 0$ for $\left\lceil n^*_{2(2)}\right\rceil \leq n\leq \left\lceil n^*_{2(2)}\right\rceil$.

Hence the proof. □

Remark 3 If (i) of Proposition 2 holds, the first constraint of (4.4) does not hold for any $n\geq 1$ and in this case the problem is meaningless. If (ii) of Proposition 2 holds, the first constraint of (4.4) hold for $\left\lceil n^*_{2(2)}\right\rceil \leq n\leq \left\lceil n^*_{2(1)}\right\rceil$ or $1\leq n\leq \left\lceil n^*_{2(1)}\right\rceil$.

Theorem 3 *If $h_2\geq h_1$ and $n^*_{2(1)}\geq 1$, then*

(i) *if $1\leq n^*_1\leq \left\lceil n^*_{2(1)}\right\rceil, n^*=n^*_1$.*

(ii) *if $n^*_1>\left\lceil n^*_{2(1)}\right\rceil, n^*=\left\lceil n^*_{2(1)}\right\rceil$.*

Proof If $h_2\geq h_1$, $\widetilde{TC_M}(n)$ is a convex function. Since n^*_1 is the minimum of $\widetilde{TC_M}(n)$ for $n\geq 1$, if $1\leq n^*_1\leq \left\lceil n^*_{2(1)}\right\rceil$ then $n^*=n^*_1$. If $n^*_1\geq \left\lceil n^*_{2(1)}\right\rceil$, then $n^*\leq \left\lceil n^*_{2(1)}\right\rceil$. Since $\widetilde{TC_M}(n)$ is decreasing on this interval, (ii)holds. Hence the proof. □

Theorem 4 *if $h_2\geq h_1$ then $K^*(n^*)>1$.*

Proof We know that $K^*(n)=\sqrt{\frac{h_2\left(\frac{A_1}{n}+A_2\right)}{A_2\left[h_1\left((n-1)(1-\frac{D}{P})+\frac{D}{P}\right)+h_2\right]}}$

(i) If $n^*=n^*_1$ and $\frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)\geq 2$ then $n^*=n^*_1=\left\lceil\sqrt{\frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)+\frac{1}{4}}-\frac{1}{2}\right\rceil$

Since $\left\lceil\sqrt{x+\frac{1}{4}}-\frac{1}{2}\right\rceil\leq\sqrt{x}+1$ holds for $x\geq 0$, and $K^*(n)$ is a decreasing function of n, to prove that $K^*(n^*)>1$ it is enough to prove that

$K^*\left(\sqrt{\frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)}+1\right)>1$

$$\text{i.e.}\quad \frac{h_2\left[\frac{A_1}{\sqrt{\frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)}+1}+A_2\right]}{A_2\left[h_1\left[\sqrt{\frac{A_1}{A_2}\left(\frac{h_2-h_1}{h_1}+\frac{2D}{P}\right)}\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]+h_2\right]}>1 \tag{4.15}$$

By a simple computation, we get

$$PA_1h_2>DA_2h_1 \tag{4.16}$$

Since A_1,A_2,h_1,h_2 are all positive and P > D, inequality (4.16) is true.

(ii) If $n^*=n^*_1=1$, $K^*(1)=\sqrt{(A_1+A_2)/A_2}$. Since A_1,A_2 are both positive, $K^*(1)>1$.

(iii) If $n^*=[n^*_{2(1)}]$ then $n^*_1>[n^*_{2(1)}]$. In view of $K^*(n^*_1)>1$, and $K^*(n)$ is a decreasing function , so $K^*([n^*_{2(1)}])\geq K^*(n)>1$.

By (i)–(iii), $K^*(n)>1$ if $h_2\geq h_1$. Hence the proof. □

4.3 Model formulation for system optimisation

If there is a common decision maker for both the buyer and the manufacturer, the objective is to minimize the total cost of the system. Let Q be the buyer's order quantity, then the manufacturer produces nQ units every time, where Q and n are decision variables. The problem for system optimisation can be formulated as follows:

$$Min\,TC_S(n,Q)=\frac{DA_1}{nQ}+\frac{h_1Q}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]+\frac{DA_2}{Q}+\frac{Qh_2}{2}$$

subject to the constraints

$$\begin{cases}\frac{nQ}{D}\leq L\\ n\geq 1\end{cases} \tag{4.17}$$

Theorem 5 *The proposed quantity discount strategy can achieve system coordination.*

Proof $TC_S(n,Q)$ is convex in Q. Let Q^* be the buyer's optimal order quantity, then Q^* satisfies

$$\left.\frac{\partial TC_S(n,Q)}{\partial Q}\right|_{Q=Q^*}=\left(-\frac{DA_1}{n}-DA_2\right)\frac{1}{(Q^*)^2}+\frac{h_1}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]+\frac{h_2}{2}=0 \tag{4.18}$$

so,

$$Q^*(n) = \sqrt{\frac{2D(\frac{A_1}{n} + A_2)}{h_1[(n-1)(1-\frac{D}{P}) + \frac{D}{P}] + h_2}} \tag{4.19}$$

Using the above equations, the problem is equivalent to

$$Min\ \widetilde{TC_s}(n) = \left[DA_1h_1\left(1-\frac{D}{P}\right) + \frac{DA_1}{n}\left(h_2 - h_1 + \frac{2Dh_1}{P}\right) + nDA_2h_1\left(1-\frac{D}{P}\right) - DA_2h_1\left(1-\frac{2D}{P}\right) + DA_2h_2\right]$$

subject to the constraints

$$\begin{cases} -A_2n^2 + n\left(\frac{L^2Dh_1}{2}\left(1-\frac{D}{P}\right) - A_1\right) + \frac{L^2D}{2}\left(h_2 - h_1 + \frac{2Dh_1}{P}\right) \geq 0 \\ n \geq 1 \end{cases} \tag{4.20}$$

