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A FUZZY COMPARISON METHOD FOR AN DETERMINISTIC INVENTORY MODEL WITHOUT SHORTAGES

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Abstract

In this paper, an inventory model without shortages has been considered and analyzed in a fuzzy environment. The Triangular, Trapezoidal, Pentagonal fuzzy numbers have been used in order to determine the optimal order quantity and optimal total cost for the inventory model. The calculation of Economic Order Quantity (EOQ) is carried out through defuzzification by using signed distance method. Ordering cost and holding cost are taken as fuzzy numbers. Our goal is to determine best of the above stated fuzzy numbers for which the total cost is minimum. To achieve this goal, the numerical results obtained from the proposed model are compared.

Keywords: Defuzzification, EOQ, Fuzzy numbers, Ordering cost, Holding cost, Signed distance method.

1. Introduction

Inventory is any stored resource that is used to satisfy current or future need. Raw materials, work-in-process and finished goods are examples of inventory. Inventory levels for finished goods are a direct function of demand. When we determine the demand for an end-product, we can use this information to determine the demand for parts, components and raw materials required to make that end-product.

All organizations have some short of inventory planning and control systems. For example a bank has a method to control its inventory of cash and a hospital has methods to control blood supply and other important items in majority of real-life inventory management problems.Decision makers face uncertainty in the key parameters of their model. For example, cost associated with holding inventories and placing an order, demand in the planning horizon and production capacity may all be uncertain. Fuzzy logic has been successfully used in various inventory management problems to rectify this uncertainty.

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J.S. Yao and J.Chiang [3] presented an inventory without backorder with fuzzy total cost and fuzzy storage cost defuzzified by centroid and signed distance method. J.S. Yao and K.M. Wu [4] proposed ranking fuzzy numbers based on decomposition principle and signed distance. J.K. Syed and L.A. Aziz [5] presented fuzzy inventory models without using signed-distance method. K.S. Park [6] proposed a fuzzy set theoretic interpretation of economic order quantity.P.K. De. and A. Rawat [8] established a fuzzy inventory model without shortages using triangular fuzzy number. D.Dutta, Pavan Kumar [9] proposed a fuzzy inventory model without shortage using trapezoidal fuzzy number with sensitivity analysis.Harish Nagar and Priyanka Surana [10] proposed a fuzzy inventory model for deteriorating items with fluctuating demand and using inventory parameters as pentagonal fuzzy numbers. Urgeletti [11] referred EOQ model in fuzzy sense and used triangular fuzzy number.

In this paper, we consider a deterministic inventory model without shortage. To minimize the total cost and obtain optimal order quantity q, we fuzzify holding cost andordering cost. The main aim is to estimate the fuzzy optimal order quantity and fuzzy optimal total cost for the inventory system under study using triangular fuzzy number, trapezoidal fuzzy number and pentagonal fuzzy number. For defuzzification we apply the signed distance method and the results are compared. Sensitivity analysis is carried out through numerical examples.

Section 2 contains definitions and preliminaries which are essential for the study. In section 3 a deterministic inventory model without shortage is described in crisp sense. Section 4, presents the inventory model in fuzzy sense. Sensitivity analysis is done in Section 5. Results are compared and conclusion of the paper is made in Section 6.

2. Definitions and Preliminaries

2.1.Definition: (Fuzzy point)

Let $\tilde{a}$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a fuzzy point if its membership function is

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, if \ x = a \\ 0, if \ x \neq a \end{cases}$$

2.2.Definition: (Level α fuzzy interval)

Let $[a, b; \alpha]$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a level α fuzzy interval,

$0 \le \alpha \le 1, a < b$ if its membership function is

$$\mu_{[a,b;\alpha]}(x) = \begin{cases} \alpha, if\ a \le x \le b \\ o, otherwise \end{cases}$$

2.3.Definition: (Triangular fuzzy number)

Let $\tilde{A} = (p,q,r), p < q < r,$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a triangular fuzzy number, if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-p}{q-p}, if\ p \le x \le q \\ \dfrac{r-x}{r-q}, if\ q \le x \le r \\ 0, \quad otherwise \end{cases}$$

2.4.Definition (Signed distance for Triangular fuzzy number)[17]

