Binomial Distribution

Binomial Distribution also know as " Bernoulli Distribution" is associated with the name of a swiss mathematician James Bernoulli also known as Jacques or Jakob (1654-1705).

Binomial Distribution is a probability distribution expressing the probability of one of dichotomous alternatives.

ie.., Success or Failure.

These assumptions are:

An experiment is performed under the same conditions for a fixed number of trials. Say. n.

In each trail. there are only two possible outcomes of the experiment. For lack of a better nomenclature they are called “Success” or “Failure”.

S = {Success, failure}

The probability of a success denoted by p remains constant from trial to trial. The probability of a failure denoted by q is equal to (1-p).

The trails are statistically independent i.e., the outcomes of any trial or sequence of trials do not affect the outcomes of subsequent trials.

If probability of a success is not the same in each trail. we will not have Binomial Distribution.

For example. If 5 balls are drawn at random from an urn containing 10 white and 20 red balls, this is a Binomial Distribution if each is replaced before another is drawn.

If the balls are drawn without replacement. The probability of drawing white ball changes each time a ball is taken from the run and we no longer have a Binomial Distribution.

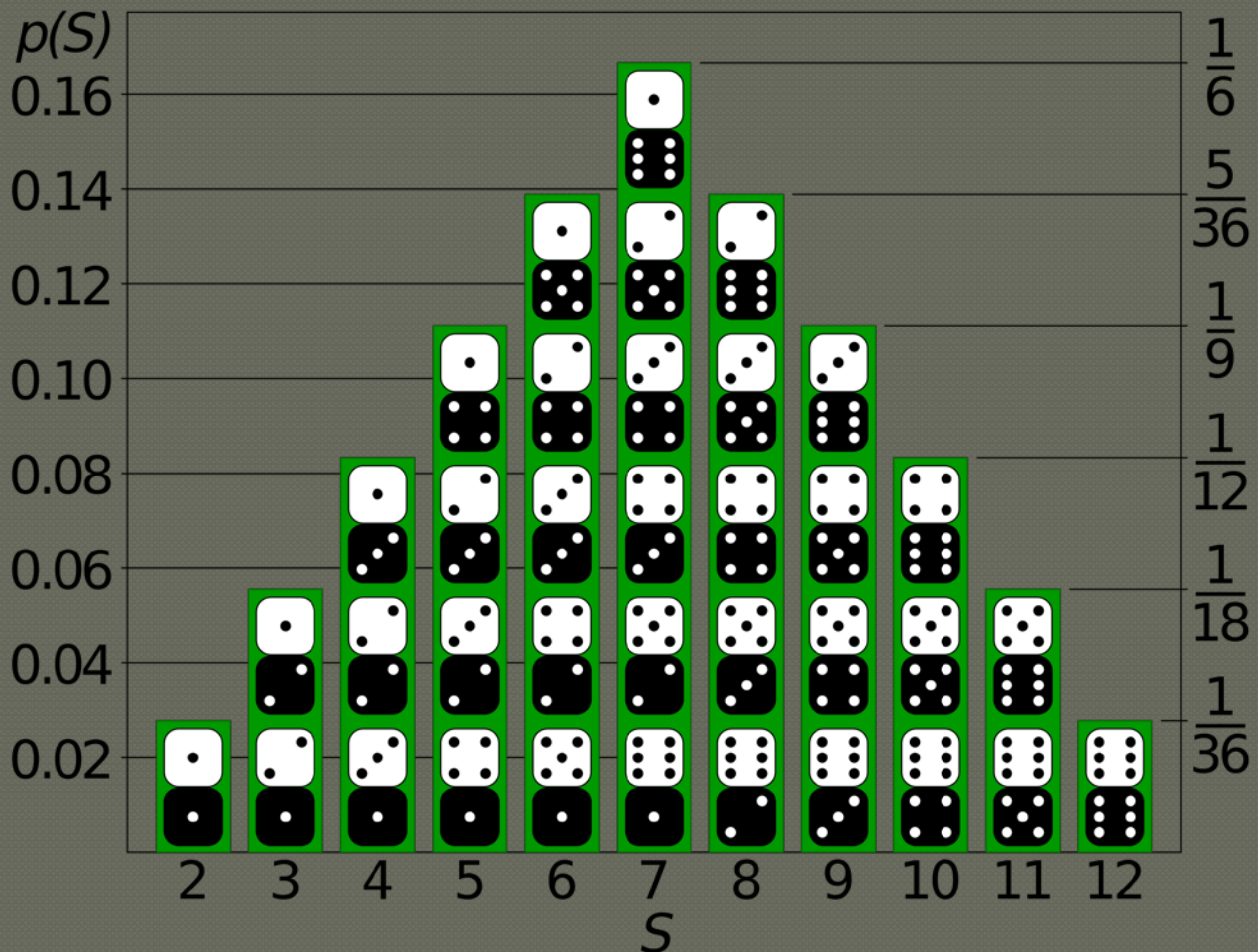
How Binomial Distribution arises can be seen from the following:

If a coin is tossed once there are two outcomes. namely tail or head. The probability of obtaining a head or $p = ½$ and the probability of obtaining a tail or $q = ½$.

Thus $(q + p) = 1$. These are terms of the binomial $(q + p)$.

$$P_n(x) = C(n,x)p^x q^{n-x}$$

$$= \frac{n!}{x!\,(n-x)!}p^x q^{n-x}$$



p(S)
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
1/6
5/36
1/9
1/12
1/18
1/36
2
3
4
5
6
7
8
9
10
11
12
S

The Binomial Distribution

- Characteristics of the Binomial Distribution:
  - A trial has only two possible outcomes – "success" or "failure", "head" or "tail", "win" or "lose", "even" or "odd"
  - There is a fixed number, n, of identical trials
  - The trials of the experiment are independent of each other
  - The probability of a success, p, remains constant from trial to trial
  - If p represents the probability of a success, then

    (1-p) = q is the probability of a failure

Probability Distributions

Frequency Distribution- *is a way of summarizing variation in observed data (outcomes of an experiment).*

Probability Distribution-

– A theoretical frequency distribution

– Describes how outcomes are expected to vary

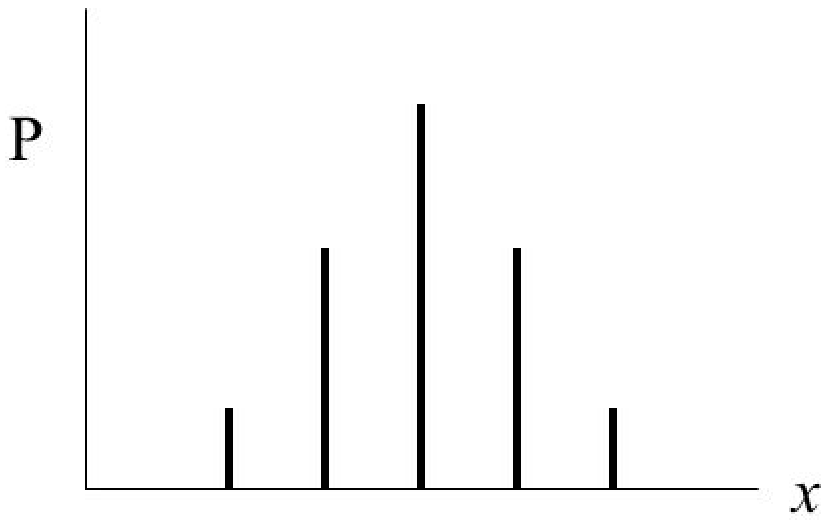
– Useful in making inferences and decisions under uncertainty

Types of Probability Distribution:

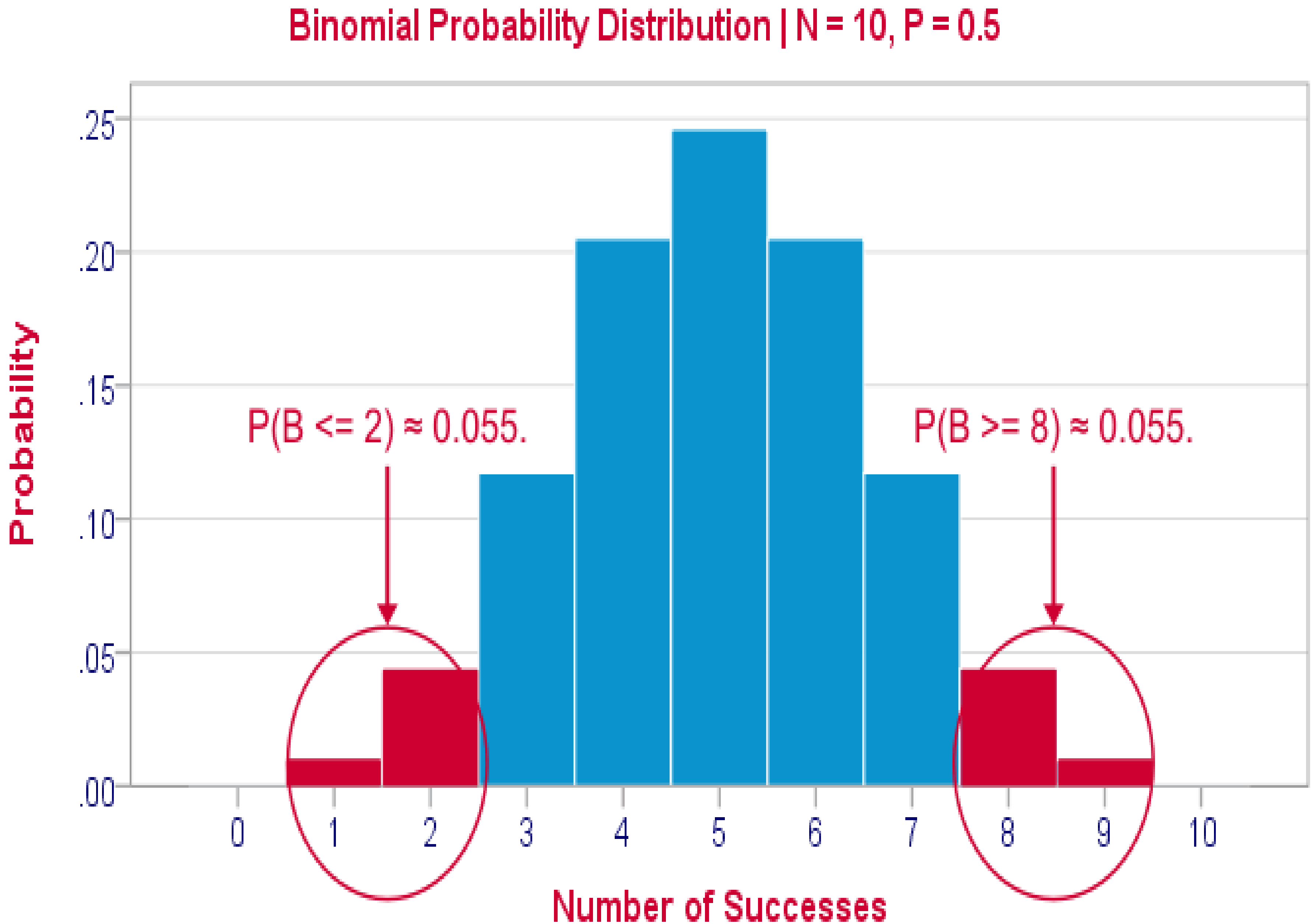
- Discrete Probability Distributions
- Continuous Probability Distributions

Binomial distribution - Graph

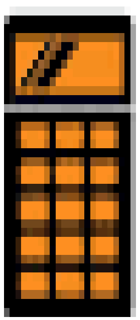
- Typical shape of a binomial distribution:
  - Symmetric, with total P(x) = 1



<u>Note</u>: this is a *theoretical* graph – how would an *experimental* one be different?