It is obvious that (4.20) is exactly the same as (4.11), so they have the same optimum n^*. Also,

$$TCs(n^*) = \overline{TC_M}(n^*) + \sqrt{2DA_2h_2} \tag{4.21}$$

where $\sqrt{2DA_2h_2}$ is the buyer's actual cost under non-coordination and the buyer's optimal order quantity under coordination is equal to that under system optimisation.

i.e.

$$K^*(n^*)Q_0 = Q^*(n^*) \tag{4.22}$$

The manufacturer's optimal order quantity is equal in these two cases.

i.e.

$$n^*K^*(n^*)Q_0 = n^*Q^*(n^*) \tag{4.23}$$

The above three equations reveal that the quantity discount contract can achieve coordination.

5 Inventory models with fuzzy cost components

Fuzzy inventory models are developed based on the preliminaries given in "Appendix". Let the ordering cost and the holding cost of the manufacturer be fuzzy and all other components be crisp constants. Let them be represented by trapezoidal fuzzy numbers as given below.

$$\begin{aligned} \tilde{A_1} &= (A_1 - \delta_1, \quad A_1 - \delta_2, A_1 + \delta_3, A_1 - \delta_4) \\ \tilde{h_1} &= (h_1 - \delta_5, h_1 - \delta_6, h_1 + \delta_7, \quad h_1 - \delta_8) \end{aligned} \tag{5.1}$$

where $\delta_i, i = 1,2,\ldots 8,$ are arbitrary positive numbers under the following restrictions: $A_1 > \delta_1 > \delta_2, \delta_3 < \delta_4$; $h_1 > \delta_5 > \delta_6, \delta_7 < \delta_8$. Then

$$\begin{aligned} &\tilde{A_{1L}}(\alpha) = A_1 - \delta_1 + (\delta_1 - \delta_2)\alpha > 0,\ \tilde{A_{1R}}(\alpha) = A_1 + \delta_4 - (\delta_4 - \delta_3)\alpha > 0 \\ &\tilde{h_{1L}}(\alpha) = h_1 - \delta_5 + (\delta_5 - \delta_6)\alpha > 0,\ \tilde{h_{1R}}(\alpha) = h_1 + \delta_8 - (\delta_8 - \delta_7)\alpha > 0 \end{aligned} \tag{5.2}$$

are the left and right limits of α cuts of $\tilde{A_1}, \tilde{A_2}, \tilde{h_1}$, and $\tilde{h_2}$.

5.1 Fuzzy inventory model without coordination

When the ordering cost and the holding cost of the manufacturer are fuzzified to be $\tilde{A_1}$, and $\tilde{h_1}$, as expressed in Eq. (5.1), the manufacturer's problem without coordination can be formulated in the fuzzy sense as

$$\begin{aligned} Min\ TC_{\widetilde{M}}(m) &= \frac{D\tilde{A_1}}{mQ_0} + \frac{\tilde{h_1}Q_0}{2}\left[(m-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] \\ &= \frac{\tilde{A_1}}{m}\sqrt{\frac{Dh_2}{2A_2}} + \tilde{h_1}\sqrt{\frac{DA_2}{2h_2}}\left[(m-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] \end{aligned}$$

subject to

$$\begin{cases} mt_0 \leq L, \\ m \geq 1 \end{cases} \tag{5.3}$$

Using Eq. (5.2), the left and right limits of the α cut,$(0 \leq \alpha \leq 1)$, of $TC_{\widetilde{M}(m)}$ are respectively,

$$TC_{ML}\widetilde{(m)}(\alpha) = \frac{\tilde{A_{1L}}(\alpha)}{m}\sqrt{\frac{Dh_2}{2A_2}} + \tilde{h_{1L}}(\alpha)\sqrt{\frac{DA_2}{2h_2}}\left[(m-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right]$$

subject to

$$\begin{cases} mt_0 \leq L, \\ m \geq 1 \end{cases} \tag{5.4}$$

and

$$TC_{MR}\widetilde{(m)}(\alpha) = \frac{\tilde{A_{1R}}(\alpha)}{m}\sqrt{\frac{Dh_2}{2A_2}} + \tilde{h_{1R}}(\alpha)\sqrt{\frac{DA_2}{2h_2}} \times \left[(m-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right]$$

subject to

$$\begin{cases} mt_0 \leq L, \\ m \geq 1 \end{cases} \tag{5.5}$$

Hence when the costs are described using trapezoidal fuzzy numbers, the defuzzified value of $TC_{\widetilde{M}(m)}$ is as given below.

$$d(TC_{\widetilde{M}}(m), \tilde{0}) = \frac{M_1}{m}\sqrt{\frac{Dh_2}{2A_2}} + M_2\sqrt{\frac{DA_2}{2h_2}}\left[(m-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right]$$

subject to

$$\begin{cases} mt_0 \leq L, \\ m \geq 1 \end{cases} \tag{5.6}$$

where

$$M_1 = A_1 + \frac{1}{4}(\delta_4 + \delta_3 - \delta_2 - \delta_1) > 0.$$

$$M_2 = h_1 + \frac{1}{4}(\delta_8 + \delta_7 - \delta_6 - \delta_5) > 0.$$

When the costs are represented by trapezoidal numbers, the defuzzified value $d(\widetilde{TC_M(m)}, \tilde{0})$ is taken as the estimate of fuzzy cost function (5.3), denoted by $FC_1^{\odot}(m)$ and is clearly a convex function of m. Since $m \geq 1$ is discrete,the optimal value of m is obtained from the following equation.

