International Journal of Information and Management Sciences

Publication details, including instruction for authors and subscription information:

http://ijims.ms.tku.edu.tw/

ISSN:1017-1819

An Inventory Model for Increasing Demand with Probabilistic Deterioration, Permissible Delay and Partial Backlogging

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International Journal of Information and Management Sciences
25 (2014), 297-316

International Journal of
Information
and
Management
Sciences

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Abstract

In this paper an inventory model for deteriorating items with time dependent quadratic demand and permissible delay in payments is developed. Shortages are allowed and are partially backlogged. An optimal policy that minimizes the total cost is developed. The objective of this study is to consider three different types of continuous probabilistic deterioration functions and to find the associated total cost. To illustrate the proposed model some numerical examples are given. Sensitivity analysis of the optimal solutions with respect to major parameters are carried out and comparison is made between the three models.

Keywords: Inventory, quadratic demand, permissible delay in payments, probabilistic deterioration, partial backlogging.

1. Introduction

In formulating inventory models deterioration of items must be given due consideration. Items like foodstuff, pharmaceutical, chemicals, etc. deteriorate significantly. The amount or value of these products decrease with time during storage period. Ghare and Schrader [9] had developed a model for an exponentially decaying inventory. Covert and Philip [6] relaxed the assumption of constant deterioration rate by considering the two-parameter weibull distribution. Further, Philip [29] extended the model by considering the varying rate of three-parameter weibull distribution. Deteriorating inventory models with trended demand were developed by Dave and Patel [7], Bahari-Kasani [1], Chung and Ting [3], Goswami and Chaudhuri [14], Hariga [15], Jalan,Giri and Chaudhuri [16], Giri, Goswami and Chaudhuri [10], Jalan and Chaudhuri [17], Lin, Tan and Lee [22] and others. Researchers [26],[30],[13],[52],[27], [41] and [42] have developed inventory models for deteriorating items. A two-ware house inventory model with increasing demand and time varying deterioration was developed by Sett and Sarkar [40]. Tripathi and Pandey [50] considered an economic quantity model for deteriorating items with weibull time dependent demand rate under trade credit scenario. Recently, Sarkar [47] had focussed on a production inventory model with probabilistic deterioration in a two-echelon supply chain management. Many inventory models with variable deterioration were developed by researchers Sarkar and Sarkar [45] and Sarkar [38]. The effect of a probabilistic deterioration in a production system was discussed by Sarkar and Sarkar [44].

Today's globally challenging environment, compels the researchers to concentrate on inventory models which helps the traders to tackle their challenges. To enhance trade, customer satisfaction bounds to play a crucial role. One of the factors which influences it is the settlement of accounts. In traditional inventory models, either in deterministic or probabilistic, it is often assumed that payment is made to the supplier for the goods immediately after receiving them. But in reality this is a rare phenomena. In order to stimulate demand, boost market share or decrease inventories of certain items, a supplier provides credit period to the customers. During that period the customer earns interest on the payment received for the goods sold and thus accumulates revenue. Therefore customers prefer to delay payment until the deadline given by the supplier. Goyal [12] was the first to develop an inventory model with permissible delay in payments. A more general economic order quantity (EOQ) model with permissible delay in payments, price-discount effect and different types of demand rate was developed by Sana and Chaudhury [34]. The works done by researchers [2], [4], [5], [28],[39], [38], [45] and [53] are to be mentioned regarding permissible delay in payments.

Sometimes stock may be inadequate to fulfill the customer's demand. During the stock out period, if the item is such that the customers can wait for its delivery, then the items are to be backlogged but if the item is very essential they may move to other suppliers and the demand is lost for ever. To reflect this phenomenon several researchers had developed models under the assumption that stockout items are partially backordered. Maihami and Abadi [23] had developed a permissible delay model for non-instantaneous deteriorating items with price and time dependent demand and partial backlogging. Recently Khanra et al. [21] formulated an inventory model with time dependent demand and shortages under trade credit policy. An economic order quantity model for deteriorating items and planned back order level was directed by Widyadana et al. [51]. There were several interesting and relevant papers such as [35], [36], [37], [45], [46] focussing on imperfect production system.

Many EOQ models were developed in the literature considering constant demand focussing on the control of inventories. In recent years, many researchers have paid their attention towards developing models with time varying demand which is more applicable in a realistic environment. Silver and Meal [48], first suggested a simple modification of the EOQ model with varying demand. Jalan and Chaudhuri [18] had developed inventory models by considering exponentially time varying demand pattern. Donaldson [8] was the first to discuss the problem of inventory replenishment with a linearly time- dependent demand analytically. Many researchers like Silver [49] , Ritchie [31]-[33], Mitra, Cox and Jesse [25] etc., made valuable contributions in this direction.

From the review of literature it is clear that the researchers have paid their attention only on two types of time dependent demand, namely linear and exponential. A linearly time-varying demand indicates a uniform change in demand rate of the item per unit time which seldom occurs in the real market. On the other hand, exponentially time varying demand indicates very rapid change in demand rate which is also unrealistic because the demand of any product cannot undergo a rate which is so high as the exponential demand.

So, an alternative realistic approach would be to consider a quadratic time dependent demand which may represent both the accelerated and retarded growth in demand. The demand for items such as aircrafts, computers, hi-tech innovative products and their spare parts and the newly introduced delicious food items, etc. show an accelerated growth in demand whereas the items like obsolete aircrafts, computers, hi-tech innovative products and their spare parts etc. show an accelerated decline in demand. The demand of seasonal products such as air conditioners, school bags, crackers etc. rises rapidly to a peak in the mid season and then falls rapidly as the season wanes out. These different types of demand can be better represented by $D(t) = a+bt+ct^2$, $a \geq 0$, $b \neq 0$, $c \neq 0$. We have $\frac{dD(t)}{dt} = b + 2ct$, $\frac{d^2D(t)}{dt^2} = 2c$. For $b > 0$, $c > 0$, as the rate of increase of the demand rate is itself an increasing function of time, $D(t)$ represents an accelerated growth in demand. For $b > 0$, $c < 0$, there is retarded growth in demand for all $t \in (0, \frac{-b}{2c})$. Thus based on the signs of b and c, $D(t)$ represents various realistic demand patterns. Inventory models with quadratic time dependent demand is more suitable for companies like Boeing, IBM, Carrier, Pizza Hut etc. Khanra and Chaudhuri [19] developed an EOQ model for perishable items, considering demand function as a quadratic function of time. Ghosh and Chaudhuri [11] investigated an EOQ model over a finite planning horizon for a deteriorating item with a quadratic time dependent demand, allowing shortages in inventory which is more realistic. Khanra et al. [20] developed an EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payments without shortages. Recently an inventory model with time dependent demand and shortages under trade credit policy was studied by Khanra et al. [21].

In this paper effort has been taken to extend the work of [24] in formulating an inventory model for deteriorating items considering time-dependent quadratic demand where shortages are partially backlogged and permissible delay in payment is allowed. In addition, three different types of continuous probabilistic deterioration functions are taken into consideration. To the authors' knowledge, this type of comparative study has not yet been considered by any of the researchers/scientists in inventory literature. Among the various time varying demand in EOQ models, the more realistic demand approach is to consider a quadratic time dependent demand rate because it represents both accelerated and retarded growth in demand. The demand rate is of the form $D(t) = a + bt + ct^2$. Here, $c = 0$ represents a linear demand rate and $b = c = 0$ represent the constant demand rate. An algorithm is presented to derive the optimal replenishment policy when the total cost is minimized. Numerical examples are provided to compare the three models. Sensitivity analysis of the optimal solutions with respect to major parameters are carried out.

