WEIGHTED GRAPHS
BIPARTITE GRAPHS

by

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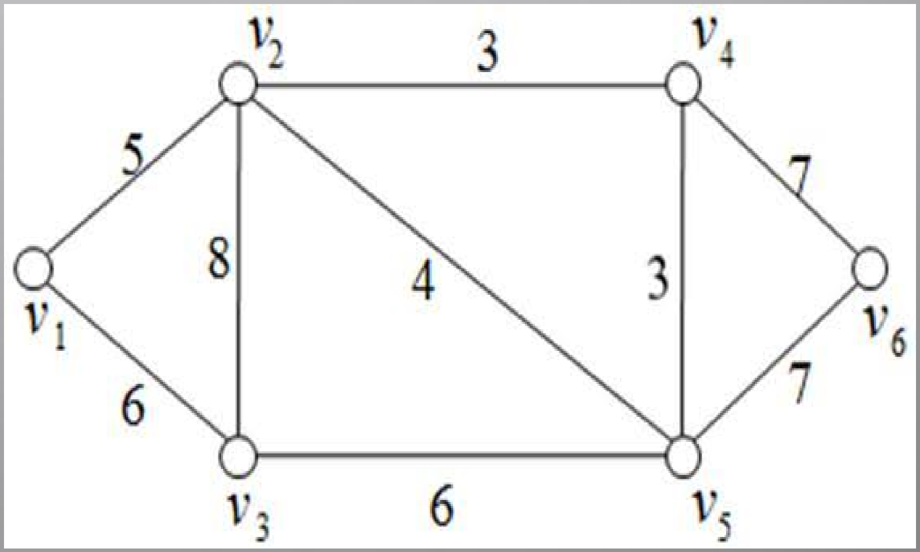
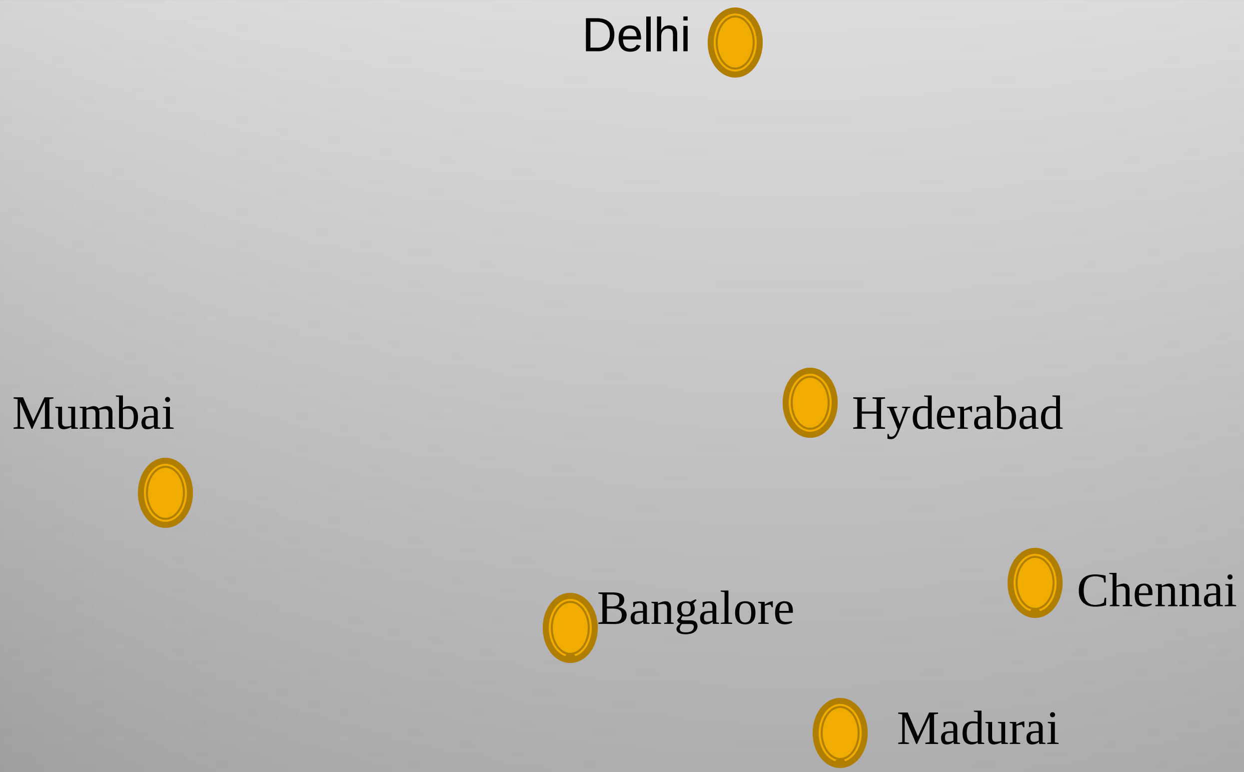
Weighted graph

- A triple (V, E, w) is called a **weighted graph** if G=(V,E) is a graph and w is a function from E to **R**.
- The number w(e) is called the **weight** of the edge **e**.