$$\begin{aligned} m^{\odot *} &= \max\left\{\min\left\{m \middle| FC_1^{\odot}(m) \leq FC_1^{\odot}(m+1)\right\}, 1\right\} \\ &= \max\left\{\min\left\{m(m+1) \geq \frac{M_1h_2}{A_2M_2(1-\frac{D}{P})}\right\}, 1\right\} \\ &= \left\lceil \sqrt{\frac{M_1h_2}{A_2M_2(1-\frac{D}{P})} + \frac{1}{4}} - \frac{1}{2} \right\rceil \geq 1 \end{aligned} \tag{5.7}$$

Equation (5.7) gives the optimal value of $m^{\odot *}$ as $m^{\odot *} = \left\lceil \sqrt{\frac{M_1h_2}{M_2A_2(1-\frac{D}{P})} + \frac{1}{4}} - \frac{1}{2} \right\rceil \geq 1$

5.2 Fuzzy inventory model with coordination

If the current order size of the buyer is changed by a factor K(K > 1), the manufacturer agrees to offer a quantity discount at a discount factor $d(K)$ so that the order quantity of the manufacturer and the buyer are nKQ_0, n is a positive integer and KQ_0 respectively. The total cost $\overline{\tilde{TC}_M}(n)$ of the manufacturer is thus given by

$$\overline{\tilde{TC}_M}(n) = \frac{D\tilde{A}_1}{nKQ_0} + \frac{KQ_0\tilde{h}_1}{2}\left[(n-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] + p_2Dd(K) \tag{5.8}$$

Thus the manufacturer's problem with coordination can be formulated in the fuzzy sense as $Min\, \overline{\tilde{TC}_M}(n)(\alpha)$ subject to the constraints

$$\begin{cases} nKt_0 \leq L \\ \dfrac{DA_2}{KQ_0} + \dfrac{KQ_0h_2}{2} - \sqrt{2DA_2h_2} \leq p_2Dd(K) \\ n \geq 1 \end{cases} \tag{5.9}$$

Using Eq. (5.2), the left and right limits of the α cut,$(0 \leq \alpha \leq 1)$, of $\widetilde{TC_M(m)}$ are respectively,

$$\begin{aligned} \overline{\tilde{TC}_{ML}}(n)(\alpha) &= \frac{DA_{1L}(\tilde{\alpha})}{nKQ_0} + \frac{KQ_0h_{1L}(\tilde{\alpha})}{2}\left[(n-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] \\ &\quad + p_2Dd(K) \\ &\quad \begin{cases} nKt_0 \leq L \\ \dfrac{DA_2}{KQ_0} + \dfrac{KQ_0h_2}{2} - \sqrt{2DA_2h_2} \leq p_2Dd(K) \\ n \geq 1 \end{cases} \end{aligned} \tag{5.10}$$

and

$$\begin{aligned} \overline{\tilde{TC}_{MR}}(n)(\alpha) &= \frac{DA_{1R}(\tilde{\alpha})}{nKQ_0} + \frac{KQ_0h_{1R}(\tilde{\alpha})}{2} \\ &\quad \left[(n-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] + p_2Dd(K) \\ &\quad \begin{cases} nKt_0 \leq L \\ \dfrac{DA_2}{KQ_0} + \dfrac{KQ_0h_2}{2} - \sqrt{2DA_2h_2} \leq p_2Dd(K) \\ n \geq 1 \end{cases} \end{aligned} \tag{5.11}$$

Hence when the costs are described using trapezoidal fuzzy numbers, the defuzzified value of $\overline{\tilde{TC}_M}(n)$ is as given below.

$$\begin{aligned} d(\overline{\tilde{TC}_M}(n), \tilde{0}) &= \frac{D\tilde{M}_1}{nKQ_0} + \frac{KQ_0\tilde{M}_2}{2}\left[(n-1)\left(1-\frac{D}{P}\right) + \frac{D}{P}\right] \\ &\quad + p_2Dd(K) \end{aligned}$$

subject to

$$\begin{cases} nKt_0 \leq L \\ \dfrac{DA_2}{KQ_0} + \dfrac{KQ_0h_2}{2} - \sqrt{2DA_2h_2} \leq p_2Dd(K) \\ n \geq 1 \end{cases} \tag{5.12}$$

When the costs are represented by trapezoidal numbers, the defuzzified value $d(\widetilde{TC_M(m)}, \tilde{0})$ is taken as the estimate of fuzzy cost function (5.8), denoted by $FC_2^{\odot}(n)$. From Eq. (5.12), it is clear that the $d(K)$ must be such that the manufacturer's inventory cost is minimized. Hence by considering the equality constraint we get,

$$d^{\odot *}(K) = \frac{\frac{DA_2}{KQ_0} + \frac{KQ_0h_2}{2} - \sqrt{2DA_2h_2}}{p_2D} \tag{5.13}$$

As the discount factor $d(K)$ is convex in K, the estimate given by Eq. (5.12) is also convex in K. Hence the unique minimum is obtained from the following equation:

$$\frac{d(FC_2^{\odot}(n))}{dK} = 0 \tag{5.14}$$

Solving Eq. (5.14) gives,

$$K^{\odot *}(n) = \sqrt{\frac{2(\frac{DM_1}{nQ_0} + \frac{DA_2}{Q_0})}{Q_0M_2[(n-1)(1-\frac{D}{P}) + \frac{D}{P}] + Q_0h_2}} \tag{5.15}$$

5.3 Fuzzy inventory model for system optimization

If there is a common decision maker for both the buyer and the manufacturer, the objective is to minimize the total cost of the system.

