OPTIMAL INVENTORY CONTROL FOR DETERIORATING ITEMS WITH DELAY IN PAYMENTS AND PARTIAL BACKLOGGING

K.F. Mary Latha[1*], R. Uthayakumar[2]

[1] Department of Mathematics, Jayaraj Annapackiam College for Women (Autonomous), Periyakulam, Theni, Tamil Nadu, India.; e-mail: kfm.latha@gmail.com

[2] Department of Mathematics, The Gandhigram Rural Institute - Deemed University, Gandhigram-624 302, Dindigul, Tamil Nadu, India.; e-mail: uthayagri@gmail.com

Abstract: This paper focuses on an inventory model with price and time dependent demand for non -instantaneously deteriorating items. Shortages are allowed and partially backlogged with time varying backlogging and deterioration rate. To enhance demand and to lure the retailers to buy more, the supplier offers a permissible delay period. The main aim of this paper is to determine the optimal replenishment policy that maximizes the total profit. Numerical examples are included to illustrate the proposed model.

Keywords: Inventory, Price and time dependent demand, Delay in payments, Time varying deterioration, Partial backlogging.

1. INTRODUCTION

Deterioration is an important aspect of inventory control. In traditional economic order quantity models deterioration of items were not given due consideration. But in real life situations it is very difficult to preserve items like medicines, volatile liquids, foodstuffs etc.. These items deteriorate over time. The first attempt to develop inventory models for these items with a constant rate of deterioration was made by Ghare and Schrader [3] and later many researchers have worked in this direction. In the realistic environment, certain items such as vegetables, fruits, and baked items remain fresh for a short span. After that period some items may start to decay. That is they do not lose their freshness or quality as soon as they enter into the inventory. Deterioration takes place only after a certain period of time for such items. So it is necessary to consider inventory problems for non-instantaneously deteriorating items. Many researchers like Sana [6], Sarkar [7], Sett et al. [10] developed their excellent works with time varying deterioration rate. Sarkar [8] developed a production inventory model for three different types of continuously distributed deterioration functions.

Classical economic order quantity models were developed under the assumption that shortages are either completely backlogged or completely lost which is an unrealistic situation. Often customers are ready to wait if the waiting time is short or else they may move to other retailers. In some inventory systems, such as fashionable items, the length of the waiting time for the next replenishment would determine whether the backlogging will be accepted or not. Hence the backlogging rate is a variable which depends on the waiting time for the next replenishment. Abad [1] focused on a pricing and lot-sizing problem for a product with a variable deterioration rate by allowing shortages and partial backlogging. Chang and Dye [2] developed an inventory model in which the backlogging rate was the reciprocal of a linear function of the waiting time.

In framing traditional inventory models, it was assumed that the retailer has to pay immediately after receiving his goods. But in today's challenging scenario, to enhance demand and to lure the retailers to buy more, the suppliers offer a delay period for settling the accounts. The effects of delay in payments in the optimal pricing and inventory control for non-instantaneously deteriorating items have been considered by some researchers.

Past research on the traditional trade credit policy assumed that a demand for the product is a known constant. However, in a competitive market and particularly in a free market economy, demand is not constant but it is dependent on price and time as they influence the pricing policy which is more responsive to structure changes in market demand over product life cycle. So it is essential to concentrate on price and time dependent natures of demand when we develop pricing and replenishment policies under permissible delay in payments. Other excellent related works can be found in Sarkar and Sarkar [9], Hou and Lin [4]. Recently Maihami and Abadi [5] have developed an inventory model for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging with a constant deterioration rate.

In this paper effort has been taken to develop an inventory model for non-instantaneously deteriorating items with time varying deterioration rate. A price and time dependent demand under permissible delay in payments is also considered. In section 2, assumptions and notations for the proposed model are included. The mathematical formulation of the model is explained in section 3. The aim of this paper is to obtain the optimal replenishment cycle time and the order quantity. In section 4 numerical examples are given and finally, we provide a summary and some suggestions for future work in section 5.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model in this paper is developed on the basis of the following assumptions and notations:

2.1. Notations

p : constant purchasing cost per unit
p_1 : selling price per unit, ($p_1 > p$)
h : holding cost per unit per unit time excluding the capital cost
s : backorder cost per unit per unit time
π : cost of lost sales per unit
t_d : length of time in which the product exhibits no deterioration
t_1 : length of time in which there is no inventory shortage
T : length of the replenishment cycle
Q : order quantity
$I_1(t)$: inventory level at any time $t[0,t_d]$
$I_2(t)$: inventory level at any time $t[t_d,t_1]$
$I_3(t)$: inventory level at any time $t[t_1,T]$
I_m : maximum inventory level
S : maximum amount of demand backlogged
I_e : interest earned per dollar
I_p : interest payable per dollar
M : trade credit period
$TP(p_1, t_1, T)$: total profit per unit time of the inventory system

2.2. Assumptions

i. A single non-instantaneously deteriorating item is considered.
ii. The deterioration function (ϕ) depends on time as $(\phi) = \gamma t$, where γ is a constant ($0 < \gamma \leq 1, t \geq 0$).
iii. The demand rate, $D(p_1,t) = (a-bp_1)e^{\lambda t}$ ($a>0$, $b>0$) is a linearly decreasing function of the price and decreases(increases) exponentially with time when $\lambda < 0 (\lambda > 0)$.
iv. Shortages are allowed and are partially backlogged. Only a fraction of demand is backlogged and is given by $B(t) = \frac{1}{1+\delta t}$ where t is the waiting time and δ is the backlogging parameter.
v. Replenishment occurs instantaneously at an infinite rate and lead time is negligible.

3. MODEL FORMULATION

The inventory system evolves as follows: The initial stock in any cycle is I_m. During the period $[0, t_d]$, the inventory level decreases owing to price dependent demand only. The inventory level decreases due to the combined effect of demand and deterioration in the interval $[t_d, t_1]$ and it reaches the zero level at $t = t_1$. During the time interval $[t_1, T]$, the demand at time t is partially backordered. The total process repeats itself after a scheduling time T. The graphical representation of the inventory system is given in figures 1 to 3. The status of the inventory is governed by the following differential equations:

$$\frac{dI_1(t)}{dt} = -D(p_1,t) = -(a-bp_1)e^{\lambda t}, 0 \leq t \leq t_d;\ I_1(0) = I_m \tag{1}$$

$$\frac{dI_2(t)}{dt} + \phi(t)I_2(t) = -D(p_1,t) = -(a-bp_1)e^{\lambda t}, t_d \leq t \leq t_1;\ I_2(t_1) = 0 \tag{2}$$

$$\frac{dI_3(t)}{dt} = -D(p_1,t)B(T-t_1) = \frac{-(a-bp_1)e^{\lambda t}}{1+\delta(T-t)} - (a-bp_1)e^{\lambda t}, t_d \leq t \leq t_1;\ I_3(t_1) = 0 \tag{3}$$

The solutions to (1) – (3) are

$$I_1(t) = \frac{(a-bp_1)}{\lambda}(1-e^{\lambda t}) + I_m, 0 \leq t \leq t_d \tag{4}$$

$$I_2(t) = (a-bp_1)e^{-\gamma t_d^2}((t_1-t) + \frac{\lambda}{2}(t_1^2-t^2) + \frac{\gamma}{6}(t_1^3-t^3) + \frac{\lambda\gamma}{8}(t_1^4-t^4)), t_d \leq t \leq t_1 \tag{5}$$

$$I_3(t) = (a-bp_1)[\frac{1}{\delta}[e^{\lambda t}\log(1+\delta(T-t)) - e^{\lambda t_1}\log(1+\delta(T-t_1)] + T(e^{\lambda t_1} - e^{\lambda t}) + te^{\lambda t} - t_1e^{\lambda t_1} + (\frac{e^{\lambda t_1} - e^{\lambda t}}{\lambda})], t_1 \leq t \leq T \tag{6}$$

(the higher powers of λ, γ, δ are neglected throughout this paper)

Using the continuity of I(t) at $t = t_d$, we know from equations (4) and (5) that the maximum inventory level for each cycle is $I_m = (a-bp_1) K$,

where $K = e^{-\gamma t_d^2}((t_1-t_d) + \frac{\lambda}{2}(t_1^2-t_d^2) + \frac{\gamma}{6}(t_1^3-t_d^3) + \frac{\lambda\gamma}{8}(t_1^4-t_d^4)) - (\frac{1-e^{\lambda t_d}}{\lambda})$

and hence $I_1(t) = (a-bp_1)[\frac{(1-e^{\lambda t})}{\lambda} + K], 0 \leq t \leq t_d$ (7)

