Fuzzy System Reliability Analysis using Pentagon Fuzzy Numbers based on Statistical Data

[1]R.Mohana, [2]G. Michael Rosario

[1, 2]Research Centre and PG Department of Mathematics
Jayaraj Annapackiam College for Women(Autonomous)
Periyakulam, Tamilnadu, India
mohana.n@vit.ac.in, tony87rio@gmail.com

Abstract--- **In this article, we find the reliability of the serial system and the parallel system of n components when the reliability of the subsystems R_j are unknown. Using statistical point estimates, the reliability of the systems are examined. Consider the statistical data for the subsystems P_j is $R_{jq} \in [0,1], q = 1,2,\ldots,n_j$. Let R_j (unknown) be the population reliability of the subsystem P_j and find the average value $\bar{R}_j$. From the statistical point of view, this $\bar{R}_j$ is a point estimate of R_j. Use $\bar{R}_j$ instead of the reliability of P_j. Since the probability distribution of the error between the point estimate $\bar{R}_j$ and R_j is unknown, use the statistical confidence interval of R_j instead. Corresponding to this confidence interval, we characterize the pentagon fuzzy number. Signed distance method is used for defuzzi cation. Finally estimation of the reliability of the serial system and the parallel system is illustrated by examples.**

Keywords--- **Fuzzy Reliability, Statistical Data, Statistical Con Dence Interval, Signed Distance, Pentagon Fuzzy Number, Point Estimation.**

I . INTRODUCTION

Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes. Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences.The reliable engineering is one of the important engineering tasks in design and development of technical system. The conventional reliability of a system is defined as the probability that the system performs its assigned function properly during a predefined period under the condition that the system behavior can be fully characterized in the context of probability measures. The reliability of a system can be determined on the basis of tests or the acquisition of operational data. However, due to the uncertainty and inaccuracy of this data, the estimation of precise values of probabilities is very difficult in many systems (e.g. power system, electrical machine, hardware etc., Hammer (2001) [6], El-Hawary (2000) [7]). For this reason the fuzzy reliability concept has been introduced and formulated in the context of fuzzy measures. The basis for this approach is constituted by the fundamental works on fuzzy set theory of Zadeh (1978) [16], Dubois and Prade (1980) [1], Zimmerman (1986) [17] and other. It is an essential tool for formulating the mathematical information of imprecise nature. The theory of fuzzy reliability was proposed and developed by several authors, Cai, Wen and Zhang (1991, 1993) [2-3]; Cai (1996) [4]; Chen, Mon (1993) [5]; Hammer (2001) [6]; El-Hawary (2000) [7], Onisawa, Kacprzyk (1995) [12]; Utkin, Gurov (1995) [14]. The recent collection of papers by Onisawa and Kacprzyk (1995) [12], gave many different approach for fuzzy reliability. This study presents a new method to analyze fuzzy system reliability using fuzzy number arithmetic operations based on statistical data. In this study statistical methodology is applied.Using statistical approach the fuzzy system is defuzzified and the reliability of the system is estimated in the fuzzy sense.

In section 2, some properties of fuzzy sets are given. Theses properties will be used in section 3 and 4. In section 3.1, a statistical point estimates are used to examine the reliability of the serial system and that of the parallel system in the crisp case. In section 3.2, the concept of the statistical confidence interval converted to a pentagon fuzzy number is applied, and then the reliability of the two systems are described in the fuzzy sense. Numerical examples are illustrated in section 4 and the results are discussed in section 5.

II. PENTAGON FUZZY NUMBERS AND SIGNED DISTANCE OF FUZZY SETS ON R

For the fuzzy system reliability analysis using the pentagon fuzzy numbers, the following definitions are needed.

1) Definition: A fuzzy number M is defined to be a Pentagon **Fuzzy Number** (PFN) if its membership function $\mu_M : R \longrightarrow [0,1]$ is defined by

$$\mu_M(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a}{b-a}\right) & for \quad x \in [a,b] \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-b}{c-b}\right) & for \quad x \in [b,c] \\ \frac{1}{2}+\frac{1}{2}\left(\frac{d-x}{d-c}\right) & for \quad x \in [c,d] \\ \frac{1}{2}\left(\frac{e-x}{e-d}\right) & for \quad x \in [d,e] \\ 0 & otherwise \end{cases}$$

This fuzzy number is denoted by (a,b,c,d,e) where $a \leq b \leq c \leq d \leq e$.

2) Definition: A fuzzy set is called a level fuzzy interval, where $0 \leq \alpha \leq 1$ and we denote it by $[p,q;\alpha]$, if its membership function is

$$\mu_{[p,q;\alpha]}(x) = \begin{cases} \alpha, & if \quad p \leq x \leq q \\ o, & otherwise \end{cases} \qquad (1)$$

3) Definition: The family of all fuzzy sets on R which we denote as F_s satisfies the following two conditions.

Let $\widetilde{D} \in F_s$. Then $\widetilde{D}$ satisfies (1) and (2) below.

i) The left and right hand side of the α - level set of $\widetilde{D}, D_l(\alpha)$ and $D_u(\alpha)$ exist. We denote it as $D(\alpha) = [\, D_l(\alpha),\ D_u(\alpha)]$.
ii) $D_l(\alpha)$ and $D_u(\alpha)$are integrable for $\alpha \in [0,1]$.

4) Definition: The left and right hand side of the α - level set of pentagon fuzzy number $\widetilde{D} = (a_1, b_1, c_1, d_1, e_1)$ are

$$D_l(\alpha) = a_1 + (c_1 - b_1)\alpha$$
$$D_u(\alpha) = e_1 - (d_1 - c_1)\alpha$$

5) Definition: Let $a, 0 \in R$. The **signed distance of the point** a from 0 is $d^*(a,0) = a$.
Let $[a,b] \in R$. The **signed distance of the closed interval** $[a,b]$ from 0 is

$$d^*([a,b],0) = 1/2(a+b).$$

By decomposition theorem, $\widetilde{D} = \bigcup_{0\leq\alpha\leq1} \alpha I_{D(\alpha)}, I_{D(\alpha)}$ is the characteristic function of the α - level set $D(\alpha)$.