Table 1: Summary of demand, deterioration, delay in payments and permissible shortages related literature.

Author	Year	Demand	Deterioration	Delay in payments	shortage
Gosh	2006	Quadratic time dependent	Time dependent	No	Fully backlogged
Khanra	2011	Quadratic time dependent	Constant	Yes	No
Sarkar [38]	2012	time dependent	time dependent	Yes	No
Reza	2012	time and price dependent	Constant	Yes	Partially backlogged
Khanra	2013	Quadratic time dependent	No	Yes	Completely backlogged
Sarkar[45]	2013	Inventory dependent	Time dependent	No	No
Sarkar [42]	2013	Stock dependent	Time varying	No	Partially backlogged
Tripathi	2013	Weibull time dependent	Constant	Yes	No
Chung	2013	Stock dependent	Constant	Yes	No
Sarkar[47]	2013	Constant	Probablistic	No	No
Present paper		Quadratic time dependent	Probablistic	Yes	Partially backlogged

2. Assumptions and Notations

The following assumptions are made in developing the model.

2.1. Assumptions

1. The demand rate for the item is represented by a quadratic and continuous function of time.
2. Time horizon is infinite.
3. The lead time is zero and the replenishment rate is infinite i.e. replenishment is instantaneous.
4. Shortages are allowed to occur. Only a fraction δ $(0 \leq \delta \leq 1)$ of it is backlogged and the remaining fraction $(1 - \delta)$ is lost.
5. Deterioration follows continuous probability distribution function as (a) uniform distribution, (b) triangular distribution, and (c) beta distribution.

2.2. Notations

$D(t)$	The time dependent demand rate is $D(t) = a + bt + ct^2$, $a > 0$, $b \neq 0$, $c \neq 0$. Here a is the initial rate of demand, b is the rate with which the demand rate increases. The rate of change in the demand rate itself changes at a rate c.
A	Cost per replenishment order.
p	Per unit cost of the item.
h_p	Inventory holding cost (excluding interest charges) per rupee of unit purchase cost per unit time.
s	Per unit shortage cost per unit time.
π	Per unit opportunity cost due to lost sales.
θ	Deterioration rate of an item.
I_p	Interest charges per rupee investment in stock per year.

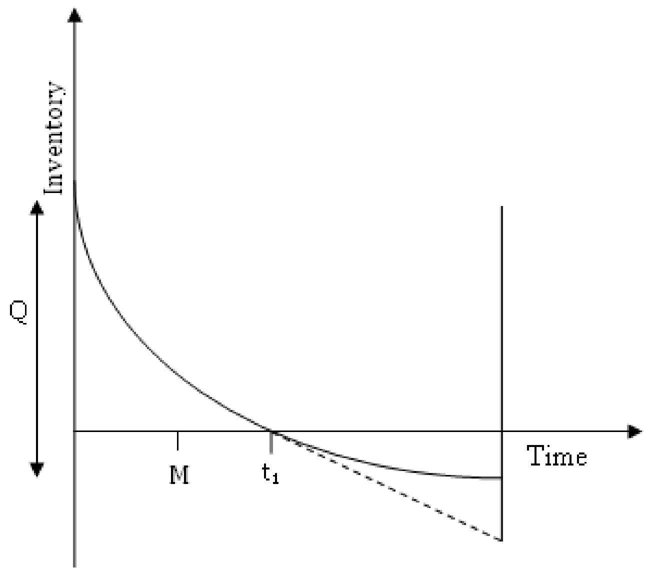

Figure 1: Graphical representation of inventory system (case1).

I_e	Interest earned per rupee in a year.
M	Permissible period of delay in settling the accounts with the supplier.
T	Time interval in year between two consecutive orders.
t_1	Time at which the inventory level becomes zero.

3. Model Formulation

Consider an inventory system in which I_m units arrive at the system at the beginning of each cycle. During the interval $[0, t_1]$ the inventory depletes due to demand and deterioration and it becomes zero at time t_1. The model is depicted in Figure 1 and Figure 2. The instantaneous inventory level at any time t during the cycle time T can be represented by the following differential equations with $I(0) = I_m$, $I(t_1) = 0$.

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt + ct^2); \quad 0 \le t \le t_1 \tag{3.1}$$

$$\frac{dI(t)}{dt} = -a\delta; \quad t_1 < t \le T \tag{3.2}$$

Using the boundary conditions the solutions of the above differential equations are

$$I(t) = \frac{1}{\theta^3}\left[\left\{\theta^2(a + bt_1 + ct_1^2) - \theta(2ct_1 + b) + 2c\right\} e^{\theta(t_1 - t)} - \left\{\theta^2\left(a + bt + ct^2\right) - \theta\left(2ct + b\right) + 2c\right\}\right] \tag{3.3}$$

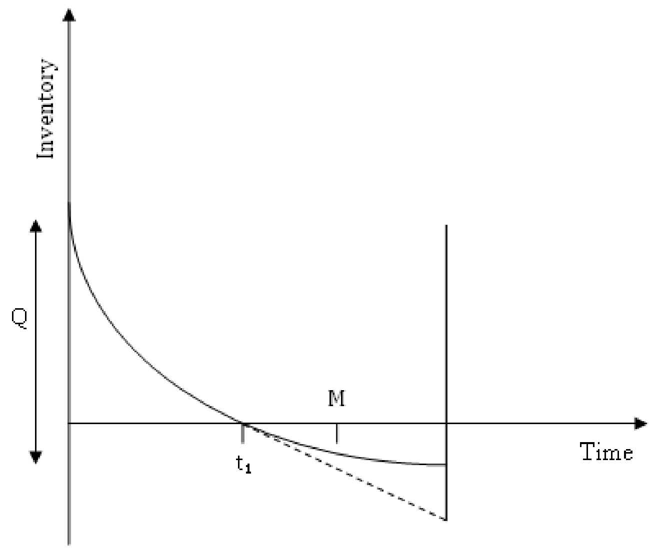

Figure 2: Graphical representation of inventory system (case2).

$$I(t) = -a\delta\,(t - t_1) \tag{3.4}$$

The maximum inventory level is given by $I_m = I(0)$

$$I_m = \frac{1}{\theta^3}\big[\,\{\theta^2(a + bt_1 + ct_1^2) - \theta(2ct_1 + b) + 2c\}\,\big(e^{\theta t_1} - 1\big) - \big\{a\theta^2 - b\theta + 2c\big\}\,\big] \tag{3.5}$$

The maximum amount of shortage to be backlogged is

$$I_b = a\delta(T - t_1) \tag{3.6}$$

Based on the assumptions and description of the model, the total annual relevant costs include the following elements.

(i) Deterioration Cost : Number of deteriorated items in $[0,\, t_1)$ is

$$\begin{aligned}
&= I(0) - \int_0^{t_1} D(t)dt \\
&= \frac{1}{\theta^3}\big[\,\{\theta^2(a + bt_1 + ct_1^2) - \theta(2ct_1 + b) + 2c\}\,e^{\theta t_1} - (a\theta^2 - b\theta + 2c)\big] \\
&\quad - \int_0^{t_1}\big(a + bt + ct^2\big)dt \\
&= \frac{1}{\theta^3}\big[\,\{\theta^2(a + bt_1 + ct_1^2) - \theta(2ct_1 + b) + 2c\}\,e^{\theta t_1} - (a\theta^2 - b\theta + 2c)\big] \\
&\quad -\big(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3}\big)
\end{aligned}$$

Deterioration cost for the cycle [0, T]

$$DC = p\Big[\frac{1}{\theta^3}\big[\,\big\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\big\}\,e^{\theta t_1}$$
$$-(a\theta^2-b\theta+2c)\big]-\big(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}\big)\Big] \tag{3.7}$$

(ii) Holding Cost: Inventory occurs during period t_1, therefore the holding cost in the interval [0, t_1) is

$$\begin{aligned} C_h &= h\int_0^{t_1} I(t)dt \\ &= h\int_0^{t_1}\frac{1}{\theta^3}\left\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\right\}\left[e^{\theta(t_1-t)}-1\right]dt \\ &= \frac{h}{\theta^4}\Big[\left\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\right\}\left[e^{\theta t_1}-1\right] \\ &\quad -\Big\{\theta^2(a+\frac{bt_1}{2}+\frac{ct_1^2}{3})-\theta(ct_1+b)+2c\Big\}\,\theta t_1\Big],\text{where } h=ph_p. \end{aligned} \tag{3.8}$$

(iii) Shortage Cost: During the shortage period there are two cases need to be considered. They are depicted in Fig. 1 and Fig. 2.