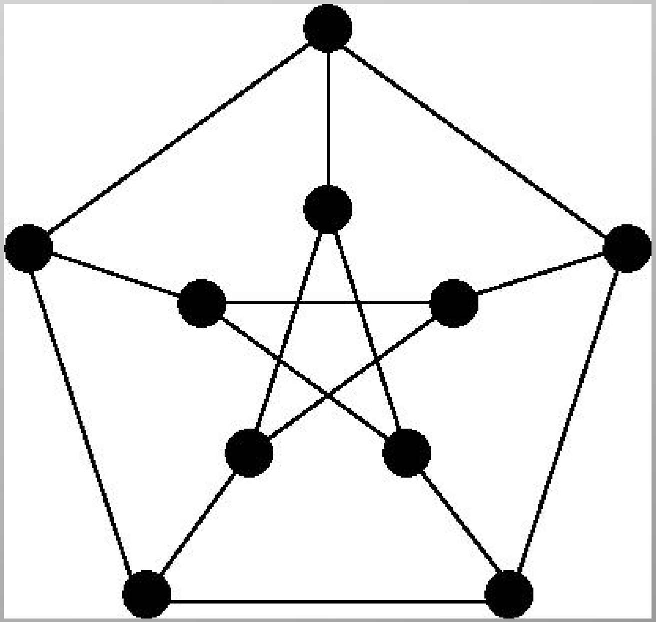
Travelling salesman problem



A salesperson begins in Bangalore, has to visit all the cities, and return to Bangalore in a shortest possible distance. (or time, or cost)

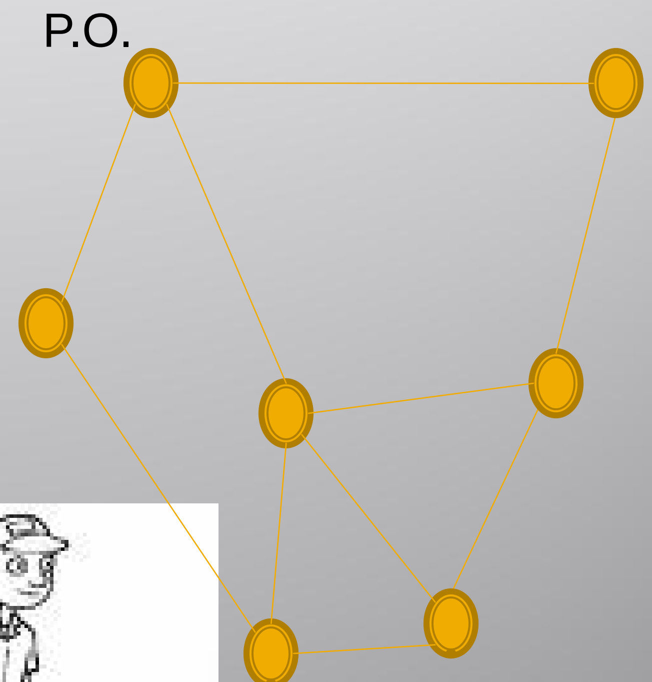
Travelling salesman problem

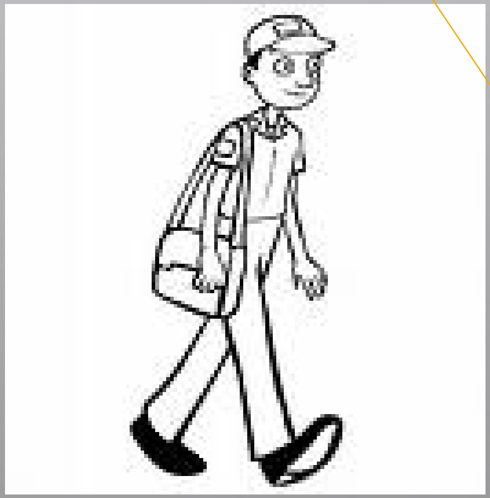
- Given a graph (or a directed graph), does there exist a cycle in the graph that contains each vertex once? (i.e. a ***Hamiltonian cycle***)?
- Given a complete weighted graph, **finding a Hamiltonian cycle of minimum weight**.