By preliminaries, we get $I_{D(\alpha)}(x) = \mu_{[D_l(\alpha),D_u(\alpha);\alpha]}(x) \forall x \in R$. Therefore, we have

$$\widetilde{D} = \bigcup_{0\leq\alpha\leq1} [D_l(\alpha), D_u(\alpha); \alpha]. \qquad (2)$$

By preliminaries, for each $\alpha \in [0,1]$, the signed distance of the interval $[D_l(\alpha), D_u(\alpha); \alpha]$ from 0 by

$$d^*([D_l(\alpha), D_u(\alpha)], 0) = 1/2[D_l(\alpha) + D_u(\alpha)] \qquad (3)$$

For each $\alpha \in [0,1]$, $[D_l(\alpha), D_u(\alpha)]$ and $[D_l(\alpha), D_u(\alpha); \alpha]$ are one-to-one mappings. Similarly, 0 and $\tilde{0}$are also one-to-one mappings. Therefore, from [3], the signed distance from $[D_l(\alpha), D_u(\alpha); \alpha]$ to $\tilde{0}$ is

$$d^*([D_l(\alpha), D_u(\alpha)], \tilde{0}) = 1/2[D_l(\alpha) + D_u(\alpha)] \qquad (4)$$

By preliminaries, $\int_0^1 D_l(\alpha)d\alpha$ and $\int_0^1 D_u(\alpha)d\alpha$ both exist.

6) Definition: Let $\widetilde{D} \in F_s$. By [2] and [4] the **signed distance of** $\widetilde{D}$ measured from $\tilde{0}$ as

$$d(\widetilde{D},\tilde{0}) = 1/2\int_0^1 (D_l(\alpha), D_u(\alpha))d\alpha$$

Let $\tilde{A} = (a_1, b_1, c_1, d_1, e_1), \tilde{B} = (a_2, b_2, c_2, d_2, e_2)$ where $0 < a_1 < b_1 < c_1 < d_1 < e_1$ and $0 < a_2 < b_2 < c_2 < d_2 < e_2$, be two pentagon fuzzy numbers. By definition 4, their left endpoints and right endpoints of the α - cut are

$$A_l(\alpha) = a_1 + (c_1 - b_1)\alpha$$
$$A_u(\alpha) = e_1 - (d_1 - c_1)\alpha$$
$$B_l(\alpha) = a_2 + (c_2 - b_2)\alpha$$
$$\text{and } B_u(\alpha) = e_2 - (d_2 - c_2)\alpha$$

respectively, where $0 \leq \alpha \leq 1$ and $0 \leq A_l(\alpha) \leq A_u(\alpha), 0 \leq B_l(\alpha) \leq B_u(\alpha)$. For _ 2 [0,1], we obtain

$$\tilde{A}\otimes\tilde{B} = \cup_{0\leq\alpha\leq1} [(a_1 + (c_1 - b_1)\alpha)(a_2 + (c_2 - b_2)\alpha) \quad (e_1 - (d_1 - c_1)\alpha)(e_2 - (d_2 - c_2)\alpha)] \quad (5)$$

III. FUZZY SYSTEM RELIABILITY USING PENTAGON FUZZY NUMBERS

In this section, the general serial system and the parallel system using the statistical data are discussed. Fig.1 represents the subsystems of a serial system, which are $P_1, P_2, \ldots, P_n$. Let $R_1, R_2, \ldots, R_n (0 \leq R_j \leq 1, j = 1,2,\ldots,n)$ are the reliabilities of $P_1, P_2, \ldots, P_n$ respectively. Then the reliability of serial system is

$$\prod_{j=1}^{n} R_j \quad (6)$$

Similarly, Fig.2 represents the subsystems of a parallel system, which are $P_1, P_2, \ldots, P_n$. Then reliability of parallel system is

$$1 - \prod_{j=1}^{n}(1 - R_j) \quad (7)$$

Suppose that $R_j, j = 1,2,\ldots,n$ are unknown and for each subsystems $P_j, j = 1,2,,\ldots,n$, we can get n records

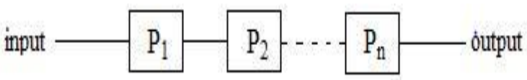

Figure 1: Configuration of serial system.

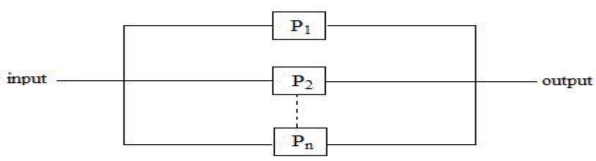

Figure 2: Configuration of parallel system

of reliability of P_j. We use statistical point estimates to examine the reliability of the serial system and that of the parallel system.

For this, let us consider that each subsystem P_j is of size $n_j, j = 1,2,\ldots,n$ and denote its statistical data by $R_{jq} \in [0,1], q = 1,2,\ldots,n_j$. Let R_j (unknown) be the population reliability of the subsystem P_j.

A. Reliability of Serial System and Reliability of Parallel System Using Point Estimates

For subsystem P_j, using the statistical data R_{jq} , $q = 1,2,\dots,n_j$, we can find their average value. Let

$\bar{R}_j = (1/n_j)\sum_{q=1}^{n_j} R_{jq} \in [0,1]$. From the statistical point of view, we know that R_j is a point estimate of $\bar{R}_j$. Therefore, we use $\bar{R}_j$ instead of the reliability of P_j.