The maximum amount of demand backlogged per cycle is

$$S = -I_3(T) = (a - bp_1)[\frac{e^{\lambda t_1}}{\delta}\log(1 + \delta(T - t_1)) - (T - t_1)e^{\lambda t_1} - (\frac{e^{\lambda t_1} - e^{\lambda T}}{\lambda})], \tag{8}$$

Therefore the order quantity per cycle is $Q = S + I_m$

$$Q = (a - bp_1)[\frac{e^{\lambda t_1}}{\delta}\log(1 + \delta(T - t_1)) - (T - t_1)e^{\lambda t_1} + \left(\frac{e^{\lambda t_1} - e^{\lambda t}}{\lambda}\right) + K] \tag{9}$$

The total profit per unit time of the inventory system includes the following components:

(i) Ordering cost = A.

(ii) HC: Holding cost $= h\left[\int_0^{t_d} I_1(t)dt + \int_{t_d}^{t_1} I_2(t)dt\right] = h(a - bp_1)[\frac{1}{\lambda}\left(t_d - \left(\frac{e^{\lambda t_d} - 1}{\lambda}\right)\right) + Kt_d + \frac{(t_1 - t_d)^2}{2}$

$$+\frac{\lambda}{6}(2t_1^3 - 3t_1^2 t_d + t_d^3) + \frac{\gamma}{24}(2t_1^4 - 4t_1^3 t_d + 4t_1 t_d^3 - 2t_d^4) + \frac{\lambda\gamma}{120}(8t_1^5 - 15t_1^4 t_d + 10t_1^2 t_d^3 - t_d^5)]$$

(iii) SC: Shortage cost due to backlog $= s\int_{t_1}^{T} -I_3(t)dt$

$$SC = s(a - bp_1)[(\frac{e^{\lambda t_1}}{\delta}\log(1 + \delta(T - t_1)) - (T - t_1)e^{\lambda t_1} - \frac{e^{\lambda t_1}}{\lambda})(T - t_1) + \frac{1}{\lambda^2}(e^{\lambda T} - e^{\lambda t_1})]$$

(iv) OC: Opportunity cost due to lost sales $= \pi(a - bp_1)\int_{t_1}^{T} D(p_1, t)(1 - B(T - t))dt$

$$OC = \delta\pi(a - bp_1)[\frac{(t_1 - T)e^{\lambda t_1}}{\lambda} + \frac{1}{\lambda^2}(e^{\lambda T} - e^{\lambda t_1})]$$

(v) PC: Purchase cost $= pQ = p(a - bp_1)[\frac{e^{\lambda t_1}}{\delta}\log(1 + \delta(T - t_1)) - (T - t_1)e^{\lambda t_1} + \left(\frac{e^{\lambda t_1} - e^{\lambda t}}{\lambda}\right) + K]$

(vi) SR: Sales revenue $= p_1\left[\int_0^{t_1} D(p_1, t)dt + S\right]$

$$SR = p_1(a - bp_1)[\left(\frac{e^{\lambda T} - 1}{\lambda}\right) + \frac{e^{\lambda t_1}}{\delta}\log(1 + \delta(T - t_1)) + (t_1 - T)e^{\lambda t_1}]$$

Since the effect of delay in payments is considered in this model there is a need for considering three different cases.

(vii) Interest Payable:

Case (i) : $0 \le M \le t_d$ In this case, payment for items is settled and the retailer starts paying the capital opportunity cost for the items in inventory with rate I_p. Thus the interest payable is $IP1 = pI_p\int_M^{t_d} I_1(t)dt + \int_{t_d}^{t_1} I_2(t)dt$

$IP1 = pI_p(a - bp_1)[\frac{t_d - M}{\lambda} + K(t_d - M) - \frac{1}{\lambda^2}(e^{\lambda T} - e^{\lambda t_1}) + \frac{(t_1 - t_d)^2}{2} + \frac{\lambda}{6}(2t_1^3 - 3t_1^2 t_d + t_d^3) + \frac{\gamma}{24}(2t_1^4 - 4t_1^3 t_d + 4t_1 t_d^3 - 2t_d^4) + \frac{\lambda\gamma}{120}(8t_1^5 - 15t_1^4 t_d + 10t_1^2 t_d^3 - t_d^5)]]$