Shortage cost over the period [t_1,T) is

$$C_s = s\int_{t_1}^{T} -I(t)dt = \frac{sa\delta}{2}(T-t_1)^2 \tag{3.9}$$

(iv) Ordering Cost: Since replenishment is done at the start of the cycle, ordering cost is

$$C_r = A \tag{3.10}$$

(v) Opportunity Cost: Opportunity cost due to lost sales during the replenishment cycle is

$$C_0 = \pi\int_{t_1}^{T} a(1-\delta)dt = \pi a(1-\delta)(T-t_1) \tag{3.11}$$

(iv) Interest Payable

Case 1: $M \le t_1$

Since the credit period is shorter than or equal to the replenishment cycle time, the product still in stock is assumed to be financed with an annual rate I_p and thus the interest payable is

$$\begin{aligned} IP_1 &= pI_p\int_M^{t_1} I(t)dt \\ &= pI_p\int_M^{t_1}\frac{1}{\theta^3}\big[\big\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\big\}\,e^{\theta(t_1-t)} \end{aligned}$$

$$- \left\{\theta^2 \left(a + bt + ct^2\right) - \theta\left(2ct + b\right) + 2c\right\}\Big] dt$$

$$= \frac{pI_p}{\theta^4}\Bigg[\left\{\theta^2(a + bt_1 + ct_1^2) - \theta(2ct_1 + b) + 2c\right\}\left[e^{\theta(t_1 - M)} - 1\right]$$

$$-\theta(t_1 - M)\left\{\theta^2(a + \frac{b}{2}(t_1 + M) + \frac{c}{3}(t_1^2 + t_1 M + M^2)) - \theta(c(t_1 + M) + b) + 2c\right\}\Bigg]$$

Case 2: $M > t_1$
In this case the buyer pays no interest for the items.

(iv) Interest Earned
Case 1: $M \leq t_1$

Since the length of the period with positive inventory stock of the item is larger than the credit period, the buyer can use the sale revenue to earn the interest with an annual rate I_e in $[\,0,\, t_1)$.
Therefore the interest earned is

$$IE_1 = pI_e \int_0^{t_1} (t - t_1)D(t)dt = pI_e \int_0^{t_1} (t - t_1)(a + bt + ct^2)dt = \frac{pI_e t_1^2}{12}(6a + 2bt_1 + ct_1^2) \tag{3.12}$$

Case 2: $M > t_1$

In this case the buyer earns the interest on the sales revenue during the period [0, M]

$$IE_2 = pI_e\Big[\int_0^{t_1} (t_1 - t)D(t)dt - (M - t_1)\int_0^{t_1} (t_1 - t)D(t)dt\Big]$$
$$= \frac{pI_e}{12}\Big[6at_1^2 + 2bt_1^3 + ct_1^4 + 2t_1(M - t_1)(6a + 3bt_1 + 2ct_1^2)\Big]$$

Therefore the total variable cost per cycle is TC(t_1, T) = ordering cost + holding cost + deteriorating cost + opportunity cost + iterest payable - interest earned. Therefore the total variable cost per unit time is

$$TC(t_1, T) = \begin{cases} TC_1(t_1, T), & \text{if } \; M \leq t_1 \\ TC_2(t_1, T), & \text{if } \; M > t_1 \end{cases} \tag{3.13}$$

where

$$TC_1(t_1, T) = \frac{1}{T}\Bigg[A + \frac{h}{\theta^4}\Bigg[\left\{\theta^2\left(a + bt_1 + ct_1^2\right) - \theta(2ct_1 + b) + 2c\right\}\left[e^{\theta t_1} - 1\right]$$

$$-\left\{\theta^2(a + \frac{bt_1}{2} + \frac{ct_1^2}{3}) - \theta(ct_1 + b) + 2c\right\}\theta t_1\Bigg]$$

$$+p\Bigg[\frac{1}{\theta^3}\Big[\left\{\theta^2(a + bt_1 + ct_1^2) - \theta(2ct_1 + b) + 2c\right\}e^{\theta t_1} - (a\theta^2 - b\theta + 2c)\Big]$$

$$-(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3})\Bigg] + \frac{sa\delta}{2}(T - t_1)^2 + \pi a(1 - \delta)(T - t_1)$$

$$+\frac{pI_p}{\theta^4}\Bigg[\left\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\right\}\left[e^{\theta(t_1-M)}-1\right]$$
$$-\theta(t_1-M)\left\{\theta^2(a+(b/2)(t_1+M)+(c/3)(t_1^2+t_1M+M^2))\right.$$
$$\left.-\theta(c(t_1+M)+b)+2c\right\}\Bigg]-\frac{pI_et_1^2}{12}(6a+2bt_1+ct_1^2)\Bigg] \tag{3.14}$$

and

$$TC_2(t_1,T)=\frac{1}{T}\Bigg[A+\frac{h}{\theta^4}\Bigg[\left\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\right\}\left[e^{\theta t_1}-1\right]$$
$$-\left\{\theta^2(a+\frac{bt_1}{2}+\frac{ct_1^2}{3})-\theta(ct_1+b)+2c\right\}\theta t_1\Bigg]$$
$$+p\Bigg[\frac{1}{\theta^3}\Big[\left\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\right\}e^{\theta t_1}-(a\theta^2-b\theta+2c)\Big]$$
$$-(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3})\Bigg]+\frac{sa\delta}{2}(T-t_1)^2+\pi a(1-\delta)(T-t_1)$$
$$-\frac{pI_e}{12}\Big[6at_1^2+2bt_1^3+ct_1^4+2t_1(M-t_1)(6a+3bt_1+2ct_1^2)\Big]\Bigg] \tag{3.15}$$

Case 1: M $\leq t_1$.