Results: Let R_j (unknown), $j = 1,2,\dots,n$ be the population reliability of the subsystem P_j. Then from the point estimate $\bar{R}_j$of R_j, we have

(1) The reliability of the serial system is

$$\prod_{j=1}^{n} \bar{R}_j \quad (8)$$

(2) The reliability of the parallel system is

$$1 - \prod_{j=1}^{n}(1 - \bar{R}_j) \quad (9)$$

B. Fuzzy System Reliability Using Pentagon Fuzzy Numbers

Because of the probability distribution of the error between the point estimate $\bar{R}_j$ and R_j is unknown, instead the statistical confidence interval of R_j used. Let

$$0 < \gamma_k < 1, \mathrm{k} = 1, 2, \; 0 < \gamma < 1 \text{ and } \gamma_1 + \gamma_2 = \gamma \quad (10)$$

The $(1 - \gamma)$ % confidence interval of $\bar{R}_j$, $j = 1,2,\dots,n$ is

$$[\bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}), \bar{R}_j + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j})], \quad (11)$$

Where ${s_j}^2 = 1/(n_j - 1)\sum_{q=1}^{n_j}(R_{jq} - \bar{R}_j)^2$. Let T be a t-distributed random variable with $n_j - 1$ degree of freedom. Then $t_{n_j-1}(\gamma_k)$ satisfies $p\left(T \geq t_{n_j-1}(\gamma_k)\right) = \gamma_k, k = 1,2$

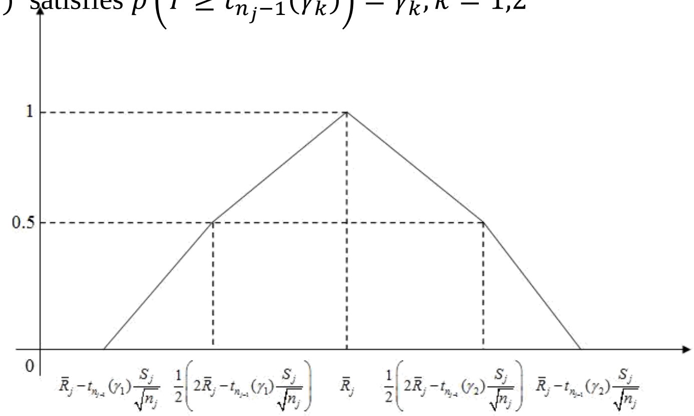

Figure 3: Pentagon Fuzzy Number R_j.

The decision maker not only chooses γ_1 and $\gamma_2$to satisfy [10], but also satisfies the following conditions for $j = 1,2,\dots,n$,

$$0 < \bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}) < 1$$

and

$$0 < \bar{R}_j - t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j}) < 1$$

The decision maker must take a suitable value from [11] as an appropriate point estimate of R_j . [11] is an interval and is not a value. Therefore, we can not consider this problem by using the statistical point of view. Instead, we use the fuzzy point of view in the following:

From the Fig.3, we consider

$$a = \bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})$$
$$b = \left(\frac{1}{2}\right)[2\bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})]$$
$$c = \bar{R}_j$$
$$d = \left(\frac{1}{2}\right)[2\bar{R}_j + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j})]$$
$$e = \bar{R}_j + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j})$$

If the decision maker takes a value that coincides with c, then the error is 0. The error is bigger, when the value deviates from c farther from both sides of c.

If the value lies at one of the two endpoints a and e the error will attain a maximum. If the value lies at any one of the points b and d, then the error will occur at the midpoint of 0 and maximum error value.

From the fuzzy point of view, we can transform the error into a confidence level. If the error is 0, then the confidence level is 1. The confidence level is lesser, when the farther the value is from both side of c.

At the two endpoints a and e, the confidence level will be minimized to 0. Since the error of b and d occur at the midpoint of 0 and maximum error value, the confidence level will attain its value between 1 and minimum. But here we set it to be 0. Therefore, at the midpoints b and d, the confidence level will be 0.5.

Corresponding to the interval in [11], we characterize the pentagon fuzzy number as follows,

$$\begin{aligned}\widetilde{R_j} &= (a,b,c,d,e)\\ &= (\bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}),\\ &\quad \left(\frac{1}{2}\right)\left[2\bar{R}_j - t_{n_j-1}(\gamma_1)\left(\frac{s_j}{\sqrt{n_j}}\right)\right], \bar{R}_j,\\ &\quad \left(\frac{1}{2}\right)\left[2\bar{R}_j + t_{n_j-1}(\gamma_2)\left(\frac{s_j}{\sqrt{n_j}}\right)\right],\\ &\quad \bar{R}_j + t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j}))\end{aligned} \quad (13)$$

From Fig.3, the membership grade of c in $\widetilde{R_j}$ is 1. The farther the point in intervals a, b from both sides of c, the membership grade is lower.

The membership grade and the confidence level have the same properties.

Therefore, if we make a correspondence between membership grade and confidence level, it is reasonable to set up a pentagon fuzzy number in [13] corresponding to [11].

By preliminaries, we get

$$R_j^* = d(\widetilde{R_j}, \bar{0}) = \widetilde{R_j} + \left(\frac{3}{8}\right)\left[t_{n_j-1}(\gamma_2) - t_{n_j-1}(\gamma_1)\right](s_j/\sqrt{n_j})$$

which belongs to the interval in [11]

This is the estimate value of R_j from a fuzzy point of view. When $\gamma_1 = \gamma_2 = \gamma/2$, then $R_j^* = \bar{R}_j$ for each $j \in \{1,2,\ldots,n\}$. The α- level set of $\bar{R}_j, 0 \le \alpha \le 1$ is

$$[R_{jl}(\alpha), R_{ju}(\alpha)], \quad (14)$$

where

$$\begin{aligned}R_{jl}(\alpha) = \bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j})\\ +\{\bar{R}_j - [2\bar{R}_j - t_{n_j-1}(\gamma_1)\left(\frac{s_j}{\sqrt{n_j}}\right)]/2\}\alpha\end{aligned}$$

$$\text{and } R_{ju}(\alpha) = \bar{R}_j - t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j}) + \{[2\bar{R}_j - t_{n_j-1}(\gamma_2)\left(\frac{s_j}{\sqrt{n_j}}\right)]/2 - \bar{R}_j\}\alpha \quad (15)$$

$$\widetilde{R_j} = \cup_{0 \le \alpha \le 1}\left[R_{jl}(\alpha), R_{ju}(\alpha); \alpha\right], j = 1,2, \dots, n. \quad (16)$$

1) Theorem: Using the pentagon fuzzy numbers $\bar{R}_j, j = 1,2,\dots,n$, in [13] we have the fuzzy system reliability as follows. (1). Fuzzy reliability of the serial system is

$$\widetilde{R_1} \otimes \widetilde{R_2} \otimes \dots \otimes \widetilde{R_n} = \cup_{0 \le \alpha \le 1}\left[\prod_{j=1}^{n} R_{jl}(\alpha), \prod_{j=1}^{n} R_{ju}(\alpha); \alpha\right], \quad (17)$$

Where $R_{jl}(\alpha), R_{ju}(\alpha)$ are given in [15].