Binomial Probability Distribution | N = 10, P = 0.5
Probability
.25
.20
.15
.10
.05
.00
P(B <= 2) ≈ 0.055.
P(B >= 8) ≈ 0.055.
0
1
2
3
4
5
6
7
8
9
10
Number of Successes

Binomial Distribution Formula



$$P(X) = {}_{n}C_{x}\, p^{x}(1-p)^{n-x}$$

The Binomial Distribution

The mean, variance, and standard deviation of a variable that follows a *binomial distribution* :

$$\text{Mean:} \quad \mu = np$$

$$\text{Variance:} \quad \sigma^2 = npq$$

$$\text{Standard Deviation:} \quad \sigma = \sqrt{npq}$$

Binomial Distribution

- Based on events for which there are only 2 alternative possibilities:

- Heads or tails
- Girl or boy
- Pregnant or not

The Goals

- List the important properties of the

t-, Chi-squared, *F*- and Lognormal distributions

- Explain when each of these distributions is particularly useful
- List the important properties of the Binomial and Poisson distributions
- Explain when the Binomial and Poisson distributions are each particularly useful

Definition of the Binomial Distribution

The Binomial Distribution occurs when:

(a) There is a fixed number (n) of trials.

(b) The result of any trial can be classified as a “success” or a “failure”

(c) The probability of a success (π or p) is constant from trial to trial.

(d) Trials are independent.

If X represents the number of successes then: $P(X = x) = {}^{n}C_{x}\, \pi^{x}(1 - \pi)^{n-x}$

Binomial Distribution Formula

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

where

n = the number of trials (or the number being sampled)

x = the number of successes desired

p = probability of getting a success in one trial

$q = 1 - p$ = the probability of getting a failure in one trial

Properties of the Binomial Distribution

The shape and location of Binomial Distribution changes as p changes for a given p. As p increase for a fixed n. the Binomial Distribution shifts to the rights.

The mode of the Binomial Distribution is equal to the value of x which has the largest probability. For example, if n=6 and p=0.3, the mode is equal to 6 .

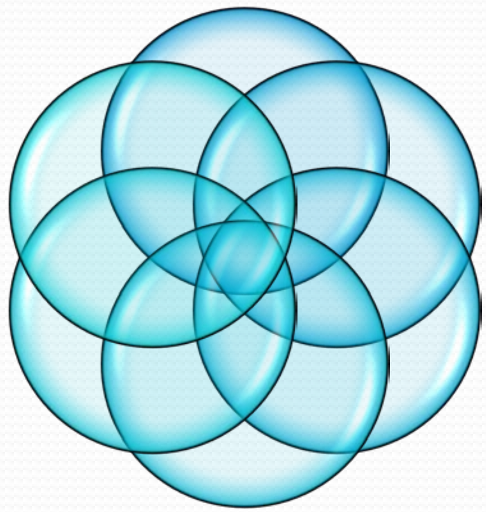
The mean and mode are equal if np is an integer. For example, when n=6 and p= 0.50, the mean and mode are both equal to 3. for fixed n, both mean and mode increase as p increase.

As **n** increases for a fixed p. The Binomial Distribution moves to the right. Flattens, and spreads out. The mean of the Binomial Distribution np, obviously increases as n increases with p held constant for larger n there are more possible outcomes of a Binomial Distribution and the probability associated with any particular outcome becomes smaller.

$z = x - np/ npq$. The approximation becomes better with increasing n proof.

If n is large and if neither p or q is too close to zero, the Binomial Distribution can be closely approximated by the normal distribution with standardized variable given by, These are the term of the binomial(q + p)2 because

(q + p)2 = q2 + 2qp + p2



(q + p)3 = q3 + 3q2p + 2qp+p2

These are the term of the binomial (q + p)3because

Importance of Binomial Distribution

The Binomial probability distribution is a discrete probability distribution that is useful in describing an enormous variety of real life events.

The Binomial Distribution can be used when:

The outcomes or results of each trail in the process are characterized as one of two types of possible outcomes.

In other words they are attributes.

The possibility of outcomes of any trial does not change and is independent of the results of previous trials.

Conclusions

- The binomial distribution is a discrete probability distribution used when there are only two possible outcomes for a random variable: success and failure..... The probability of failure, q, is equal to 1- p; therefore, the probabilities of success and failure are complementary.