Let Q be the buyer's order quantity, then the manufacturer produces nQ units every time, where Q and n are decision variables. The problem for system optimisation can be formulated as follows:

$$Min\ \widetilde{TC_S}(n,Q) = \frac{D\tilde{A_1}}{nQ} + \frac{\tilde{h_1}Q}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + \frac{DA_2}{Q} + \frac{Qh_2}{2}$$

subject to the constraints

$$\begin{cases} \frac{nQ}{D} \le L \\ n \ge 1 \end{cases} \tag{5.16}$$

Using Eq. (5.2), the left and right limits of the α cut,$(0 \le \alpha \le 1)$, of $\overline{\tilde{TC_S}}(n,Q)$ are respectively,

$$\widetilde{TC_S}(n,Q)(\alpha) = \frac{D\tilde{A_{1L}}(\alpha)}{nQ} + \frac{\tilde{h_{1L}}(\alpha)Q}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + \frac{DA_2}{Q} + \frac{Qh_2}{2}$$

subject to the constraints

$$\begin{cases} \frac{nQ}{D} \le L \\ n \ge 1 \end{cases} \tag{5.17}$$

$$\widetilde{TC_{SR}}(n,Q)(\alpha) = \frac{D\tilde{A_{1R}}(\alpha)}{nQ} + \frac{\tilde{h_{1R}}(\alpha)Q}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + \frac{DA_2}{Q} + \frac{Qh_2}{2}$$

subject to the constraints

$$\begin{cases} \frac{nQ}{D} \le L \\ n \ge 1 \end{cases} \tag{5.18}$$

Hence when the costs are described using trapezoidal fuzzy numbers, the defuzzified value of $\overline{\tilde{TC_S}}(n,Q)$ is as given below.

$$d(\overline{\tilde{TC_S}}(n,Q),\tilde{0}) = \frac{D\tilde{M_1}}{nQ} + \frac{\tilde{M_2}Q}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] + \frac{DA_2}{Q} + \frac{Qh_2}{2}$$

subject to the constraints

$$\begin{cases} \frac{nQ}{D} \le L \\ n \ge 1 \end{cases} \tag{5.19}$$

Thus the defuzzified value $d(\overline{\tilde{TC_S}}(n,Q),\tilde{0})$ is taken as the estimate of fuzzy cost function (5.16), when the costs are represented by trapezoidal numbers, and is denoted by $FC_3^{\odot}(n,Q)$. As $FC_3^{\odot}(n,Q)$ is a convex function of Q, the optimal value of Q obtained from

$$\frac{\partial FC_3^{\odot}(n,Q)}{\partial Q} = 0 \tag{5.20}$$

is

$$Q^{\odot *}(n) = \sqrt{\frac{2D(\frac{M_1}{n}+A_2)}{M_2[(n-1)(1-\frac{D}{P})+\frac{D}{P}]+h_2}} \tag{5.21}$$

6 Numerical example

To illustrate the applicability of the proposed quantity discount strategy consider the following values of parameters in appropriate unit for the crisp model : D = 10,000 units per year, P = 25,000 units per year, $p_2 = \$30$ per unit, $\alpha = 0.5$, L = 0.25 year, $A_1 = \$300$ per order, $A_2 = \$100$ per order, $h_1 = \$10$, $h_2 = \$12$. The solution to this numerical example is obtained by using MATLAB software. The optimal solution is $TCB = \$4898.97$, $TCM(m^*) = \$5715.48$, $TCM(n^*) = \$5589.10$. The graphical representation of the total cost curve of the system is given in Fig. 3.

6.1 Sensitivity analysis

The sensitivity analysis is performed by changing the value of the key parameters one at a time and keeping the remaining parameters unchanged. The savings in percentage of the buyer and the manufacturer are respectively $SIPB = 100\alpha(TCM(m^*) - \overline{TCM}(n^*))/TCB(m^*)$ and SIPM1 = $100(1-\alpha)(TCM(m^*) - \overline{TCM}(n^*))/TCM(m^*)$.

If the manufacturer does not share his saving with the buyer, his savings is

SIPM2 = $100(TCM(m^*) - \overline{TCM}(n^*))/TCM(m^*)$. Computational results for different values of A_2, h_1, h_2 are specified in Table 1 and Table 2. Observations obtained from the tables are as follows:

(i) Table 1 reveals that as the value of h_2 i.e. the buyer's holding cost increases, the savings percentage of the buyer, manufacturer, and the system increases. The coordination strategy is significant and the manufacturer and the buyer can be benefitted from the proposed coordination strategy.

(ii) As the value of h_1 increases, the savings percentage of the buyer, manufacturer, and the system decreases and the coordination strategy is insignificant in this case.