(2). Fuzzy reliability of the parallel system is

$$\tilde{1} \ominus [(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \dots \otimes (\tilde{1} \ominus \widetilde{R_n})] = \cup_{0 \le \alpha \le 1}\left[1 - \prod_{j=1}^{n} Q_{ju}(\alpha), 1 - \prod_{j=1}^{n} Q_{jl}(\alpha); \alpha\right], \quad (18)$$

where

$$Q_{jl}(\alpha) = 1 - \{\bar{R}_j - t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j}) - \{[2\bar{R}_j - t_{n_j-1}(\gamma_2)\left(\frac{s_j}{\sqrt{n_j}}\right)]/2 - \bar{R}_j\}\alpha\}$$

$$\text{and } Q_{ju}(\alpha) = 1 - \bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}) + \{\bar{R}_j - [2\bar{R}_j - t_{n_j-1}(\gamma_1)\left(\frac{s_j}{\sqrt{n_j}}\right)]/2\}\alpha\}$$

$0 \le \alpha \le 1, j = 1,2,\dots,n$.

Proof :

(1). From Equations (12) and (15), for each $j = 1,2,\dots,n$ we have $0 < R_{jl}(\alpha) \le Q_{ju}(\alpha)$, for all $\alpha \in [0,1]$. We get (1).

(2). We have $\tilde{1} = (1,1,1,1,1)$. By [15],

we obtain

$$\tilde{1} \ominus \widetilde{R_j} = \bigcup_{0 \le \alpha \le 1}\left[Q_{jl}(\alpha), Q_{ju}(\alpha); \alpha\right], \; j = 1,2,\dots,n,$$

where

$$Q_{jl}(\alpha) = 1 - \{\bar{R}_j - t_{n_j-1}(\gamma_2)(s_j/\sqrt{n_j}) - \{[2\bar{R}_j - t_{n_j-1}(\gamma_2)\left(\frac{s_j}{\sqrt{n_j}}\right)]/2 - \bar{R}_j\}\alpha\}$$

$$\text{and } Q_{ju}(\alpha) = 1 - \bar{R}_j - t_{n_j-1}(\gamma_1)(s_j/\sqrt{n_j}) + \{\bar{R}_j - [2\bar{R}_j - t_{n_j-1}(\gamma_1)\left(\frac{s_j}{\sqrt{n_j}}\right)]/2\}\alpha\}$$

$0 \le \alpha \le 1$.

From [12], we get $0 < Q_{jl}(\alpha) < Q_{ju}(\alpha)$, for all $\alpha \in [0,1]$ and $j = 1,2,\dots,n$.

We have

$$[(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \dots \otimes (\tilde{1} \ominus \widetilde{R_n})] = \cup_{0 \le \alpha \le 1}\left[1 - \prod_{j=1}^{n} Q_{jl}(\alpha), 1 - \prod_{j=1}^{n} Q_{ju}(\alpha); \alpha\right].$$

We obtain

$$[(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \dots \otimes (\tilde{1} \ominus \widetilde{R_n})]$$

$$= \cup_{0 \leq \alpha \leq 1} [1 - \prod_{j=1}^{n} Q_{ju}(\alpha), 1 - \prod_{j=1}^{n} Q_{jl}(\alpha); \alpha].$$

This completes the proof of (2).

2) Theorem: Under the condition of Theorem 3.1, we have the following results.

(1). By definition 6, we can defuzzify [17] to get the estimate reliability of serial system in the fuzzy sense as follows,

$$d(\widetilde{R_1} \otimes \widetilde{R_2} \otimes \dots \otimes \widetilde{R_n}, \tilde{0}) = (1/2) \int_0^1 [\prod_{j=1}^{n} R_{jl}(\alpha) + \prod_{j=1}^{n} R_{ju}(\alpha); \alpha] d\alpha$$

(2). By definition 6, we can defuzzify [18] to get the estimate reliability of parallel system in the fuzzy sense as follows,

$$d(\tilde{1} \ominus [(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \dots \otimes (\tilde{1} \ominus \widetilde{R_n}), \tilde{0}])$$

$$= (1/2) \int_0^1 [(1 - \prod_{j=1}^{n} Q_{ju}(\alpha) + 1 - \prod_{j=1}^{n} Q_{jl}(\alpha))] d\alpha$$

Proof: By Theorem 3.1 and definition 6, we have desired result.

IV. NUMERICAL EXAMPLES

1) Example: Consider the following serial system and parallel system.

In Jing et al [8], let $\gamma = 0.02, \gamma_1 = 0.011, \gamma_2 = 0.009$.

Here $\bar{R}_j = (1/n_j) \sum_{q=1}^{n_j} R_{jq}$ and $s_j^2 = (1/n_j - 1) \sum_{q=1}^{n_j} (R_{jq} - \bar{R}_j{}^2)$

From the table of the t-distribution with $n_j - 1$ degrees of freedom, $j = 1,2,3$, we get the following data.

$$t_9(\gamma_1) = 2.7017, t_{19}(\gamma_1) = 2.5212, t_{14}(\gamma_1) = 2.5921, t_9(\gamma_2) = 2.9068, t_{19}(\gamma_2) = 2.6034,$$
$$t_{14}(\gamma_2) = 2.6946.$$

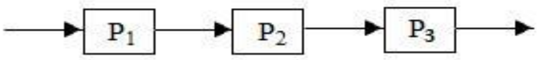

Figure 4: Serial System

Subsystem	Sample Size	Sample Mean	Sample Standard Deviation
P_1	$n_1 = 10$	$\bar{R}_1 = 0.8$	$s_1 = 0.02$
P_2	$n_2 = 20$	$\bar{R}_2 = 0.75$	$s_2 = 0.03$
P_3	$n_3 = 15$	$\bar{R}_3 = 0.9$	$s_3 = 0.01$