Let $\tilde{A} = (a,b,c)$ be a triangular fuzzy number. Then the signed distance of A measured from O is defined by

$$d(A, \tilde{O}\ ) = \frac{1}{4}(a + 2b + c)$$

2.5. Definition (Trapezoidal fuzzy number)

A trapezoidal fuzzy number $\tilde{A} = (p,q,r,s), p < q < r < s,$ is represented with membership function $\mu_{\tilde{A}}(x)$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-p}{q-p}, if\ p \le x \le q \\ 1, if\ q \le x \le r \\ R(x) = \dfrac{s-x}{s-q}, if, r \le x \le s \\ 0, otherwise \end{cases}$$

The α -cut of $\tilde{A} = (p,q,r,s)$, $0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

Where $A_L(\alpha) = a + (b-a)\alpha$ and $A_R(\alpha) = d - (d-c)\alpha$ are the left and right endpoints of $A(\alpha)$.

2.6. Definition: (Signed distance for Trapezoidal fuzzy number)[16]

Let $\tilde{D}$ define the signed distance of $\tilde{D}$ measured from $\tilde{O}$ as

$$d(\tilde{D},\tilde{O}) = \frac{1}{2}\int_0^1 [D_L(\alpha) + D_R(\alpha)]d\alpha$$.

2.7. Definition: (Pentagonal fuzzy number)

A pentagonal fuzzy number $\tilde{A} = (a,b,c,d,e)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}} = \begin{cases} L_1(x) = \dfrac{x-a}{b-a}, & a \le x \le b \\ L_2(x) = \dfrac{x-b}{c-b}, & b \le x \le c \\ 1, & x = c \\ R_1(x) = \dfrac{d-x}{d-c}, & c \le x \le d \\ R_2(x) = \dfrac{e-x}{e-d}, & d \le x \le e \\ 0, & otherwise \end{cases}$$

The α -cut of $\tilde{A} = (a,b,c,d,e)$, $0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$

where $A_{L_1}(\alpha) = a + (b-a)\alpha = L_1^{-1}(\alpha)$ and $A_{L_2}(\alpha) = b + (b-a)\alpha = L_2^{-1}(\alpha)$

and $A_{R_1}(\alpha) = d - (d-c)\alpha = R_1^{-1}(\alpha)$ and $A_{R_2}(\alpha) = e - (e-d)\alpha = R_2^{-1}(\alpha)$

2.8. Definition: (Signed distance for Pentagonal fuzzy number)

Let $\tilde{D}$ define the signed distance of $\tilde{D}$ measured from $\tilde{O}$ as

$$d(\tilde{D},\tilde{O}) = \frac{1}{12}(a + 3b + 4c + 3d + e)$$

3. Model Description

3.1.Notations

h	-	Holding cost per unit quantity per unit time
s	-	Set up cost per order
q	-	Order quantity per cycle
T	-	Length of the plan
D	-	Total Demand over the time period [0,T]
t_q	-	length of a cycle
$\bar{h}$	-	Fuzzy holding cost
$\bar{s}$	-	Fuzzy set up cost

3.2. Assumptions

In this paper, the following assumptions are considered

- Total demand is considered as a constant
- Time plan is constant
- Shortage cost is not allowed
- Fuzzifying holding cost andordering cost only

4. Proposed Inventory Model in Crisp Sense

First we deal an inventory model without shortages, in crisp environment. In this model, the economic lot size is obtained by the following equation:

$$q = \sqrt{\frac{2sD}{hT}}, \text{ Where } \frac{q}{t_q} = \frac{D}{q}.$$

The total cost for the period [0, T] is given by

Total cost= $F(q)$ =carrying cost+ordering cost

$$F_q(c,o) = F_q = \frac{hTq}{2} + \frac{sD}{q} \qquad \text{------------------------------- (4.1)}$$

