COMPUTER ORIENTED NUMERICAL METHODS - ALGEBRAIC AND TRANSCENDENTAL FUNCTION

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Introduction

- There are two types of functions: (i) Algebraic function and (ii) Transcendental function
- An **algebraic function** is informally a function that satisfies a polynomial equation
- A function which is not algebraic is called a **transcendental function**.
- The values of x which satisfy the equation $f(x) = 0$ are called **roots** of $f(x)$.
- If $f(x)$ is quadratic, cubic or bi-quadratic expression, then algebraic formulae are available for getting the solution.
- If $f(x)$ is a higher degree polynomial or transcendental function then algebraic methods are not available.

Bisection Method

- Consider a continuous function $f(x)$.
- Numbers $a < b$ such that $f(a)$ and $f(b)$ have opposite signs.
- Let $f(a)$ be negative and $f(b)$ be positive for $[a, b]$.
- Then there exists at least one point(root), say x, $a < x < b$ such that $f(x) = 0$.
- Now according to Bisection method, bisect the interval $[a, b]$, $x_1 = \dfrac{a+b}{2}(a < x_1 < b)$.
- If $f(x_1) = 0$ then x_1 be the root of the given equation.
- Otherwise the root lies between x_1 and b if $f(x_1) < 0$.
- OR the root lies between a and x_1 if $f(x_1) > 0$.
- Then again bisect this interval to get next point x_2.
- Repeat the above procedure to generate $x_1, x_2, \ldots$ till the root upto desired accuracy is obtained.

Characteristics:

1. This method always slowly converge to a root.
2. It gives only one root at a time on the the selection of small interval near the root.
3. In case of the multiple roots of an equation, other initial interval can be chosen.
4. Smallest interval must be selected to obtain immediate convergence to the root, .

Bisection Method- Example

Ex. Find real root of $x^3 - x - 1 = 0$ correct upto three decimal places using Bisection method

Bisection Method- Example

Sol. Let $f(x) = x^3 - x - 1 = 0$

$f(0) = -1 < 0$

$f(1) = -1 < 0$

$f(2) = 5 > 0$

$\therefore$ since $f(x)$ is continuous function there must be a root in the lying in the interval $(1, 2)$

Now according to Bisection method, the next approximation is obtained by taking the midpoint of $(1, 2)$

$$c = \frac{1+2}{2} = 1.5,\ f(1.5) =$$

No. of iterations	a $(f(a) < 0)$	b $(f(b) > 0)$	$c = \frac{a+b}{2}$	f(c) $(< 0, > 0)$
1	1	2	1.5	-

Bisection Method- Example

Ex. Find real root of $x^2 - lnx - 12 = 0$ correct upto three decimal places using Bisection method

Newton-Raphson Method(N-R Method)

Graphical derivation of the Method:

- Consider the portion of the graph $y = f(x)$ which crosses $X - axis$ at R corresponding to the equation $f(x) = 0$.
- Let B be the point on the curve corresponding to the initial guess x_0 at A.
- The tangent at B cuts the $X - axis$ at C which gives first approximation x_1. Thus $AB = f(x_0)$ and $AC = x_0 - x_1$
- Now $\angle ACB = \alpha$, $tan\alpha = \dfrac{AB}{AC}$

$$\therefore f'(x_0) = \frac{f(x_0)}{x_0 - x_1}$$

$$\therefore x_0 - x_1 = \frac{f(x_0}{f'(x_0)} \Rightarrow x_0 - \frac{f(x_0}{f'(x_0)} = x_1$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

- In general $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$; where $n = 0, 1, 2, 3, \ldots$

Secant Method

- In N-R method two functions f and f' are required to be evaluate per step.
- Also it requires to evaluate derivative of f and sometimes it is very complicated to evaluate f'.
- Often it requires a very good initial guess.
- To overcome these drawbacks, the derivative of f' of the function f is approximated as $f'(x_n) = \dfrac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n}$
- Therefore formula of N-R method becomes

$$x_{n+1} = x_n - \frac{f(x_n)}{\left(\frac{f(x_{n-1})-f(x_n)}{x_{n-1}-x_n}\right)}$$

$$\therefore\ x_{n+1} = x_n - f(x_n)\frac{x_{n-1} - x_n}{f(x_{n-1}) - f(x_n)}$$

where $n = 1, 2, 3, \ldots,\ f(x_{n-1}) \neq f(x_n)$

General Features:

- The secant method is an open method and may not converge.
- It requires fewer function evaluations.
- In some problems the secant method will work when Newton's method does not and vice-versa.
- The method is usually a bit slower than Newton's method. It is more rapidly convergent than the bisection method.
- This method does not require use of the derivative of the function.
- This method requires only one function evaluation per iteration.

THANK YOU