Now for minimizing the total average cost per unit time, the optimal values of T and t_1 can be obtained by solving the following simultaneous equations:

$$T\frac{\partial TC_{11}(t_1,T)}{\partial T}-TC_{11}(t_1,T)=0 \text{ and } \frac{\partial \mathrm{TC}_{11}(\mathrm{t}_1,\mathrm{T})}{\partial \mathrm{t}_1}=0 \tag{3.16}$$

where $TC_{11}(t_1,T)=[A+C_h+DC+C_s+C_r+C_o+IP_1-IE_1]$ provided they satisfy the sufficient conditions (see Appendix A) $\frac{1}{T}\frac{\partial^2TC_{11}(t_1,T)}{\partial t_1^2}>0$; $\frac{1}{T}\frac{\partial^2TC_{11}(t_1,T)}{\partial T^2}>0$; and $\frac{\partial^2TC_{11}(t_1,T)}{\partial t_1^2}\frac{\partial^2TC_{11}(t_1,T)}{\partial T^2}>\frac{1}{T^2}(T\frac{\partial^2TC_{11}(t_1,T)}{\partial T\partial t_1}-\frac{\partial TC_{11}(t_1,T)}{\partial t_1})^2$

Equation (3.16) is equivalent to

$$T\big[sa\delta(T-t_1)\big]-\big[A+\frac{h}{\theta^4}\big[\left\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\right\}\left[e^{\theta t_1}-1\right]$$
$$-\{\theta^2(a+\frac{bt_1}{2}+\frac{ct_1^2}{3})-\theta(ct_1+b)+2ct\}\theta t_1\big]+p\big[\frac{1}{\theta^3}\big[\left\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\right\}e^{\theta t_1}$$
$$-(a\theta^2-b\theta+2c)\big]-\big(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}\big)\big]+\frac{sa\delta}{2}(T-t_1)^2-\pi a(1-\delta)t_1$$
$$+\frac{pI_p}{\theta^4}\big[\left\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\right\}\left[e^{\theta(t_1-M)}-1\right]$$
$$-\theta(t_1-M)\{\theta^2(a+\frac{b}{2}(t_1+M)+\frac{c}{3}(t_1^2+t_1M+M^2))-\theta(c(t_1+M)+b)+2c\}\big]$$
$$-\frac{pI_et_1}{2}(6at_1^2+2bt_1^3+ct_1t)+2(M-t_1)(6a+3bt_1+2ct_1^2)\big]=0$$

and

$$\left(\frac{h}{\theta}(e^{\theta t_1}-1)+\frac{p}{\theta}\right)(a+bt_1+ct_1^2)-p(a+bt_1+3ct_1^2)-sa\delta(T-t_1)-\pi a(1-\delta)$$
$$+I_p\frac{p}{\theta^2}[\theta(e^{\theta(t_1-M)}-1)(a+bt_1+ct_1^2)+(b+2ct_1)]-pI_e(a+bt_1^2/2+ct_1^3/3)=0$$

Case 2: M $> t_1$.

Now for minimizing the total average cost per unit time, the optimal values of T and t_1 can be obtained by solving the following simultaneous equations:

$$T\frac{\partial TC_{21}(t_1,T)}{\partial T}-TC_{21}(t_1,T)=0 \text{ and } \frac{\partial \mathrm{TC}_{21}(\mathrm{t}_1,\mathrm{T})}{\partial \mathrm{t}_1}=0 \tag{3.17}$$

where $TC_{21}(t_1,T)=[A+C_h+DC+C_s+C_r+C_o+IP_2-IE_2]$ provided they satisfy the sufficient conditions (see Appendix A) $\frac{1}{T}\frac{\partial^2 TC_{21}(t_1,T)}{\partial t_1^2}>0$; $\frac{1}{T}\frac{\partial^2 TC_{21}(t_1,T)}{\partial T^2}>0$; and $\frac{\partial^2 TC_{211}(t_1,T)}{\partial T^2}\frac{\partial^2 TC_{21}(t_1,T)}{\partial t_1^2}>\frac{1}{T^2}\left(T\frac{\partial^2 TC_{21}(t_1,T)}{\partial t_1\partial T}-\frac{\partial TC_{21}(t_1,T)}{\partial t_1}\right)^2$
Equation (3.17) is equivalent to

$$T\big[sa\delta(T-t_1)\big]-\big[A+\frac{h}{\theta^4}\big[\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\}\,\big[e^{\theta t_1}-1\big]$$
$$-\left\{\theta^2(a+\frac{bt_1}{2}+\frac{ct_1^2}{3})-\theta(ct_1+b)+2c\right\}\theta t_1\big]+p\big[\frac{1}{\theta^3}\big[\,\{\theta^2(a+bt_1+ct_1^2)$$
$$-\theta(2ct_1+b)+2c\}\,e^{\theta t_1}-(a\theta^2-b\theta+2c)\big]-\big(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3})\big]+\frac{sa\delta}{2}(T-t_1)^2$$
$$-\pi a(1-\delta)t_1-\frac{pI_et_1^2}{12}(6a+2bt_1+ct_1^2)\big]=0$$

Algorithm

- Step 1: *Determine T_1^* and t_{11}^* from equation* (3.16). *If $M\le t_{11}^*$ evaluate $TC_1(t_{11}^*,T_1^*)$ using equation* (3.14).
- Step 2: *Determine T_2^* and t_{12}^* from equation* (3.17). *If $M>t_{12}^*$ evaluate $TC_2(t_{12}^*,T_2^*)$ using equation* (3.15).
- Step 3: *If the condition $M\le t_{11}^*$ and $M>t_{12}^*$ is satisfied go to step* 4, *otherwise go to step* 5.
- Step 4: *Compare $TC_1(t_{11}^*,T_1^*)$ and $TC_2(t_{12}^*,T_2^*)$ and find the minimum cost.*
- Step 5: *If $M\le t_{11}^*$ is satisfied but $M<t_{12}^*$, then $TC_1(t_{11}^*,T_1^*)$ is the minimum cost, else if $t_{11}^*<M$ but $M>t_{12}^*$ then $TC_2(t_{12}^*,T_2^*)$ is the minimum cost.*

Using the optimal value solution procedure described above, we can find the optimal order quantity to be

$$Q=\frac{1}{\theta^3}\big[\,\{\theta^2(a+bt_1+ct_1^2)-\theta(2ct_1+b)+2c\}\,e^{\theta t_1}-\{a\theta^2-b\theta+2c\}\,\big]+a\delta(T-t_1) \tag{3.18}$$

In our model, we consider the deterioration θ which follows three different types of probability distribution function as $\theta = E[f(x)]$ where $f(x)$ follows (1) uniform distribution, (2) triangular distribution, (3) beta distribution. We show a numerical comparison between the three models.

3.1. Deterioration follows uniform distribution

We consider that the deterioration θ follows uniform distribution as $\theta = E[f(x)] = \frac{\alpha+\beta}{2}, \alpha > 0, \beta > 0, \alpha < \beta$. Therefore, the equation of $TC_1(t_1, T)$ can be written as

$$
\begin{aligned}
TC_1(t_1,T) = \frac{1}{T}\Bigg[A &+ \frac{h}{\left(\frac{\alpha+\beta}{2}\right)^4}\Bigg[\Bigg\{\left(\frac{\alpha+\beta}{2}\right)^2(a+bt_1+ct_1^2) - \left(\frac{\alpha+\beta}{2}\right)(2ct_1+b)+2c\Bigg\}w_0 \\
&- \left\{\left(\frac{\alpha+\beta}{2}\right)^2\left(a+\frac{bt_1}{2}+\frac{ct_1^2}{3}\right) - \left(\frac{\alpha+\beta}{2}\right)(ct_1+b)+2c\right\}\left(\frac{\alpha+\beta}{2}\right)t_1\Bigg] \\
&+ p\Bigg[\frac{1}{\left(\frac{\alpha+\beta}{2}\right)^3}\Bigg[\Bigg\{\left(\frac{\alpha+\beta}{2}\right)^2(a+bt_1+ct_1^2) - \left(\frac{\alpha+\beta}{2}\right)(2ct_1+b)+2c\Bigg\}e^{\left(\frac{\alpha+\beta}{2}\right)t_1} \\
&- \left(a\left(\frac{\alpha+\beta}{2}\right)^2 - b\left(\frac{\alpha+\beta}{2}\right)+2c\right)\Bigg] - \left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}\right)\Bigg] + w_1 \\
&+ \frac{pI_p}{\left(\frac{\alpha+\beta}{2}\right)^4}\Bigg[\left\{\left(\frac{\alpha+\beta}{2}\right)^2(a+bt_1+ct_1^2) - \left(\frac{\alpha+\beta}{2}\right)(2ct_1+b)+2c\right\}w_2 \\
&- \left(\frac{\alpha+\beta}{2}\right)(t_1-M)\left\{\left(\frac{\alpha+\beta}{2}\right)^2\left(a+\frac{b}{2}(t_1+M)+\frac{c}{3}\left(t_1^2+t_1M+M^2\right)\right)\right] \\
&- \left(\frac{\alpha+\beta}{2}\right)(c(t_1+M)+b)+2c\Bigg\}\Bigg] - \frac{pI_et_1^2}{12}(6a+2bt_1+ct_1^2)\Bigg]
\end{aligned}
$$

where $w_0 = e^{\left(\frac{\alpha+\beta}{2}\right)t_1} - 1$, $w_1 = \frac{sa\delta}{2}(T-t_1)^2 + \pi a(1-\delta)(T-t_1)$ and $w_2 = e^{\left(\frac{\alpha+\beta}{2}\right)(t_1-M)} - 1$