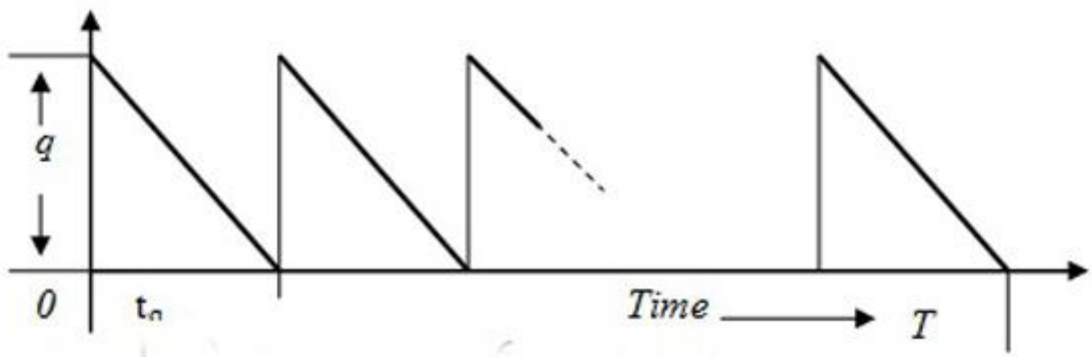
The crisp optimal solution is given by [8]

Optimal order quantity $q^* = \sqrt{\frac{2sD}{hT}}$ ------------------------------ (4.2)

Minimum total cost

F(q*)= $\sqrt{2hsDT}$ ------------------------------ (4.3)

Diagrammatic representation:



5. Proposed Inventory Model in Fuzzy Environment

5.1. Triangular fuzzy total cost:

Within this model, we consider the carrying and ordering costs as imprecise quantities and are represented as triangular fuzzy numbers. The total demand and time of plan are taken as constants. Therefore, the fuzzy total cost becomes

$$FTC = \frac{\tilde{h}Tq}{2} + \frac{\tilde{s}D}{q}$$

Let $\tilde{h}$ denotes the fuzzy carrying cost and $\tilde{s}$ denotes fuzzy ordering cost which are characterized by the triangular fuzzy numbers by $\tilde{h} = (h_1, h, h_2)$, $\tilde{s} = (s_1, s, s_2)$ respectively in LR form.

$$FTC = F_q(\tilde{h},\tilde{s}) = [\tilde{h}\otimes\frac{Tq}{2}]\oplus[\tilde{s}\otimes\frac{D}{q}]$$
$$=[(h-h_1,h,h+h_2)\otimes(\frac{Tq}{2})]\oplus[(s-s_1,s,s+s_2)\otimes(\frac{D}{q})]$$
$$=[F_q(h,s)-(\frac{h_1Tq}{2}+\frac{s_1D}{q}),F_q(h,s),F_q(h,s)+(F_q(h,s)+(\frac{h_2Tq}{2}+\frac{s_2D}{q})]$$

The signed distance method for defuzzification is given by the following equation,

$$d(F_{(q,s)}(\tilde{a},\tilde{b},\tilde{c}),0)=\frac{1}{2}(a+2b+c)$$

Using Signed distance Method and

Applying defuzzification

$$d(F_q(\tilde{h},\tilde{s}),0)=F_q(h,s)+\frac{1}{4}[(\frac{h_2Tq}{2}+\frac{s_2D}{q})-(\frac{h_1Tq}{2}+\frac{s_1D}{q})] \quad \text{------------------ (5.1.1)}$$
$$=F_q(h,s)+\frac{1}{4}[\frac{Tq}{2}(h_2-h_1)-\frac{D}{q}(s_2-s_1)]$$
$$=F_d(q)$$

$F_d(q)$ is minimum when $F_d\,'(q)=0$, and $F_d\,''(q)$ is positive

$$\Rightarrow F_d(q)=\frac{d}{dq}F_d(q)=\frac{hT}{2}-\frac{sD}{q^2}-\frac{1}{4}[\frac{Tq}{2}(h_2-h_1)-\frac{D}{q}(s_2-s_1)]=0$$

Thus,

$$q_d{*}=\sqrt{\frac{2D}{T}}(\frac{4s+s_2-s_1}{4h+h_2-h_1}) \quad \text{--------------------- (5.1.2)}$$

Also

$$\frac{d^2}{dq^2}F_d(q_d{*})=\frac{2s}{q^3}(D+(h_2-h_1))>0$$

This shows that $F_d(q)$ is minimum at q_d*.