Case (ii) : $t_d \le M \le t_1$ This case is similar to case (i). Thus the interest payable is $IP2 = pI_p(a - bp_1)\left[\frac{(t_1 - M)^2}{2} + \frac{\lambda}{6}(2t_1^3 - 3t_1^2 M + M^3) + \frac{\gamma}{24}(2t_1^4 - 4t_1^3 M + 4t_1 M^3 - 2M^4) + \frac{\lambda\gamma}{120}(8t_1^5 - 15t_1^4 M + 10t_1^2 M^3 - 3M^5)\right]$

Case (iii) : $t_1 \le M \le T$ In this case there is no opportunity cost. Therefore IP3 = 0.

(viii) Interest earned: When the account is not settled, the retailer accumulates sales revenue by selling goods and earns interest with rate I_e.

Case (i) : $0 \le M \le t_d$ $IE1 = p_1 I_e\int_M^{t_1} t D(p_1, t)dt = \frac{p_1 I_e(a - bp_1)}{\lambda^2}[1 + (\lambda M - 1)e^{\lambda M}]$

Case (ii) : $t_d \le M \le t_1$ $IE2 = p_1 I_e\int_M^{t_1} t D(p_1, t)dt = \frac{p_1 I_e(a - bp_1)}{\lambda^2}[1 + (\lambda M - 1)e^{\lambda M}]$

Case (iii) : $t_1 \le M \le T$ $IE3 = p_1 I_e[\int_0^{t_1} t D(p_1, t)dt + (M - t_1)\int_0^{t_1} D(p_1, t)dt]$

$$= p_1 I_e(a - bp_1)[\frac{1 + (\lambda t_1 - 1)e^{\lambda t_1}}{\lambda^2} + (M - t_1)\left(\frac{e^{\lambda t_1} - 1}{\lambda}\right)]$$

Hence the total profit per unit time during the replenishment cycle is

$$TP(p_1, t_1, T) = \begin{cases} TP_1(p_1, t_1, T) & \text{if } 0 \le M \le t_d \\ TP_2(p_1, t_1, T) & \text{if } t_d \le M \le t_1 \\ TP_3(p_1, t_1, T) & \text{if } t_1 \le M \le T \end{cases} \tag{10}$$

where $TP_i(p_1, t_1, T) = [SR - A - HC - SC - OC - PC - IP1 + IE1] / T$; $i = 1, 2, 3$.

For a given p_1, the criteria for maximizing the value of $TP_i(p_1, t_1, T)$ are $\frac{\partial TPi(p_1, t_1, T)}{\partial t_1} = 0$; $\frac{\partial TPi(p_1, t_1, T)}{\partial T} = 0$; (11)

Now for maximizing the total profit of the system, the optimal values of t_1, T can be obtained by solving the above simultaneous equations provided they satisfy the sufficiency conditions.

$$\frac{\partial^2 TP_i}{\partial T^2} < 0; \frac{\partial^2 TP_i}{\partial t^2} < 0; \text{and } \frac{\partial^2 TP_i}{\partial T^2}\frac{\partial^2 TP_i}{\partial t^2} - (\frac{\partial^2 TP_i}{\partial T \partial t_1})^2 > 0 \tag{12}$$

Due to the complexity of the equations the optimal values are obtained using MATLAB software.

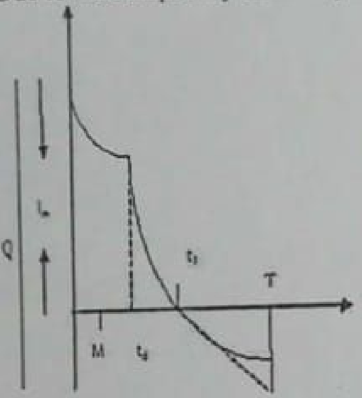

Fig. 1 Graphical representation of the system (case1)