Table 1 Effects of changes in the system parameters of the crisp model when $A_2 = 100$

h_1	h_2	$K^*(n)$	$d(K^*)$	SIPM1	SIPB	SIPM2	SIPS
10	10	1.1180	0.0000929	0.5573	0.6966	1.1146	0.6912
	11	1.1443	0.0001423	0.8255	0.9944	1.6511	0.9021
	12	1.1677	0.0001967	1.1055	1.2897	2.2110	1.1905
	13	1.1887	0.0002546	1.3883	1.5752	2.7766	1.4759
14	17	1.1709	0.0002423	1.4590	1.3312	2.2917	1.2316
15		1.1524	0.0001959	0.9181	1.0936	1.8362	0.9982
16		1.1348	0.0001557	0.7230	0.8825	1.4459	0.7948
17		1.1180	0.0001211	0.5573	0.6966	1.1146	0.6192
15	15	1.1180	0.0001138	0.5573	0.6966	1.1146	0.6192
	16	1.1359	0.0001533	0.7343	0.8949	1.4685	0.8067
	17	1.1524	0.0001959	0.9181	1.0936	1.8362	0.9982
	18	1.1677	0.0002409	1.1055	1.2897	2.2110	1.1905
20	24	1.1677	0.0002782	1.1055	1.2897	2.2110	1.1905
21		1.1547	0.0002393	0.9447	1.1218	1.8894	1.0257
22		1.1421	0.0002041	0.8005	0.9673	1.6010	0.8760
23		1.1299	0.0001723	0.6717	0.8257	1.3435	0.7408
24		1.1180	0.0001439	0.5573	0.6966	1.1146	0.6192

Table 2 Effects of changes in the system parameters of the crisp model when $A_2 = 200$

h_1	h_2	$K^*(n)$	$d(K^*)$	SIPM1	SIPB	SIPM2	SIPS
10	10	1.1336	0.0008922	0.2384	0.2086	0.4767	0.2225
	11	1.1443	0.0001423	0.8255	0.9944	1.6511	0.9021
	12	1.1677	0.0001967	1.1055	1.2897	2.2110	1.1905
	13	1.1887	0.0002546	1.3883	1.5752	2.7766	1.4759
14	17	1.4005	0.0016000	0.1113	0.0875	0.2225	0.0980
15		1.3899	0.0015000	1.0711	0.8742	2.1422	0.9627
16		1.3795	0.0014000	1.9703	1.6660	3.9405	1.8054
17		1.3693	0.0013000	2.8148	2.4629	5.6295	2.6271
15	15	1.3693	0.0013000	2.8148	2.4629	5.6295	2.6271
	16	1.3801	0.0014000	1.9157	1.6164	3.8315	1.7534
	17	1.3899	0.0015000	1.0711	0.8742	2.1422	0.9627
	18	1.3988	0.0016000	0.2757	0.2183	0.5515	0.2437
20	24	1.3988	0.0019000	0.2757	0.2183	0.5515	0.2437
21		1.3912	0.0018000	0.9546	0.7756	1.9091	0.8558
22		1.3838	0.0017000	1.6026	1.3355	3.2052	1.4569
23		1.3765	0.0017000	2.2220	1.8979	4.4440	2.0472
24		1.3693	0.0016000	2.8148	2.4629	5.6295	2.6271

(iii) It is observed from Table 2 that as the value of A_2 and h_1 increases, the savings percentage of the buyer, manufacturer, and the system increases and the coordination strategy is significant.

(iv) Table 2 also reveals that as the value of A_2 and h_2 increases, the savings percentage of the buyer, manufacturer, and the system increases and the coordination strategy is significant when h_1 =\$10 whereas the savings percentage of the buyer, manufacturer, and the system decreases when h_1 =\$15.

Figure 4 shows the effect of h_1 and h_2 on the savings of the manufacturer, the buyer and the system when $A_2 = \$100$ and Fig. 5 shows the effect of h_1 and h_2 on the savings of the manufacturer, the buyer and the system when $A_2 = \$200$.

The impact of the level of fuzziness in the cost components over the decision variables is analyzed by assigning trapezoidal fuzzy numbers to the input parameters (A_1, h_1) of the fuzzy models. Signed distance method is adopted to defuzzify the values of the parameters and the corresponding percentage difference from the crisp values are denoted by ($\hat{A_1}$, $\hat{h_1}$,) and is given in Table 3. The optimal total cost for the three fuzzy models are given in Table 4. The following conclusions can drawn from Tables 5, 6, 7 and 8.

(i) As the value of the ordering cost and holding cost of the manufacturer and the buyer increases, there is a corresponding increase in the total cost of the manufacturer when there is no coordination and when there is coordination between the manufacturer and the buyer. The total cost of the system also increases.

(ii) Keeping the ordering cost of the manufacturer $\hat{A_1}$ fixed and as the value of the other parameters increases, the total cost of the system increases. The total cost of the manufacturer in a coordinating system is slightly less than that in a non-coordinating system.

(iii) Keeping the ordering cost of the buyer A_2 fixed and as the value of the other parameters increases, the total cost of the system in all the three cases increases. The total cost of the manufacturer in a coordinating system is less than that in a non-coordinating system.

(iv) Keeping the holding cost of the manufacturer $\hat{h_1}$ fixed and as the value of the other parameters increases, the total cost of the optimization system increases whereas it decreases in the other two cases. The total cost of the manufacturer in a coordinating system is less than that in a non-coordinating system.

(v) Keeping the holding cost of the buyer h_2 fixed and as the value of the other parameters increases, the total cost increases in all the three cases. The total cost of the manufacturer in a coordinating system is greater than that in a non-coordinating system.