5.2. Trapezoidal Fuzzy Total Cost:

Within this model, we consider the carrying costs and ordering costs as imprecise quantities and are represented as trapezoidal fuzzy numbers. The total demand and time of plan are taken as constants. Therefore, the fuzzy total cost becomes

$$FTC = \frac{\tilde{h}Tq}{2} + \frac{\tilde{s}D}{q}$$

Let $\tilde{h} = (h_1, h_2, h_3, h_4)$ and $\tilde{s} = (s_1, s_2, s_3, s_4)$ denote the fuzzy carrying cost and fuzzy ordering cost which are characterized by the trapezoidal fuzzy numbers respectively in LR form.

$$FTC = F_q(\tilde{h}, \tilde{s}) = [\tilde{h} \otimes \frac{Tq}{2}] \oplus [\tilde{s} \otimes \frac{D}{q}]$$

$$= [(h_1, h_2, h_3, h_4) \otimes (\frac{Tq}{2})] \oplus [(s_1, s_2, s_3, s_4) \otimes (\frac{D}{q})]$$

$$= [\frac{h_1 Tq}{2} + \frac{s_1 D}{q}], [\frac{h_2 Tq}{2} + \frac{s_2 D}{q}], [\frac{h_3 Tq}{2} + \frac{s_3 D}{q}], [\frac{h_4 Tq}{2} + \frac{s_4 D}{q}]$$

The signed distance method for defuzzification is given by the following equation,

$$d(F_q(\tilde{a}, \tilde{b}), 0) = \frac{1}{2}\int_0^1 (F_{q_L(\alpha)} + F_{q_R(\alpha)}) d\alpha$$

Applyingdefuzzification

$$d(F_q(\tilde{h}, \tilde{s}), 0) = \frac{1}{2}\int_0^1 (F_{q_L(\alpha)} + F_{q_R(\alpha)}) d\alpha$$

$$= \frac{1}{2}[\frac{1}{2}(\frac{Tq}{2}(h_1 + h_2 + h_3 + h_4)) + \frac{1}{2}(\frac{D}{q}(s_1 + s_2 + s_3 + s_4))]$$

$$= \frac{Tq}{8}(h_1 + h_2 + h_3 + h_4) + \frac{D}{4q}(s_1 + s_2 + s_3 + s_4)$$

$$= F_d(q) \qquad \text{----------------- (5.2.1)}$$

$$F_d(q)' = \frac{d}{dq}F_d(q) = \frac{1}{4}[\frac{Tq}{2}(h_1 + h_2 + h_3 + h_4) - \frac{D}{q}(s_1 + s_2 + s_3 + s_4)] = 0$$

$$\Rightarrow q_d * = \sqrt{\frac{2D}{T}(\frac{s_1 + s_2 + s_3 + s_4}{h_1 + h_2 + h_3 + h_4})} \text{ ----------------- (5.2.2)}$$

Also

$$\frac{d^2}{dq^2} F_d\left(q_d *\right) = \frac{1}{4}[\frac{2D}{q^3}] > 0$$

This shows that $F_d(q)$ is minimum at $q_d *$.

5.3. Pentagonal Fuzzy Total Cost:

Within this model, we consider the carrying and ordering costs as imprecise quantities and are represented as pentagonal fuzzy numbers. The total demand and time of plan are taken as constants.

Let $\tilde{h} = (h_1, h_2, h_3, h_4, h_5)$ and $\tilde{s} = (s_1, s_2, s_3, s_4, s_5)$ denote the fuzzy carrying cost and fuzzy ordering cost which are characterized by the pentagonal fuzzy numbers respectively in LR form.

$$FTC = F\ (\tilde{h}, \tilde{s}) = [\tilde{h} \otimes \frac{Tq}{2}] \oplus [\tilde{s} \otimes \frac{D}{q}]$$

$$= [(h_1, h_2, h_3, h_4, h_5) \otimes \frac{Tq}{2}] \oplus [(s_1, s_2, s_3, s_4, s_5) \otimes \frac{D}{q}]$$

$$= [\frac{h_1 Tq}{2} + \frac{s_1 D}{q}], [\frac{h_2 Tq}{2} + \frac{s_2 D}{q}], [\frac{h_3 Tq}{2} + \frac{s_3 D}{q}], [\frac{h_4 Tq}{2} + \frac{s_4 D}{q}], [\frac{h_5 Tq}{2} + \frac{s_5 D}{q}]$$