Chinese Postman Problem

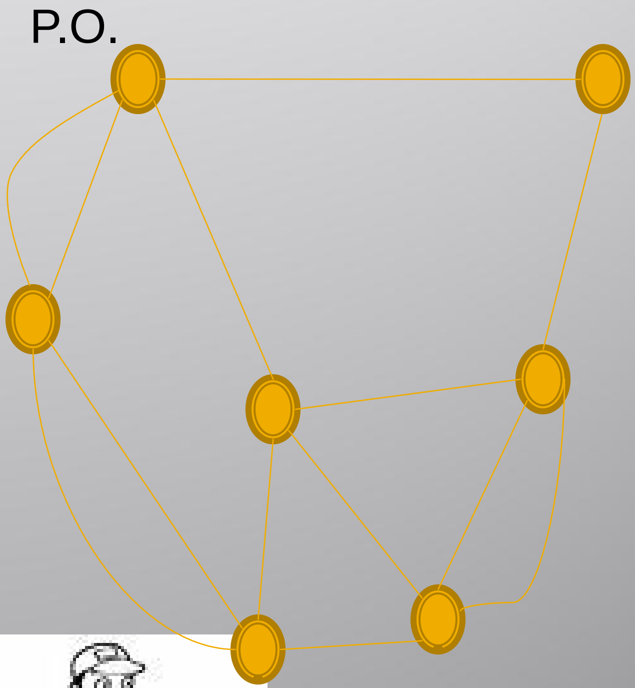
- A postman begins in the post office, has to traverse all the streets, and returns to the post office in a shortest possible distance.

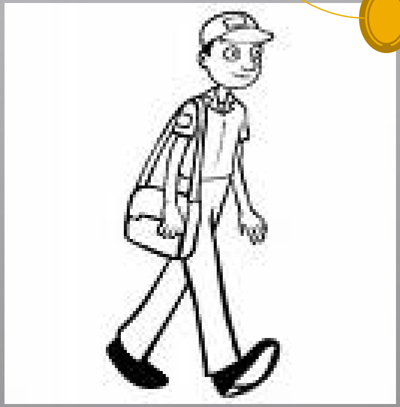




Chinese Postman Problem

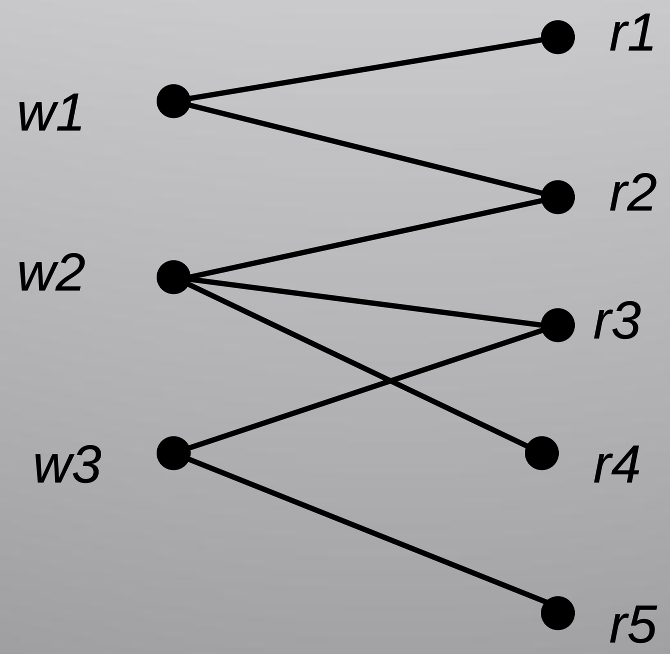
- Definition: an ***Eulerian trail*** is a closed walk that traverses all the edges in the graph.
- Given a graph finding a minimum Chinese Postman Tour.





Bipartite Graphs

- A graph is **bipartite** if V(G) is partitioned into nonempty subsets V_1 and V_2, such that if (x , y) is in E(G), where x and y belong to different subsets.



A graph G is a bipartite graph iff it contains no odd cycles

v_i

Necessity :
let G be a bipartite graph with bipartition [A, B].
Let C be a cycle of length ‘n’ in G.
The vertices alternately belong to A and B.
so ‘n’ is even.
Sufficiency :
Let G be a graph with no odd cycles
we prove this by induction on q
if q=0 or 1 , then G is bipartite
so assume that if q = m-1, then G is bipartite
Let q = m (>1)

Cont…

Let (u , v) be an edge in G
Consider H = G - (u , v)
since G has no odd cycles, H too had no odd cycles.
1. u and v are connected in H
suppose u and v belong to the same set,
    P (u , v) is of even length.
But P (u , v) , (u , v) , u is of odd length.
Hence u ε A and v ε B.
2. **u and v are not connected in H**
let C be the component in H which contains u
[A , B] & [D , E]- bipartition of C and H-V(C)
assume that u ε A and v ε D
Then [A∪ D, B ∪ E] is a bipartition of G.

THANK YOU