3.2. Deterioration follows triangular distribution

We consider that deterioration θ follows triangular distribution as $\theta = E[f(x)] = (\alpha+\beta+\gamma)/3$ where f(x) is the probability density function of the triangular distribution with lower limit α, upper limit β and mode γ as well as $\alpha < \beta$ and $\alpha \le \gamma \le \beta$.

Therefore, the equation of $TC_1(t_1, T)$ can be written as

$$
\begin{aligned}
&TC_1(t_1,T) \\
&= \frac{1}{T}\Bigg[A + \frac{h}{(\frac{\alpha+\beta+\gamma}{3})^4}\Bigg[\Bigg\{(\frac{\alpha+\beta+\gamma}{3})^2(a+bt_1+ct_1^2) - (\frac{\alpha+\beta+\gamma}{3})(2ct_1+b)+2c\Bigg\}w_3 \\
&- \Bigg\{(\frac{\alpha+\beta+\gamma}{3})^2(a+\frac{bt_1}{2}+\frac{ct_1^2}{3}) - \frac{\alpha+\beta+\gamma}{3}(ct_1+b)+2c\Bigg\}\frac{\alpha+\beta+\gamma}{3}t_1\Bigg]
\end{aligned}
$$

$$+p\Big[\frac{1}{(\frac{\alpha+\beta+\gamma}{3})^3}\Big[\Big\{(\frac{\alpha+\beta+\gamma}{3})^2(a+bt_1+ct_1^2)-(\frac{\alpha+\beta+\gamma}{3})(2ct_1+b)+2c\Big\}w_4$$
$$-(a(\frac{\alpha+\beta+\gamma}{3})^2-b\frac{\alpha+\beta+\gamma}{3}+2c)\Big]-(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3})\Big]+w_1$$
$$+\frac{pI_p}{(\frac{\alpha+\beta+\gamma}{3})^4}\Big[\Big\{(\frac{\alpha+\beta+\gamma}{3})^2(a+bt_1+ct_1^2)-(\frac{\alpha+\beta+\gamma}{3})(2ct_1+b)+2c\Big\}w_5$$
$$-(\frac{\alpha+\beta+\gamma}{3})(t_1-M)\Big\{(\frac{\alpha+\beta+\gamma}{3})^2(a+\frac{b}{2}(t_1+M)+\frac{c}{3}(t_1^2+t_1M+M^2))$$
$$-\left(\frac{\alpha+\beta+\gamma}{3}\right)(c(t_1+M)+b)+2c\Big\}\Big]-\frac{pI_et_1^2}{12}(6a+2bt_1+ct_1^2)\Big]$$

where $w_3=e^{(\frac{\alpha+\beta+\gamma}{3})t_1}-1$, $w_4=e^{(\frac{\alpha+\beta+\gamma}{3})t_1}$ and $w_5=e^{(\frac{\alpha+\beta+\gamma}{3})(t_1-M)}-1$

3.3. Deterioration follows beta distribution

We consider that Deterioration θ follows beta distribution as $\theta=E[f(x)]=\frac{\alpha}{\alpha+\beta}$ where f(x) follows a continuous probability distribution defined on the interval (0,1) parameterized by two positive parameters denoted by α and β. Therefore, the equation of $TC_1(t_1,T)$ can be written as

$$TC_1(t_1,T)=\frac{1}{T}\Bigg[A+\frac{h}{(\frac{\alpha}{\alpha+\beta})^4}\Bigg(\Big\{(\frac{\alpha}{\alpha+\beta})^2(a+bt_1+ct_1^2)-(\frac{\alpha}{\alpha+\beta})(2ct_1+b)+2c\Big\}w_6$$
$$-\Big\{(\frac{\alpha}{\alpha+\beta})^2(a+\frac{bt_1}{2}+\frac{ct_1^2}{3})-(\frac{\alpha}{\alpha+\beta})(ct_1+b)+2c\Big\}(\frac{\alpha}{\alpha+\beta})t_1\Bigg)$$
$$+p\Big[\frac{1}{(\frac{\alpha}{\alpha+\beta})^3}\Big[\Big\{(\frac{\alpha}{\alpha+\beta})^2(a+bt_1+ct_1^2)-(\frac{\alpha}{\alpha+\beta})(2ct_1+b)+2c\Big\}e^{(\frac{\alpha}{\alpha+\beta})t_1}$$
$$-(a(\frac{\alpha}{\alpha+\beta})^2-b(\frac{\alpha}{\alpha+\beta})+2c)\Big]-(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3})\Big]+w_1$$
$$+\frac{pI_p}{(\frac{\alpha}{\alpha+\beta})^4}\Big[\Big\{(\frac{\alpha}{\alpha+\beta})^2(a+bt_1+ct_1^2)-(\frac{\alpha}{\alpha+\beta})(2ct_1+b)+2c\Big\}w_7$$
$$-(\frac{\alpha}{\alpha+\beta})(t_1-M)\Big\{(\frac{\alpha}{\alpha+\beta})^2(a+\frac{b}{2}(t_1+M)+\frac{c}{3}(t_1^2+t_1M+M^2))$$
$$-(\frac{\alpha}{\alpha+\beta})(c(t_1+M)+b)+2c\Big\}\Big]-\frac{pI_et_1^2}{12}(6a+2bt_1+ct_1^2)\Bigg]$$

where $w_6=e^{(\frac{\alpha}{\alpha+\beta})t_1}-1$ and $w_7=e^{(\frac{\alpha}{\alpha+\beta})(t_1-M)}-1$.

4. Numerical Example

We use the solution procedure described above and MatLab software to find the numerical results. **Example 1 (case 1).** Let us consider an inventory system with the following data:
$A = 185$, $h = 0.10$, $s = 50$, $p = 76.5$, $\pi = 40$, $a = 35$, $b = 12$, $c = 0.30$, $\theta = 0.20$, $M = 0.3918$, $\delta = 0.56$, $I_p = 0.013$, $I_e = 0.012$ in appropriate units.
The minimum average cost is $TC(t_1^*, T^*) = 549.07$ and the optimal values of the cycle length, shortage period and the lotsize are $T^* = 0.6743$ and $t_1^* = 0.5241$, Q = 50 respectively. Based on these values, the convexity of the total cost function is shown in Figure 3.

(Sub case 1) The data are same except that θ follows a uniform distribution and α =0.10, $\beta = 0.30$.
(Sub case 2) The data are same except that θ follows a beta distribution and $\alpha = 0.10$, $\beta = 0.30$.
(Sub case 3) The data are same except that θ follows a uniform distribution and $\alpha = 0.10$, $\beta = 0.30$, $\gamma = 0.20$.