Table 4.1. Statistical Data.

j	Degrees Of Freedom (k)	a	b	d	e
1	9	0.7829	0.7914	0.8092	0.8184
2	19	0.7331	0.7415	0.7587	0.7675
3	14	0.8933	0.8967	0.9035	0.9070

Table 4.2. Two Endpoints and Midpoints of Pentagon Fuzzy Numbers

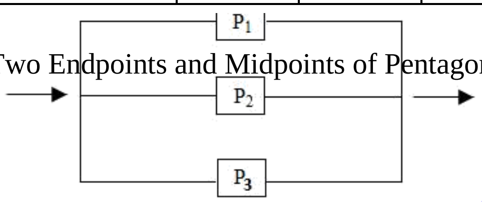

Figure 5: Parallel System

where a, b, c, d, e are represent in [13].

From Tables 4.1 and 4.2 we have the following pentagon fuzzy numbers,

$$\widetilde{R_1} = (0.7829, 0.7914, 0.8, 0.8092, 0.8184), \widetilde{R_2} = (0.7331, 0.7415, 0.75, 0.7587, 0.7675),$$

$$\widetilde{\widetilde{R_3}} = (0.8933, 0.8967, 0.9, 0.9035, 0.9070).$$

From [15], the α -level fuzzy sets of $\widetilde{R_j}, j = 1,2,3$ for $0 \leq \alpha \leq 1$ are

$$\bar{R}_{1l}(\alpha) = 0.7829 + 0.0086, \bar{R}_{1u}(\alpha) = 0.8184 - 0.0092, \bar{R}_{2l}(\alpha) = 0.7331 + 0.0085,$$
$$\bar{R}_{2u}(\alpha) = 0.7675 - 0.0087, \bar{R}_{3l}(\alpha) = 0.8933 + 0.0033, \bar{R}_{3u}(\alpha) = 0.9070 - 0.0035.$$

(1) Since $0 < R_{jl}(\alpha) < R_{ju}(\alpha)$ for all $0 \leq \alpha \leq 1, j = 1,2,3$, from Theorem 3.1(1), the fuzzy reliability of the serial system is

$$\widetilde{R_1} \otimes \widetilde{R_2} \otimes \widetilde{R_3} = \cup_{0 \leq \alpha \leq 1} \left[\prod_{j=1}^{3} R_{jl}(\alpha), \prod_{j=1}^{3} R_{ju}(\alpha); \alpha\right].$$

From Theorem 3.2(1), we get the estimate reliability of the serial system in the fuzzy sense as

$$d(\widetilde{R_1} \otimes \widetilde{R_2} \otimes \ldots \otimes \widetilde{R_n}, \tilde{0}) = (1/2) \int_0^1 \left[\prod_{j=1}^{3} R_{jl}(\alpha) + \prod_{j=1}^{3} R_{ju}(\alpha); \alpha\right] d\alpha$$

$$= 0.5386.$$

From [8], we get the crisp case reliability of the serial system to be

$$\prod_{j=1}^{3} \bar{R}_j = 0.54$$

(2) From Theorem 3.1(2), we have

$$Q_{1l}(\alpha) = 0.1816 + 0.0092\alpha, Q_{1u}(\alpha) = 0.21710 - 0086\alpha,$$
$$Q_{2l}(\alpha) = 0.2325 + 0.0087\alpha, Q_{2u}(\alpha) = 0.26690 - 0085\alpha,$$
$$Q_{3l}(\alpha) = 0.0930 + 0.0035\alpha, Q_{3u}(\alpha) = 0.10670 - 0033\alpha,$$

Since $0 < Q_{jl}(\alpha) < Q_{ju}(\alpha)$for all $0 \leq \alpha \leq 1, j = 1,2,3$. Fuzzy reliability of the parallel system is

$$\tilde{1} \ominus [(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \ldots \otimes (\tilde{1} \ominus \bar{R}_n)] = \bigcup_{0 \leq \alpha \leq 1} \left[1 - \prod_{j=1}^{3} Q_{ju}(\alpha), 1 - \prod_{j=1}^{3} Q_{jl}(\alpha); \alpha\right]$$

From Theorem 3.2(2), we get the estimate reliability of the parallel system in the fuzzy sense as

$$d(\tilde{1} \ominus [(\tilde{1} \ominus \widetilde{R_1}) \otimes (\tilde{1} \ominus \widetilde{R_2}) \otimes \ldots \otimes (\tilde{1} \ominus \bar{R}_n)], \tilde{0})$$

$$= (1/2) \int_0^1 \left[\left(1 - \prod_{j=1}^{3} Q_{ju}(\alpha) + 1 - \prod_{j=1}^{3} Q_{jl}(\alpha)\right)\right] d\alpha = 0.9949$$

From [9], we find the crisp case reliability of the parallel system to be

2) Example: In Mon and Cheng [11], they considered the following problem.

Two grinding machines, are working next to each other. Let us find the probability that people coming into the vicinity of the machines are injured mainly by getting a chip into their eye using

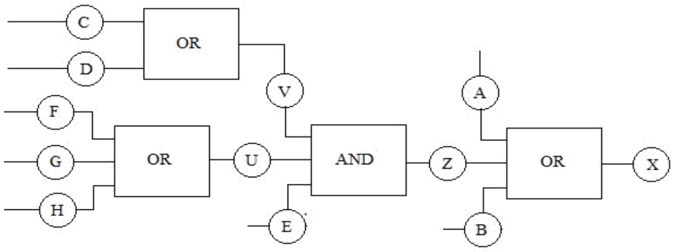

Figure 6: The System

pentagon fuzzy numbers. The most endangered persons are the operators, who are obliged to wear safety glasses but often fail to do this.