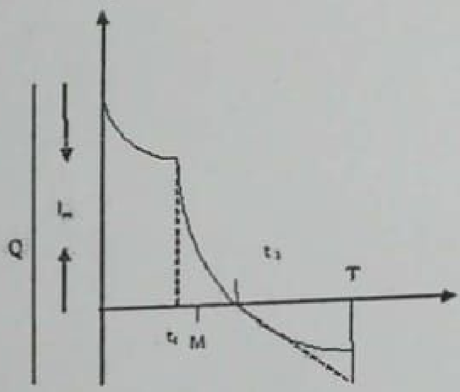

Fig2 Graphical representation of the system (case2)

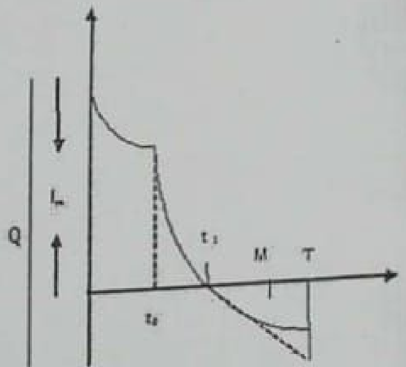

Fig 3 Graphical representation of the system (case3)

4. NUMERICAL EXAMPLE : To illustrate the applicability of the model, let us consider the following data: The solutions are obtained using MATLAB.

Example 1 A=250; h=1; p=20; p_1=200; t_d=0.0833; s=30; δ = 0.02; λ=-0.98; γ=0.02; I_p=0.17; I_e=0.10; π=1 in appropriate units. The optimal values of t_1, T, Q are $t_1^*=0.5336$, $T^*=2.4532$, $Q^*=758$.

Example 2 Again the data are same as in example 1 except that M=0.25. The optimal values of t_1, T, Q are $t_1^*=0.6236$, $T^*=2.3623$, $Q^*=696$.

Example 3 Again the data are same as in example 1 except that M = 0.25. The optimal values of t_1, T, Q are $t_1^*=0.8331$, $T^*=2.3623$, $Q^*=621$.

5. CONCLUSION : In this paper an inventory model is developed for non-instantaneously deteriorating items with varying deterioration rate. A permissible delay in payments is considered as a type of price reduction and it can attract new customers and thereby increase sales. A demand which is price and time dependent in nature is focused to suit a realistic environment. Owing to the competitive and challenging environment, shortages are allowed and partially backlogged. The backlogging rate is assumed to be a variable and dependent on the waiting time for the next replenishment. The aim of this paper is to determine the optimal replenishment time and the order quantity which maximizes the profit function. The proposed model can be extended to include some more practical situations such as quantity discounts, time value of money and inflation.

6. REFERENCES

[1] Abad, P.L., Optimal pricing and lot sizing under conditions of perishability and partial backordering, Management Science 42, 1093-1104,1996.

[2] Chang, H.J., and Dye, C.Y., An EOQ model for deteriorating items with time varying demand and partial backlogging, Journal of the Operational Research Society 50, 1176-1182, 1999.

[3] Ghare, P.M., and Schrader, G.H., A model for exponentially decaying inventory system, International Journal of Production Research 21, 449-460,1963.

[4] Hou, K.L., and Lin, L.C., Optimal pricing and ordering policies for deteriorating items with multivariate demand under trade credit and inflation Opsearch 50, 404-417,2013.

[5] Maihami, R., Abadi, I.N.K., Joint control of inventory and pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging, Mathematical and Computer Modelling 55, 1722-1733, 2012.

[6] Sana, S.S., Optimal selling price and lot size with time varying deterioration and partial backlogging, Applied Mathematics and Computation 217,185-194, 2010b.

[7] Sarkar, B., An EOQ model with delay in payments and time varying deterioration rate, Mathematical and Computer Modelling 55, 367-377, 2012a.

[8] Sarkar, B., A production - inventory model with probabilistic deterioration in two-echelon supply chain management, Applied Mathematical Modelling 37, 3138–3151, 2012.

[9] Sarkar, B., and Sarkar, S., An improved inventory model with partial backlogging, time varying deterioration and stock dependent demand Economic Modelling 30, 924-932, 2013.

[10]Sett, B.K., Sarkar, B., and Goswami, A., A two ware house inventory model with increasing demand and time varying deterioration, Scientia Iranica, E 19, 1969-1977, 2012.