Table 2: Computational results of Example 1

θ	t_1	T	TC
Beta	0.5373	0.6192	562.92
Triangular	0.5241	0.6743	549.07
Uniform	0.5047	0.7386	556.33

The optimal solution is given in Table 2. From the table it is clear that the total cost is minimum when the the deterioration function follows a triangular distribution. The comparison between the three probabilistic deteriorated models is done with the help of graphical representation. The graph contains a combination of three figures which are due to the change of the three probabilistic deterioration functions. The above plot is for beta distribution, middle plot is for triangular distribution and the downside plot is for uniform distribution. (see Figure 4.)

Example 2 (case 2). The data are same as in Example 1 except that M = 0.5479 ($M > t_1$).
Using the solution procedure described above, and using MatLab software, the minimum average cost is $TC(t_1^*, T^*) = 551.06$ and the optimal values of the cycle length, shortage period and the lotsize are $T^* = 0.6590$ and $t_1^* = 0.5112$, Q = 50 respectively. (see Figure 5.)

5. Sensitivity Analysis

The sensitivity analysis of the key parameters of the model has been discussed below. The sensitivity analysis is performed by changing the values of the key parameters one at a time and keeping the remaining parameters unchanged.

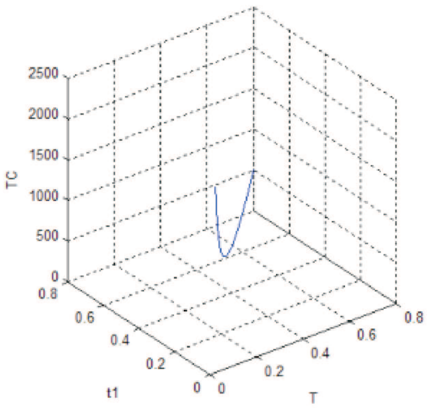

Figure 3: Graphical representation of the total cost(case1).

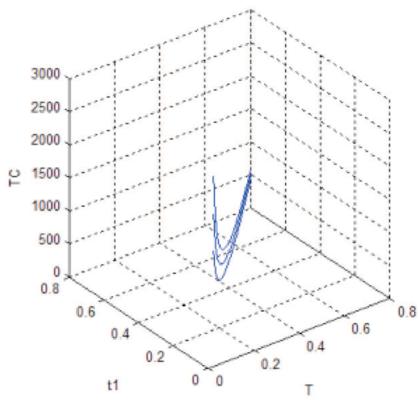

Figure 4: Comparison of three probabilistic models(case1).

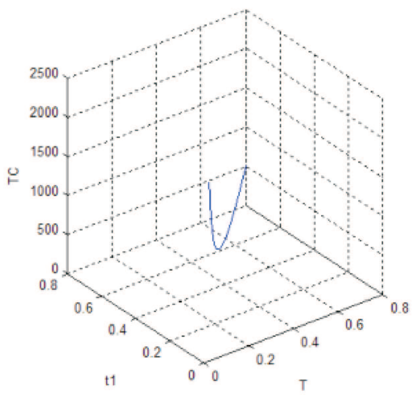

Figure 5: Graphical representation of the total cost(case2).

Table 3: Effects of changes in the system parameters of the model(case 1).

	I_e	0.014			0.016			0.018		
		uniform	beta	triangular	uniform	beta	triangular	uniform	beta	triangular
I_p 0.06	t_1	0.5089	0.5394	0.5271	0.5077	0.5388	0.5262	0.5063	-	-
	T	0.7489	0.6246	0.6816	0.7509	0.6261	0.6834	0.7529	0.0282	-
	TC1	555.75	561.52	548.12	556.62	560.79	548.07	557.54	299.69	-
	Q	119	16	50	119	16	50	120	31	-
0.07	t_1	0.4978	0.5178	-	0.4671	0.5170	-	0.4655	0.5161	-
	T	0.6376	0.5936	-	0.6869	0.5948	-	0.6883	0.5960	-
	TC1	555.23	566.86	-	567.63	566.17	-	568.71	565.49	-
	Q	50	16	-	120	16	-	120	16	-

- indicates an infeasible solution.

The sensitivity analysis based on example 1 as shown in Table 3 and Table 4 indicates the following observations:

1. From Table 4, it is clear that when the value of parameters a, b, c, M increase, the optimal total cost increases for the three models.
2. When the value of parameters a, b, c, M increase, the optimal length of the replenishment cycle decreases whereas the optimal length of positive inventory stock increases for the three different probabilistic deterioration functions.
3. As the value of a increases, the optimal order quantity decreases but the optimal order quantity increases as the value of b, c, M increases.
4. From Table 3 it is clear that the optimal cost is less sensitive to changes in I_e and I_p. For certain changes in the values of the parameters a, b, c, M, I_e and I_p, the models have no feasible solution. This may be due to the choice of the particular values in this numerical example.

6. Conclusion

In this paper we have developed a time-dependent quadratic inventory model for deteriorating items with permissible delay in payments. Shortages are allowed and partially backlogged. An analytic formulation of the model and an optimal solution procedure to find the optimal replenishment policy were also presented. Moreover sensitivity analysis of the optimal solutions with respect to various parameters were carried out. A comparison between the three models is done with the help of a graphical representation. A rapidly increasing demand can be represented by an exponential function of time. The assumption of an exponential rate of change in demand is high and the fluctuation or variation of any commodity in the real market cannot be so high. So, a quadratic demand seems to be a better representation of time-varying demands. Offering a certain credit period without interest enthuse the consumers to order more quantities, as delayed payment indirectly reduces the purchase cost. A possible future research issue is to consider two level trade credit or trade credit linked with order quantity with time dependent quadratic demand. A similar comparative study could be done by developing a multi - echelon economic order quantity model with time dependent quadratic demand by considering shortages, inflation and delay in payments.

Table 4: Effects of changes in the system parameters of the model

			uniform	beta	triangular
a	35	t_1	0.5047	0.5373	0.5241
		T	0.7386	0.6192	0.6743
		TC1	556.33	562.91	549.07
		Q	119	16	50
	36	t_1	0.4794	-	0.5034
		T	0.7250	-	0.6648
		TC1	581.38	-	567.30
		Q	114	-	46
	38	t_1	0.4278	0.4833	0.4620
		T	0.6966	0.5974	0.6454
		TC1	638.53	606.21	608.19
		Q	103	6	37
b	13	t_1	0.5029	0.5359	0.5226
		T	0.7336	0.6246	0.6696
		TC1	556.53	564.19	549.68
		Q	161	30	72
	15	t_1	0.4996	0.5334	0.5197
		T	0.7241	0.6066	0.6606
		TC1	556.89	566.76	550.87
		Q	243	58	118
	17	t_1	0.4968	0.5312	0.5172
		T	0.7151	0.5987	0.6521
		TC1	557.10	569.37	552.06
		Q	325	86	162
c	0.01	t_1	-	0.5378	0.5248
		T	-	0.6202	0.6754
		TC1	-	562.71	548.89
		Q	49	115	
	0.04	t_1	0.5050	0.5378	0.5247
		T	0.7390	0.6200	0.6753
		TC1	556	562.73	548.91
		Q	174	45	109
	0.05	t_1	0.5050	0.5377	-
		T	0.7390	0.6200	-
		TC1	556	562.74	-
		Q	174	44	-
M	0.2822	t_1	0.5249	0.5375	-
		T	0.6720	0.6170	-
		TC1	549.12	563.81	-
		Q	50	16	-
	0.3096	t_1	0.2115	0.5373	0.5245
		T	0.5671	0.6174	0.6725
		TC1	983.60	563.56	549.12
		Q	63	16	50
	0.3370	t_1	-	0.5372	0.5243
		T	-	0.6174	0.6730
		TC1	-	563.56	549.11
		Q	-	16	50

- indicates an infeasible solution.