j	Symbols	Basic Event	Populations Reliability (unknown)	Sample Size	Sample Mean	Sample Standard Deviation
1	A	Operator 1 fails to wear safety glasses	R_A	$n_A = 10$	$\bar{R}_A$=0.2	s_A=0.004
2	B	Operator 2 fails to wear safety glasses	R_B	$n_B = 10$	$\bar{R}_B$=0.2	s_B=0.004
3	C	Machine 1 is operating	R_C	$n_C = 10$	$\bar{R}_C$=0.8	s_C=0.004
4	D	Machine 2 is operating	R_D	$n_D = 10$	$\bar{R}_D$=0.6	s_D=0.01
5	E	Persons entering the area without safety glasses	R_E	$n_E = 10$	$\bar{R}_E$=0.9	s_E=0.02
6	F	Persons entering the endangered area bringing material	R_F	$n_F = 10$	$\bar{R}_F$=0.5	s_F=0.004
7	G	Persons entering the area carrying away made product	R_G	$n_G = 10$	$\bar{R}_G$=0.6	s_G=0.004
8	H	Persons entering the area for other reasons	R_H	$n_H = 10$	$\bar{R}_H$=0.5	s_H=0.001

Table 4.3. The Basic Events Contributing to The Accident.

Further endangered are persons coming into the vicinity of the machines, the persons bringing and carrying away items, further those entering the area for other reasons.

Let γ=0.02, γ_1=0.011, γ_2=0.009. From Example 4.1, $t_9(\gamma_1)$=2.7017, $t_9(\gamma_2)$=2.9068.

j	Symbols	a	b	d	d
1	A	0.0966	0.0983	0.1018	0.1037
2	B	0.1966	0.1983	0.2018	0.2037
3	C	0.7966	0.7983	0.8018	0.8037
4	D	0.5915	0.5957	0.6046	0.6092
5	E	0.8829	0.8915	0.9092	0.9184
6	F	0.4966	0.4983	0.5018	0.5037
7	G	0.5966	0.5983	0.6018	0.6037
8	H	0.4992	0.4996	0.5005	0.5009

Table 4.4. Two Endpoints and Midpoints of Pentagon Fuzzy Numbers

Where a, b, c, d, e are represents in [13].

From Table 4.3 and 4.4 ,we get the following pentagon fuzzy numbers,

$\widetilde{R_1}$=(0.0966,0.0983,0.1,0.1018,0.1037)(= $\widetilde{R_A}$), $\widetilde{R_2}$=(0.1966,0.1983,0.2,0.2018,0.2037)(= $\widetilde{R_B}$),
$\widetilde{R_3}$=(0.7966,0.7983,0.8,0.8018,0.8037)(= $\widetilde{R_C}$), $\widetilde{R_4}$=(0.5915,0.5957,0.6,0.6046,0.6092)(= $\widetilde{R_D}$),
$\widetilde{R_5}$=(0.8829,0.8915,0.9,0.9092,0.9184)(= $\widetilde{R_E}$), $\widetilde{R_6}$=(0.4966,0.4983,0.5,0.5018,0.5037)(= $\widetilde{R_F}$),
$\widetilde{R_7}$=(0.5966,0.5983,0.6,0.6018,0.6037)(= $\widetilde{R_G}$), $\widetilde{R_8}$=(0.4992,0.4996,0.5,0.5005,0.5009)(=$\widetilde{R_H}$)

From[15], the α –level sets of $\bar{R}_j$, j =1,2,……,8, $0 \le \alpha \le 1$ are

$R_{1l}(\alpha)$=0.0966+0.0017 α , $R_{1u}(\alpha)$=0.1037-0.0018 α , $R_{2l}(\alpha)$=0.1966+0.0017 α ,
$R_{2u}(\alpha)$=0.2037-0.0018 α , $R_{3l}(\alpha)$=0.7966+0.0017 α , $R_{3u}(\alpha)$=0.8037-0.0018 α ,
$R_{4l}(\alpha)$=0.5915+0.0043 α , $R_{4u}(\alpha)$=0.6092-0.0046 α , $R_{5l}(\alpha)$=0.8829+0.0085 α ,
$R_{5u}(\alpha)$=0.9184-0.0092 α , $R_{6l}(\alpha)$=0.4966+0.0017 α , $R_{6u}(\alpha)$=0.5037-0.0018 α ,
$R_{7l}(\alpha)$=0.5966+0.0017 α , $R_{7u}(\alpha)$=0.6037-0.0018 α , $R_{8l}(\alpha)$=0.4992+0.0004 α ,
$R_{8u}(\alpha)$=0.5009-0.0005 α .

From Fig.6,
U = F + G + H;
V = C + D;
Z = E × U × V;
X = A + B + Z (19)

Since $0 \le R_{jl}(\alpha) \le R_{ju}(\alpha)$ for all $0 \le \alpha \le 1, j = 1,2,3,.....,8$, by [19], Theorem 3.1(2), we have the following results.

From [15], the α – cut of $\tilde{R}_j$ at the left and right endpoints are $R_{jl}(\alpha)$, $R_{ju}(\alpha)$ and $Q_{jl}(\alpha)$, $Q_{ju}(\alpha)$. By Theorem 3.1(2), we obtain

$$Q_{jl}(\alpha) = 1 - R_{ju}(\alpha) \text{ and } Q_{ju}(\alpha) = 1 - R_{jl}(\alpha).$$

For [19], U = F+G+H, through [18] in Theorem 3.1(2), we have

$Q_{6l}(\alpha)$ = 0.4963 + 0.0018α ; $Q_{6u}(\alpha)$ = 0.5034- 0.0017α ;
$Q_{7l}(\alpha)$ = 0.3963 + 0.0018α ; $Q_{7u}(\alpha)$ = 0:4034- 0.0017α ;
$Q_{8l}(\alpha)$ = 0.4991 + 0.0005α ; $Q_{8u}(\alpha)$ = 0.5008- 0.0004α ;

and we get $\tilde{R}_U = \bigcup_{0\le\alpha\le1}[1-\prod_{j=3}^{4}Q_{ju}(\alpha), 1-\prod_{j=3}^{4}Q_{jl}(\alpha);\alpha]$