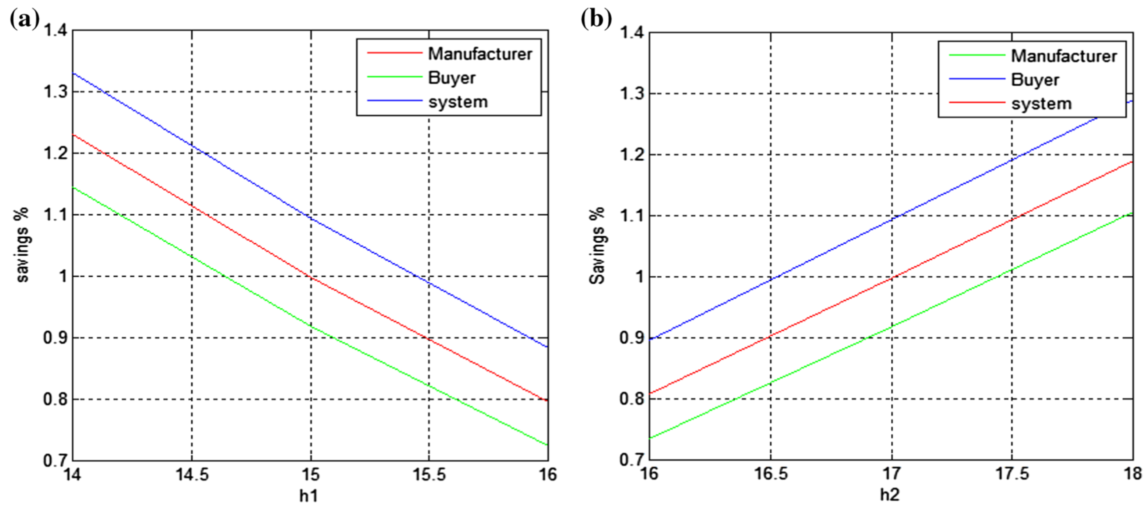

Fig. 4 Effects of h_1 and h_2 on the optimal solution when $A_2 = \$100$. **a** Effects of h_1 on the optimal solution when $h_2 = \$17$. **b** Effects of h_2 on the optimal solution when $h_1 = \$15$

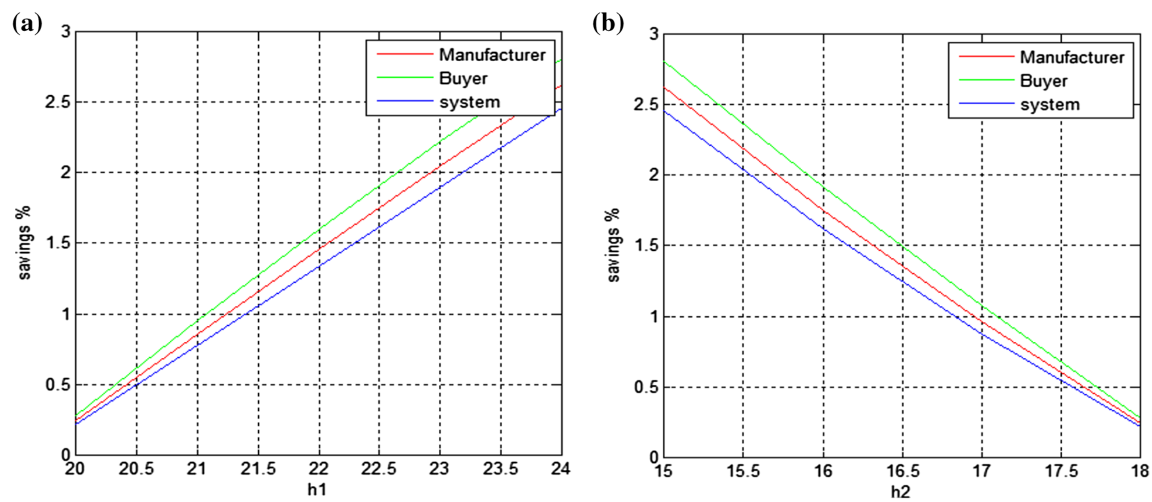

Fig. 5 Effects of h_1 and h_2 on the optimal solution when $A_2 = \$200$. **a** Effects of h_1 on the optimal solution when $h_2 = \$24$. **b** Effects of h_2 on the optimal solution when $h_1 = \$15$

Table 3 Fuzzy trapezoidal values for the input parameters of the model

$\tilde{A}_1$	$d(\tilde{A}_1,\tilde{0})$	$\hat{A}_1$	$\tilde{h}_1$	$d(\tilde{h}_1,\tilde{0})$	$\hat{h}_1$
(200, 250, 440, 470)	340.0	13.33	(2, 6, 16, 17)	10.25	2.5
(230, 270, 500, 510)	377.5	25.83	(5, 7, 14, 16)	10.50	5.0
(240, 260, 500, 520)	380.0	26.67	(4, 6, 15, 18)	10.75	7.5
(240, 260, 500, 520)	380.0	26.67	(3, 8, 15, 18)	11.0	10.0

7 Conclusion

In this paper we have developed a two echelon supply chain model consisting of a single manufacturer and a single buyer for a fixed lifetime product under fuzzy environment. The objective was to analyse the effectiveness of quantity discount strategy in a single manufacturer single buyer supply chain environment. Decentralized crisp models with and without coordination were developed and compared with the centralized model. This model also reveals that the the manufacturer and the buyer gains the most when their holding costs are equal. From the computational results it is clear that the savings of the manufacturer, the buyer and the system is high when the holding cost of the buyer increases. In the realistic environment, certain uncertainties in cost components will affect the

Table 4 Optimal total cost for the fuzzy models

$\hat{A_1}$	A_2	$\hat{h_1}$	h_2	$FC_1^{\odot}(m)$	$FC_2^{\odot}(m)$	$FC_3^{\odot}(n,Q)$
13.33	100	2.5	12	530.64	529.26	5428.24
25.83	100	5.0	12	1040.95	1040.95	5935.69
26.67	100	7.5	12	1265.65	1265.65	6164.50
26.67	100	10	12	1469.78	1467.68	6366.66