Acknowledgements

The research work has been supported by University Grants Commission (UGC - SAP), New Delhi, India.

Appendix A

To prove this appendix, we first prove the following lemma.

Lemma 1. *If a function* $G(x,y) = \frac{F(x,y)}{y}$ *where* $F(x,y)$ *is a twice differentiable function of* x *and* y*, then the minimum value of* $G(x,y)$ *exists at* $x = x*$ *and* $y = y*$ *if*

$$\frac{1}{y}\frac{\partial^2 F}{\partial y^2} > 0, \frac{1}{y}\frac{\partial^2 F}{\partial x^2} > 0 \quad \text{and} \quad \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 F}{\partial y^2} > \frac{1}{y^2}\Big(y\frac{\partial^2 F}{\partial y \partial x} - \frac{\partial F}{\partial x}\Big)^2$$

at $x = x*$ *and* $y = y*$.

Proof. We have $G(x,y) = \frac{F(x,y)}{y}$. For maximum or minimum, the necessary conditions are $\frac{\partial G}{\partial x} = 0$ and $\frac{\partial G}{\partial x} = 0$. Now

$$\frac{\partial G}{\partial x} = 0 \Rightarrow y\frac{\partial F}{\partial y} - F(x,y) = 0$$
$$\frac{\partial G}{\partial x} = 0 \Rightarrow \frac{\partial F}{\partial x} = 0.$$

Let these equations be satisfied at $x = x*$ and $y = y*$.

At $x = x*$ and $y = y*$,

$$\frac{\partial^2 G}{\partial x^2} = \frac{1}{y}\frac{\partial^2 F}{\partial x^2}; \quad \frac{\partial^2 G}{\partial y^2} = \frac{1}{y}\frac{\partial^2 F}{\partial y^2}; \quad \frac{\partial^2 G}{\partial y \partial x} = \frac{1}{y^2}[y\frac{\partial^2 F}{\partial y \partial x} - \frac{\partial F}{\partial x}].$$

We know that the sufficient conditions for the existence of a minimum of G(x,y) at $x = x*$ and $y = y*$ are

$$(\frac{\partial^2 G}{\partial x^2})(\frac{\partial^2 G}{\partial y^2}) - (\frac{\partial^2 G}{\partial x \partial y})^2 > 0 \quad \text{and} \quad \frac{\partial^2 G}{\partial x^2} > 0, \frac{\partial^2 G}{\partial y^2} > 0.$$

Hence the lemma.

Here $TC_1(t_1,T) = \frac{1}{T}TC_{11}(t_1,T)$ where $TC_{11}(t_1,T) = [A + C_h + DC + C_s + C_r + C_o + IP_1 - IE_1]$. For maximum or minimum value of $TC_1(t_1,T)$ we have the necessary conditions

$$T\frac{\partial TC_{11}(t_1,T)}{\partial T} - TC_{11}(t_1,T) = 0 \ \text{ and } \ \frac{\partial TC_{11}(t_1,T)}{\partial t_1} = 0$$

Suppose the values obtained by solving the above simultaneous equation are $T = T^*$ and $t_1 = t_1^*$ then the value of $TC_1(t_1^*,T^*)$ is the minimum if it satisfies the sufficiency conditions

$$\frac{1}{T}\frac{\partial^2 TC_{11}(t_1,T)}{\partial T^2} > 0; \quad \frac{1}{T}\frac{\partial^2 TC_{11}(t_1,T)}{\partial t_1^2} > 0;$$

and

$$\frac{\partial^2 TC_{11}(t_1,T)}{\partial T^2}\frac{\partial^2 TC_{11}(t_1,T)}{\partial t_1^2} > \frac{1}{T^2}(T\frac{\partial^2 TC_{11}(t_1,T)}{\partial t_1 \partial T} - \frac{\partial TC_{11}(t_1,T)}{\partial t_1})^2.$$

Appendix B

Here $TC_2(t_1,T) = \frac{1}{T} TC_{21}(t_1,T)$ where $TC_{21}(t_1,T) = [A + C_h + DC + C_s + C_r + C_o + IP_2 - IE_2]$. For maximum or minimum value of $TC_2(t_1,T)$ we have the necessary conditions

$$T\frac{\partial TC_{21}(t_1,T)}{\partial T} - TC_{21}(t_1,T) = 0 \text{ and } \frac{\partial TC_{21}(t_1,T)}{\partial t_1} = 0$$

Suppose the values obtained by solving the above simultaneous equation are $T = T^*$ and $t_1 = t_1^*$ then the value of $TC_2(t_1^*,T^*)$ is the minimum if it satisfies the sufficiency conditions

$$\frac{1}{T}\frac{\partial^2 TC_{21}(t_1,T)}{\partial T^2} > 0; \quad \frac{1}{T}\frac{\partial^2 TC_{21}(t_1,T)}{\partial t_1^2} > 0;$$

and

$$\frac{\partial^2 TC_{21}(t_1,T)}{\partial T^2}\frac{\partial^2 TC_{21}(t_1,T)}{\partial t_1^2} > \frac{1}{T^2}\Big(T\frac{\partial^2 TC_{21}(t_1,T)}{\partial t_1 \partial T} - \frac{\partial TC_{21}(t_1,T)}{\partial t_1}\Big)^2.$$

References

[1] Bahari-Kashani, H. (1989). *Replenishment schedule for deteriorating items with time-proportional demand*, Journal of Operational Research Society, Vol.40, 75-81.

[2] Chen, S. H., Cardenas-Barron, L. E. and Teng, J. T. (2014). *Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity*, International Journal of Production Economics, http://dx.doi.org/10.1016/j.ijpe.2013.05.032.

[3] Chung, K. J. and Ting, P. S. (1993). *A heuristic for replenishment of deteriorating items with a linear trend in demand*, Journal of Operational Research Society, Vol.44, 1235-1241.

[4] Chung, K. J., Cardenas-Barron, L. E. and Ting, P. S. (2014). *An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit*, International Journal of Production Economics, http://dx.doi.org/10.1016/j.ijpe.2013.12.033.

[5] Chung, K. J. and Cardenas-Barron, L. E. (2013). *The simplified solution procedure for deteriorating items under stock - dependent demand and two - level trade credit in the supply chain management*, Applied Mathematical Modelling, Vol.37, 4653-4660.

[6] Covert, R. P. and Philip, G. C. (1973). *An EOQ model for items with weibull distribution deterioration*, AIIE Transactions, Vol.5, 323-326.

[7] Dave, U. and Patel, L. K. (1981). *(T, Si) policy inventory model for deteriorating items with time proportional demand*, Journal of Operations Research Society, Vol.32, 137-142.

[8] Donaldson, W. A. (1977). *Inventory replenishment policy for a linear trend in demand - an analytical solution*, Operational research Quartely, Vol.28, 663-670.

[9] Ghare, P. M. and Schrader, G. H. (1963). *A model for an exponentially decaying inventory*, Journal of Industrial Engineering, Vol.14, 238-243.

[10] Giri, B. C., Goswami, A. and Chaudhuri, K. S. (1996). *An EOQ model for deteriorating items with time varying demand and costs*, Journal of Operational Research Society, Vol.47, 1398-1405.

[11] Ghosh, S. K. and Chaudhuri, K. S. (2006). *An EOQ model with a quadratic demand, time - proportional deterioration and shortages in all cycles*, International Journal of Systems Sciences, Vol.37, 663-672.

[12] Goyal, S. K. (1985). *EOQ under conditions of permissible delay in payments*, Journal of Operations Research Society, Vol.36, 335-338.