For [19], V=C+D, through [18] in Theorem 3.1(2), we find

$Q_{3l}(\alpha) = 0.1963 + 0.0018\alpha$; $Q_{3u}(\alpha) = 0.2034- 0.0017\alpha$;
$Q_{4l}(\alpha) = 0.3908 + 0.0046\alpha$; $Q_{4u}(\alpha) = 0.4085- 0.0043\alpha$;

and we have
$\widetilde{R_V} = \bigcup_{0\le\alpha\le1}[1-\prod_{i=3}^{4}Q_{ju}(\alpha), 1-\prod_{i=3}^{4}Q_{jl}(\alpha);\alpha]$

For [19], Z = E×U×V, by [17] in theorem 3.1(1) and

$0 \le R_{5l}(\alpha) \le R_{5u}(\alpha), 0 < 1-\prod Q_{iu}(\alpha) < 1-\prod Q_{il}(\alpha)$

and $0 < 1-\prod_{j=3}^{4}Q_{ju}(\alpha) < 1-\prod_{j=3}^{4}Q_{jl}(\alpha)$ we obtain
$\widetilde{R_Z} = \bigcup_{0\le\alpha\le1}[R_{5l}(\alpha)\left(1-\prod_{j=6}^{8}Q_{ju}(\alpha)\right)\left(1-\prod_{j=3}^{4}Q_{ju}(\alpha)\right), R_{5u}(\alpha)(1-\prod_{j=6}^{8}Q_{ju}(\alpha))(\left(1-\prod_{j=3}^{4}Q_{ju}(\alpha)\right);\alpha),$
For equation 19, X = A + B + Z, by Equation, (3.18) in Theorem 3.1(2), we have

$Q_{1l}(\alpha) = 0.8963 + 0.0018\alpha$; $Q_{1u}(\alpha) = 0.9034- 0.0017\alpha$;
$Q_{2l}(\alpha) = 0.7963 + 0.0018\alpha$; $Q_{2u}(\alpha) = 0.8034- 0.0017\alpha$;
Since $\tilde{1} \ominus \widetilde{R_A} = \bigcup_{0\le\alpha\le1}Q_{1l}(\alpha), Q_{1u}(\alpha):\alpha]$ and

And $\tilde{1} \ominus \widetilde{R_B} = \bigcup_{0\le\alpha\le1}[Q_{2l}(\alpha), Q_{2u}(\alpha);\alpha]$

Since $\tilde{1} \ominus \widetilde{R_Z} = \bigcup_{0\le\alpha\le1}[1-R_{5u}(\alpha)(1-\prod_{j=6}^{8}Q_{jl}(\alpha))(1-\prod_{j=6}^{8}Q_{jl}(\alpha)),$

$$1-R_{5l}(\alpha)(1-\prod_{j=6}^{8}Q_{ju}(\alpha))(1-\prod_{j=6}^{8}Q_{ju}(\alpha);\alpha)]$$

Because $0 \le Q_{jl}(\alpha) < Q_{ju}(\alpha), j = 1,2$ and

$$0 \le 1-R_{5u}(\alpha)(1-\prod_{j=6}^{8}Q_{jl}(\alpha))(1-\prod_{j=6}^{8}Q_{jl}(\alpha)) < 1-R_{5l}(\alpha)(1-\prod_{j=6}^{8}Q_{ju}(\alpha))(1-\prod_{j=6}^{8}Q_{ju}(\alpha);\alpha)$$

For all, $\alpha \in [0,1]$

We get, $(\tilde{1} \ominus \widetilde{R_A}) \otimes (\tilde{1} \ominus \widetilde{R_B}) \otimes (\tilde{1} \ominus \widetilde{R_Z}) = \bigcup_{0\le\alpha<1}[LH(\alpha), RH(\alpha);\alpha],$

Where $LH(\alpha) = Q_{1l}(\alpha)Q_{2l}(\alpha)[1-R_{5u}(\alpha)\left(1-\prod_{j=6}^{8}Q_{jl}(\alpha)\right)\left(1-\prod_{j=3}^{4}Q_{jl}(\alpha)\right)],$

$RH(\alpha) = Q_{1u}(\alpha)Q_{2u}(\alpha)[1-R_{5l}(\alpha)\left(1-\prod_{j=6}^{8}Q_{ju}(\alpha)\right)\left(1-\prod_{j=3}^{4}Q_{ju}(\alpha)\right)]$

Therefore, from [18] in Theorem 3.1(2), we obtain the fuzzy reliability of X

$$\widetilde{R_X} = [\left(\tilde{1} \ominus (\tilde{1} \ominus \widetilde{R_A})\right) \otimes (\tilde{1} \ominus \widetilde{R_B}) \otimes (\tilde{1} \ominus \widetilde{R_Z})] = \bigcup_{0\le\alpha\le1}[1-RH(\alpha), 1-LH(\alpha),;\alpha], \quad (23)$$

From [23] and definition 6, we have the estimate reliability of the system X in the fuzzy sense as follows

$$d(\widetilde{R_X},\tilde{0}) = \left(\frac{1}{2}\right)\int_0^1 (1-RH(\alpha), 1-LH(\alpha))d\alpha = 1-(1/2)\int_0^1 Q_{lu}(\alpha)Q_{2u}(\alpha) - Q_{lu}(\alpha)Q_{2u}(\alpha)\,Q_{5l}(\alpha)$$
$$+Q_{lu}(\alpha)Q_{2u}(\alpha)Q_{5l}(\alpha)\left(\prod_{j=6}^{8}Q_{ju}(\alpha)+\prod_{j=3}^{4}Q_{ju}(\alpha)\right) - Q_{lu}(\alpha)Q_{2u}(\alpha)Q_{5l}(\alpha)\left(\prod_{j=6}^{8}Q_{ju}(\alpha)+\prod_{j=3}^{4}Q_{ju}(\alpha)\right)$$

$$-(1/2)\int_0^1 Q_{1l}(\alpha)Q_{2l}(\alpha) - Q_{1l}(\alpha)Q_{2l}(\alpha)\,Q_{5u}(\alpha)$$

$$+Q_{1l}(\alpha)Q_{2l}(\alpha)Q_{5u}(\alpha)\left(\prod_{j=6}^{8} Q_{jl}(\alpha) + \prod_{j=3}^{4} Q_{jl}(\alpha)\right) - Q_{1l}(\alpha)Q_{2l}(\alpha)Q_{5u}(\alpha)\left(\prod_{j=6}^{8} Q_{jl}(\alpha) + \prod_{j=3}^{4} Q_{jl}(\alpha)\right) \quad (24)$$