The signed distance method for defuzzification is given by the following equation,

$$d(F_q(\tilde{a}, \tilde{b}), 0) = \frac{1}{2}\int_0^1 (F_{q_L(\alpha)} + F_{q_R(\alpha)}) d\alpha$$

Applying defuzzification,

$$d(F_q(\tilde{h}, \tilde{s}), 0) = \frac{1}{2}\int_0^1 (F_{q_L(\alpha)} + F_{q_R(\alpha)}) d\alpha$$

$$= \frac{1}{12}[\frac{Tq}{2}(h_1 + 3h_2 + 4h_3 + 3h_4 + h_5) + \frac{D}{q}(s_1 + 3s_2 + 4s_3 + 3s_4 + s_5)]$$

$$= F_d(q) \text{ --------------------- (5.3.1)}$$

$$F_d(q)' = \frac{d}{dq}F_d(q) = \frac{1}{12}[\frac{Tq}{2}(h_1+3h_2+4h_3+3h_4+h_5)+\frac{D}{q}(s_1+3s_2+4s_3+3s_4+s_5)] = 0$$

$$\Rightarrow \; q_d* = \sqrt{\frac{2D}{T}(\frac{s_1+3s_2+4s_3+3s_4+s_5}{h_1+3h_2+4h_3+3h_4+h_5})} \text{ -------------------- (5.3.2)}$$

Also,

$$\frac{d^2}{dq^2}F_d(q_d*) = \frac{1}{4}[\frac{2D}{q^3}(s_1+3s_2+4s_3+3s_4+s_5)] > 0$$

This shows that $F_d(q)$ is minimum at q_d*.

5.4. Numerical Analysis

5.4.1. Crisp Model:

Let h=20; s=12; D=500 ; T=6

Using Equations (4.2) & (4.3) and we get the values of minimum order quantity $q_d*$and minimum total cost TC.

Table 1

Demand(D)	q_d*	*TC*
450	15.8114	1138.4
475	16.2447	1169.6
500	16.6667	1200
525	17.0783	1229.92
550	17.4801	1250.68

5.4.2. Fuzzy model using Triangular Fuzzy number:

Let $\tilde{h} = (11,12,13)$; $\tilde{s} = (18,20,22)$; T=6; D=500

Using Equations (5.1.1) & (5.1.2) and we get the values of minimum order quantity q_d * and minimum fuzzy total cost(FTC).

Table 2

Demand(D)	q_d *	FTC
450	15.8745	1190.5881
475	16.3095	1223.21298
500	16.7332	1254.9900
525	17.1464	1285.9821
550	17.5499	1316.2447

5.4.3. Fuzzy model using Trapezoidal Fuzzy number:

Let $\tilde{s} = (15,18,22,24)$; $\tilde{h} = (8,11,13,16)$; T=6; D=500

Using Equations (5.2.1) & (5.2.2) and we get the values of minimum order quantity q_d * and minimum fuzzy total cost (FTC).

Table 3

Demand(D)	q_d *	FTC
450	15.7121	1131.2825
475	16.1426	1162.2822
500	16.5622	1192.4764
525	16.9710	1221.9247
550	17.3704	1250.6798

5.4.4.Fuzzy model using Pentagonal Fuzzy number:

Let $\tilde{h} = (8,11,12,13,15)$; $\tilde{s} = (15,18,20,22,24)$; T=6; D=500

Using Equations (5.3.2) & (5.3.1) and we get the following,

Table 4

Demand(D)	q_d *	FTC
450	15.7778	1136.0458
475	16.2102	1167.1761
500	16.6319	1197.4974
525	17.0420	1227.0697
550	17.4431	1255.9459

6. Conclusion

In this paper we have used signed distance method for defuzzying holding cost and setup cost. These costs are taken as triangular fuzzy number, trapezoidal fuzzy number and pentagonal fuzzy number. We conclude that using trapezoidal fuzzy number and pentagonal fuzzy number we get minimum total cost compared to the crisp model. But still using trapezoidal fuzzy number value of total cost is much lesser than all other models. Numerical example are given to illustrate the results.

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