Table 5 Optimal total cost for the fuzzy models when $\hat{A_1}$ is fixed

$\hat{A_1}$	A_2	$\hat{h_1}$	h_2	$FC_1^{\odot}(m)$	$FC_2^{\odot}(m)$	$FC_3^{\odot}(n,Q)$
25.83	150	5.0	14	1020.90	1020.30	7501.04
	200	7.5	16	1266.60	1263.60	9263.66
	250	10.0	18	1544.18	1529.75	11,016.59

Table 6 Optimal total cost for the fuzzy models when A_2 is fixed

A_2	$\hat{A_1}$	$\hat{h_1}$	h_2	$FC_1^{\odot}(m)$	$FC_2^{\odot}(m)$	$FC_3^{\odot}(n,Q)$
200	25.83	5.0	14	1017.76	1017.60	8500.92
	26.67	7.5	16	1283.40	1280.87	9280.87
	26.67	10.0	18	1508.57	1501.45	9986.73

Table 7 Optimal total cost for the fuzzy models when $\hat{h_1}$ is fixed

$\hat{h_1}$	$\hat{A_1}$	A_2	h_2	$FC_1^{\odot}(m)$	$FC_2^{\odot}(m)$	$FC_3^{\odot}(n,Q)$
2.5	25.83	150	14	789.45	782.11	7262.85
	26.67	200	16	783.40	778.83	8778.83
	26.67	250	18	769.55	766.68	10,253.52

Table 8 Optimal total cost for the fuzzy models when h_2 is fixed

$\hat{h_1}$	$\hat{A_1}$	A_2	h_2	$FC_1^{\odot}(m)$	$FC_2^{\odot}(m)$	$FC_3^{\odot}(n,Q)$
14	13.33	150	5.0	750.87	784.75	7229.50
	25.83	200	7.5	1285.02	1279.23	8762.55
	26.67	250	10.0	1567.99	1613.44	9980.04

optimal decisions. In such situations fuzzy methodologies play a vital role to find the solution suitable to the real world. The impact of the level of fuzziness in the cost components over the decision variables is analyzed by developing fuzzy models under three scenarios in which trapezoidal fuzzy numbers are assigned to the input parameters. The proposed model is best suited for manufacturing systems which produces fixed lifetime products and faces uncertainities in the cost components. A future research direction might be to extend this study to other demand patterns, and to analyse the effectiveness of other coordination mechanism for fixed lifetime products. In addition, shortages with complete or partial backlogging could also be considered. This research can also be extended to multi-echelon supply chain systems.

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Appendix

Preliminaries

The fuzzy set theory was introduced to deal with problems in which fuzzy phenomena exist. In a universe of discourse X, a fuzzy subset $\tilde{a}$ of X is defined by the membership function $\mu_{\tilde{a}}(x)$ which maps each element x in X to a real number in the interval [0, 1]. The function value of $\mu_{\tilde{a}}(x)$ denotes the grade of membership.

Definition 1 *Fuzzy normal* (Vijayan and Kumaran 2008) A fuzzy set is normal if the largest grade obtained by any element in that set is 1.

Definition 2 *Fuzzy covex* (Vijayan and Kumaran 2008) A fuzzy set $\tilde{a}$ on X is convex iff $\mu_{\tilde{a}}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\tilde{a}}(x_1), \mu_{\tilde{a}}(x_2))$

Definition 3 *Fuzzy point* (Pu and Liu 1980) Let $\tilde{a}$ be a fuzzy set on R = $(-\infty, \infty)$. It is called a fuzzy point if its membership function is

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, & x = a \\ 0, & x \neq a \end{cases} \tag{8.1}$$

Definition 4 *Fuzzy number* (Vijayan and Kumaran 2008) A fuzzy number is a fuzzy subset of the real line which is both normal and convex. For a fuzzy number $\tilde{A}$, its membership function is denoted by

$$\mu_{\tilde{a}}(x) = \begin{cases} l(x), & x < m \\ 1, & m \leq x \leq n \\ u(x), & x > n \end{cases} \tag{8.2}$$

where l(x) is upper semi continuous, strictly increasing for $x < m$ and there exist $m_1 < m$ such that $l(x) = 0$ for $x \leq m_1$, u(x) is continuous, strictly decreasing function for $x > n$ and there exist $n_1 \geq n$ such that $u(x) = 0$ for $x \geq n_1$, l(x) and u(x) are called left and right reference functions respectively.

Definition 5 *Trapezoidal fuzzy number* (Zimmerman 1991) The fuzzy number $\tilde{A}$ is said to be a trapezoidal fuzzy number if it is fully determined by (a_1, a_2, a_3, a_4) of crisp numbers such that $a_1 < a_2 < a_3 < a_4$, whose membership function, representing a trapezoid, can be denoted by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4 \\ 0, & otherwise \end{cases} \tag{8.3}$$

where a_1, a_2, a_3 and a_4 are the lowerlimit, lower mode, upper mode and upper limit respectively of the fuzzy number $\tilde{A}$. The interval $[a_1, a_4]$ is called the support of the fuzzy number and it gives the range of all possible values of $\tilde{A}$ that are least marginally possible or plausible. The interval $[a_2, a_3]$ corresponds to the core of fuzzy number and gives the range of most plausible values. The intervals $[a_1, a_2]$ and $[a_3, a_4]$ are called penumbra of the fuzzy number $\tilde{A}$.