[13] Goyal, S. K. and Giri, B. C. (2001). *Recent trends in modeling of deteriorating inventory*, European Journal of Operations Research, Vol.134, 1-16.

[14] Goswami, A. and Chaudhuri, K. S. (1991). *An EOQ model for deteriorating items with a linear trend in demand*, Journal of Operational Research Society, Vol.42, 1105-1110.

[15] Hariga, M. (1995). *An EOQ model for deteriorating items with shortages and time-varying demand*, Journal of Operational Research Society, Vol.46, 398-404.

[16] Jalan, A. K., Giri, R. R. and Chaudhuri, K. S. (1996). *EOQ model for items with weibull distribution deterioration, shortages and trended demand*, International Journal of Systems Sciences, Vol.27, 851-855.

[17] Jalan, A. K. and Chaudhuri, K. S. (1999). *Structural properties of an inventory system with deterioration and trended demand*, International Journal of Systems Sciencesm Vol.30, 627-633.

[18] Jalan, A. K. and Chaudhuri, K. S. (1999). *An EOQ model for deteriorating items in a declining market with SFI policy*, Korean Journal of Computational and Applied Mathematics, Vol.6, 437-449.

[19] Khanra, S. and Chaudhuri, K. S. (2003). *A note on an order level inventory model for a deteriorating item with time dependent quadratic demand*, Computer and Operations Research, Vol.30, 1901-1916.

[20] Khanra, S., Ghosh, S. K. and Chaudhuri, K. S. (2011). *An EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment*, Applied Mathematics and Computation, Vol.218, 1-9.

[21] Khanra, S., Mandal, B. and Sarkar, B. (2013). *An inventory model with time dependent demand and shortages under trade credit policy*, Economic Modelling, Vol.35, 349-355.

[22] Lin, C., Tan, B. and Lee, W. C. (2000). *An EOQ model for deteriorating items with shortages*, International Journal of Systems Sciences, Vol.31, 391-400.

[23] Maihami, R. and Abadi, I. N. K. (2012). *Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging*, Mathematical and Computer Modelling, Vol.55, 1722-1733.

[24] Mary Latha, K. F. and Uthayakumar, R. (2012). *An ordering policy for deteriorating items with quadratic demand, permissible delay and partial backlogging*, Lecture notes in Mechanical Engineering, 673-681.

[25] Mitra, A., Cox, J. F. and Jesse, R. R. (1984). *A note on deteriorating order quantities with a linear trend in demand*, Journal of Operational Research Society, Vol.35, 141-144.

[26] Nahmias, S. (1982). *Perishable inventory theory: A review*, Operations Research, Vol.30, 680-708.

[27] Ouyang, L. Y., Wu, K. S. and Yang, C. T. (2006). *A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments*, Computers and Industrial Engineering, Vol.51, 637-651.

[28] Ouyang, L. Y., Yang, C. T., Chan, Y. L. and Cardenas-Barron, L. E. (2013). *A comprehensive extension of the optimal replenishment decisions under two levels of trade credit policy depending on the order quantity*, Applied Mathematics and Computation, Vol.224, 268-277.

[29] Philip, G. C. (1974). *A generalised EOQ model for items with weibull distribution*, AIIE Transactions, Vol.6, 159-162.

[30] Raffat, F. (1991). *Survey of literature on continuously deteriorating inventory model*, European Journal of Operations Research Society, Vol.42, 27-37.

[31] Ritchie, E. (1980). *Practical inventory replenishment policies for a linear trend in demand followed by a period of steady demand*, Journal of Operational Research Society, Vol.31, 605-613.

[32] Ritchie, E. (1984). *The EOQ for linear increasing demand: A simple optimal solution*, Journal of Operational Research Society, Vol.35, 949-952.

[33] Ritchie, E. (1985). *Stock replenishment quantities for unbounded linear increasing demand: an interestingconsequence of the optimal policy*, Journal of Operational Research Society, Vol.36, 737-739.

[34] Sana, S. S. and Chaudhuri, K. S. (2008). *A deterministic EOQ model with delay in payments and price discount offers*, European Journal of Operational Research, Vol.184, 509-533.

[35] Sarkar, B., Sana, S. S. and Chaudhri, K. S. (2010). *Optimal reliability, production lotsize and safety stock in an imperfect production system*, International Journal of Mathematics in Operational Research, Vol.2, 467-490.

[36] Sarkar, B., Sana, S. S. and Chaudhri, K. S. (2010). *Optimal reliability, production lotsize and safety stock:An economic manufacturing quantity model*, International Journal of Management Science and Engineering Management, Vol.5, 192-202.

[37] Sarkar, B. (2012). *An inventory model with reliability in an imperfect production process*, Applied Mathematics and Computation, Vol.218, 4881-4891.

[38] Sarkar, B. (2012). *An EOQ model with delay-in payments and time-varying deterioration rate*, Mathematical and Computer Modelling , Vol.55, 367-377.

[39] Sarkar, B. (2012). *An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production*, Applied Mathematics and Computation, Vol.218, 8295-8308.

[40] Sett, B. K., Sarkar, B. and Goswami, A. (2012). *A two-warehouse inventory model with increasing demand and time varying deterioration*, Scientia Iranica, Transaction E: Industrial Engineering, Vol.19, 306-310.

[41] Sarkar, B., Saren, S. and Wee, H. M. (2013). *An inventory model with variable demand component cost and selling price for deteriorating items*, Economic Modelling, Vol.30, 306-310.

[42] Sarkar, B. and Sarkar, S. (2013). *An improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand*, Economic Modelling, Vol.30, 924-932.

[43] Sarkar, B. and Sarkar, S. (2013). *Variable deterioration and demand - An inventory model* Economic Modelling, Vol.31, 548-556.

[44] Sarkar, M. and Sarkar, B. (2013). *An economic manufacturing quantity model with probabilistic deterioration in a production system*, Economic Modelling, Vol.31, 245-252.

[45] Sarkar, B., Sana, S. S. and Chaudhari, K. S. (2013). *Inventory model with finite replenishment rate,trade credit policy and price -discount offer*, Journal of Industrial Engineering, Vol.2013, 1-18.

[46] Sarkar, B. and Moon, I. (2013). *Improved quality, setup cost reduction, and variable backorder costs in an imperfect production process*, International Journal of Production Economics, Vol.155, 204-213.

[47] Sarkar, B. (2013). *A production-inventory model with probabilistic deterioration in two-echelon supply chain management*, Applied Mathematical Modelling, Vol.37, 3138-3151.

[48] Silver, E. A. and Meal, H. C. (1969). *A simple modification of the EOQ for the case of a varying demand rate*, Production Inventory Management, Vol.10, 52-65.

[49] Silver, E. A. (1979). *A simple inventory replenishment decision rule for a linear trend in demand*, Journal of Operational Research Society, Vol.30, 71-75.

[50] Tripathi, R. P. and Pandey, H. S. (2013). *An EOQ model for deteriorating items with Weibull time dependent demand rate under trade credits*, International Journal of Information and Management Sciences, Vol.24, 329-347.

[51] Widyadana, G. A., Cardenas-Barron, L. E. and Wee, H. M. (2011). *Economic order quantity model for deteriorating items and planned backorder level*, Mathematical and Computer Modelling, Vol.54, 1569-1575.

[52] Wu, K. S., Ouyang, L. Y. and Yang, C. T. (2006). *An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging*, International Journal of Production Economics, Vol.101, 369-384.

[53] Wu, J., Ouyang, L. Y, and Cardenas-Barron, L. E. and Goyal, S. K. (2014). *Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing*, European Journal of Operational Research, Vol.237, 898-908.

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(Received January 2014; accepted September 2014)