By $R_{5l}(\alpha), R_{5u}(\alpha), [20,], Q_{jl}(\alpha), Q_{ju}(\alpha), j = 6,7,8, [21], Q_{jl}(\alpha), Q_{ju}(\alpha), j = 3,4, [22], and\ Q_{jl}(\alpha), Q_{ju}(\alpha), j = 1,2,$ [24] can found as

$$d(\widetilde{R_X}, \tilde{0}) = 0.746758.$$

This result is the estimate reliability of the system X using pentagon fuzzy numbers.

V. CONCLUSION

In this study, statistical data is used to estimate the reliability of a system which is serial or parallel. By fuzzy methods, confidence intervals can be transformed to any fuzzy number. Decomposition theorem is used to write fuzzy set as the union of level α- intervals and signed distance method is used to defuzzify the fuzzy reliability and to obtain the estimate reliability of the system.

This is illustrated by an example which have 3 subsystems as an extension, we also illustrated by an example which have 7 subsystems, which is a combination of serial and parallel system.

When the number of subsystems are increasing, to obtain optimal value of system reliability, triangular fuzzy number can be extended to pentagon fuzzy number. We can also verify whether these results are accepted or rejected.

Jing et al [8] used triangular fuzzy number to estimate the reliability of system and using statistical data. In their study, The reliability of the serial system is $R_* = 0.5405$. From the crisp case, the reliability of the serial system is $R' = 0.54$. Similarly, the reliability of the parallel system is $R_{**} = 0.994977$. From the crisp case, the reliability of the parallel system is $R'' = 0.995$.

Hence the results are accepted for the reliability of serial system and the parallel system using triangular fuzzy number in Jing et al [8]. In this study, we use pentagon fuzzy number to estimate the reliability of serial system and that value is $R_{1*} = 0.5386$. But crisp value for the reliability of the serial system is $R' = 0.54$.

Then $\frac{(R' - R_{1*})}{R'} \times 100\% = 0.2593\%$.

Thus $R_{1*} = 0.5386$ is near to the crisp case $R' = 0.54$. That is to say, defuzzification is significant in this study. Similarly, the reliability of the parallel system is $R_{1**} = 0.9945$. From the crisp case, the reliability of the parallel system is $R'' = 0.995$.

and also $\frac{(R'' - R_{1**})}{R''} \times 100\% = 0.01\%$.
Thus $R_* = 0.9945$ is near to the crisp case $R' = 0.995$. That is to say, defuzzification is significant. Hence the results are accepted for the reliability of serial system and the parallel system using pentagon fuzzy number.

REFERENCES

[1] D. Dubois, and H. Prade, "Fuzzy Sets and Systems: Theory and Applications". Academic Press, 1980.

[2] K.-Y. Cai, C.-Y. Wen, and M.-L. Zhang, "Fuzzy variables as a basic for a theory of fuzzy reliability in the possibility context", Fuzzy Sets and Systems, 42, pp. 145-172, 1991.

[3] K.-Y. Cai, C.-Y. Wen, and M.-L. Zhang, "Fuzzy states as for a theory of fuzzy reliability" Microelectron Reliability, 33, pp. 2253-2263, 1993.

[4] K.-Y. Cai, "System failure engineering and fuzzy methodology", An introductory overview, Fuzzy Sets and Systems, 83, pp. 113-133, 1996

[5] C.-H. Chen, and L. Mon, "Fuzzy system reliability analysis by interval of con dence" Fuzzy Sets and Systems, 56, pp. 29-35, 1993.

[6] M. Hammer, "Application of fuzzy theory to electrical machine reliability", Zeszyty-Naukowe Politechniki Slaskiej, seria, Elektryka, 177, pp. 161-166, 2001.

[7] M.E. El-Hawary, "Electric power applications of fuzzy systems", IEEE Press Series on Power Engineering, 2000.

[8] Jing-Shing Yao, Jin-Shieh Su and Teng-San Shih, "Fuzzy system reliability analysis using triangular fuzzy numbers based on statistical data", Journal of information science and engineering, 24, pp. 1521-1535, 2008.

[9] G. Michael Rosario, and et al., “Fuzzy Reliability Evaluation of weaving machine in Textile Industry”, International Journal of Mathematical Archive, 9, 1, pp. 20-25, 2018.

[10] G. Michael Rosario, and et al., “An inventory model for deteriorating items with time dependent demand in both fuzzy and crisp sense”, Mathematical Sciences International Research Journal, Spl. Issue 1, ISSN: 2278-8697, 7, 2018.

[11] D.L. Mon, and C.H. Cheng, “A general and easy computational method for function of fuzzy numbers”, Int. J. Systems Sci., 26 (4), 983-990, 1995.

[12] T. Onisawa, and J. Kacprzyk, “Reliability and safety analysis under fuzziness.Physica”, Verlag Heidelberg, 1995.

[13] D. Singer, “A fuzzy set approach to fault tree and reliability analysis”, Fuzzy Sets and Systems, 34, pp. 145-155, 1990.

[14] L.V. Utkin, and S.V. Gurov, “A general formal approach for fuzzy reliability analysis in the possibility context”, Fuzzy Sets and Systems, 83, pp. 203-213, 1995.

[15] J.V. Virant, and N. Zimic, “Attention to time in fuzzy logic”, Fuzzy Sets and Systems, 82, pp. 39-49, 1996.

[16] L. A. Zadeh, “Fuzzy set as a basis for theory of possibility”, Fuzzy Sets and Systems, 1, pp. 3-28, 1978.

[17] H.J. Zimmermann, “Fuzzy Set Theory And Its Applications”, KluwerNijho, Boston, 1986.