Let $\tilde{A_1} = (a_{11}, a_{12}, a_{13}, a_{14})$, $\tilde{A_2} = (a_{21}, a_{22}, a_{23}, a_{24})$ be two trapezoidal fuzzy numbers, then $\tilde{A_1} + \tilde{A_2} = (a_{11} + a_{21}$, $a_{12} + a_{22}$, $a_{13} + a_{23}$, $a_{14} + a_{24})$ and for all $b \ge 0$, $b\tilde{A_1} = (ba_{11}, ba_{12}, ba_{13}, ba_{14})$.

The set $\tilde{A}(\alpha) = \{x : \mu_{\tilde{A}}(x) \ge \alpha\}$, where $\alpha\varepsilon[0, 1]$ is called the α cut of $\tilde{A}$. $\tilde{A}(\alpha)$ is a nonempty bounded closed interval contained in the set of real numbers and it can be denoted by $\tilde{A}(\alpha) = [\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]$. $\tilde{A}_L(\alpha)$ and $\tilde{A}_R(\alpha)$ are respectively the left and right limits of $\tilde{A}(\alpha)$ and are usually known as the left and right α cuts of $\tilde{A}$. $\tilde{A}_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $\tilde{A}_R(\alpha) = a_4 - (a_4 - a_3)\alpha$ for a trapezoidal number $\tilde{A} = (a_1, a_2, a_3, a_4)$.

Definition 6 *Level* α *Fuzzy interval* (Chiang et al. 2005) Let [a, b; α] be a fuzzy set on $R = (-\infty, \infty)$. It is called a level α fuzzy interval, $0 \le \alpha \le 1$, $a < b$, if its membership function is

$$\mu_{[a,b;\alpha]}(x) = \begin{cases} \alpha, & if\ a \le x \le b \\ 0, & otherwise \end{cases} \tag{8.4}$$

Signed distance method (Yao and Wu 2000)

The signed distance between the real numbers a and 0, denoted by $d_0(a, 0)$ is given by $d_0(a, 0) = a$. Hence the signed distance of $\tilde{A}_L(\alpha)$ and $\tilde{A}_R(\alpha)$ measured from 0 are $d_0(\tilde{A}_L(\alpha), 0) = \tilde{A}_L(\alpha)$ and $d_0(\tilde{A}_R(\alpha), 0) = \tilde{A}_R(\alpha)$.

The signed distance of the interval $(\tilde{A}_L(\alpha), \tilde{A}_R(\alpha))$ measured from the origin 0 by

$$\begin{aligned} d_0((\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)), 0) &= \frac{1}{2}[d_0(\tilde{A}_L(\alpha), 0) + d_0(\tilde{A}_R(\alpha)] \\ &= \frac{1}{2}(\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha)) \end{aligned} \tag{8.5}$$

where $\tilde{A}_L(\alpha)$ and $\tilde{A}_L(\alpha)$ exist and are integrable for $\alpha\varepsilon[0, 1]$.

For each $\alpha\,\varepsilon\,[0, 1]$, the crisp interval $[\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]$ and the level α fuzzy interval $[[\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]; \alpha]$ are in one to one correspondence. The signed distance from $[\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]$ to $\tilde{0}$ (where $\tilde{0}$ is the 1 level fuzzy point which maps to the origin) is

$$([\tilde{A}_L(\alpha), \tilde{A}_R(\alpha); \alpha], \tilde{0}) = d_0((\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)), 0) = \frac{1}{2}(\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha)) \tag{8.6}$$

The signed distance of $\tilde{A}$ measured from 0 is defined as

$$d(\tilde{A}, \tilde{0}) = \frac{1}{2}\int_0^1 (\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha))d\alpha \tag{8.7}$$

Lemma 1 (Linearity property of the operator d (Vijayan and Kumaran 2008))

Let $\tilde{A}_i, i = 1, 2, \ldots N$ be N fuzzy numbers and $b_i, i = 1, 2, \ldots N$ are real crisp constants. Then

$$d\left(\sum_{i=1}^{N} b_i\tilde{A}_i, \tilde{0}\right) = \sum_{i=1}^{N} b_i d(\tilde{A}_i, \tilde{0}) \tag{8.8}$$

Proof By definition,

$$\begin{aligned} d(\sum_{i=1}^{N} b_i\tilde{A}_i, \tilde{0}) &= \frac{1}{2}\int_0^1 \left(\left(\sum_{i=1}^{N} b_i\tilde{A}_i\right)_L(\alpha) + \left(\sum_{i=1}^{N} b_i\tilde{A}_i\right)_R(\alpha)\right)d\alpha \\ &= \frac{1}{2}\int_0^1 \left(\sum_{i=1}^{N} b_i\tilde{A}_{iL}(\alpha) + \sum_{i=1}^{N} b_i\tilde{A}_{iR}(\alpha)\right)d\alpha \\ &= \frac{1}{2}\int_0^1 \sum_{i=1}^{N} b_i(\tilde{A}_{iL}(\alpha) + \tilde{A}_{iR}(\alpha))d\alpha \\ &= \sum_{i=1}^{N} b_i \frac{1}{2}\int_0^1 (\tilde{A}_{iL}(\alpha) + \tilde{A}_{iR}(\alpha))d\alpha \\ &= \sum_{i=1}^{N} b_i d(\tilde{A}_i, \tilde{0}) \end{aligned}$$

where $\tilde{A}_{iL}(\alpha)$ and $\tilde{A}_{iL}(\alpha)$ are respectively the left and right α cuts of the fuzzy number $\tilde{A}_i$. Hence the lemma is proved. □

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